Logos & Episteme

an international journal
of epistemology

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RESEARCH ARTICLES

SCEPTICISM WITHOUT KNOWLEDGE-ATTRIBUTIONS

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ABSTRACT: The sceptic says things like "nobody knows anything at all," "nobody knows that they have hands," and "nobody knows that the table exists when they aren't looking at it." According to many recent anti-sceptics, the sceptic means to deny ordinary knowledge attributions. Understood this way, the sceptic is open to the charge, made often by Contextualists and Externalists, that he doesn't understand the way that the word "knowledge" is ordinarily used. In this paper, I distinguish a form of Scepticism that is compatible with the truth of ordinary knowledge attributions and therefore avoids these criticisms. I also defend that kind of Scepticism against the suggestion that it is philosophically uninteresting or insignificant.

KEYWORDS: scepticism, epistemology, knowledge, meaning

I. Introduction

The sceptic says things like "nobody knows anything at all," "nobody knows that they have hands," and "nobody knows that the table exists when they aren't looking at it." According to many recent anti-sceptics, the sceptic means to deny ordinary knowledge attributions. Everyday people judge themselves and others as knowing things. You might take it that you know that it is Thursday. You might say that you know what time it is. You judge that I know some things about Philosophy. Most people take themselves to know established scientific theories to be true. Sceptics hold that these assertions are literally false, or so we are told by some anti-sceptics.

The sceptic, supposedly, makes a crucial mistake, however. The sceptic does not understand the way that the word "know" is ordinarily used.[1] If he did

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[1] Christopher Hill, "Process Reliabilism and Cartesian Scepticism," in *Skepticism*, eds. Keith DeRose and Ted Warfield (New York: Oxford University Press, 1999), Keith DeRose, "Solving the Skeptical Problem," *Philosophical Review* 104, 1 (1995): 1-52, Stewart Cohen, "Contextualism and Scepticism," *Nous* 34, SUPPL. 1 (2000): 94-107, Mark Kaplan, "To What Must an Epistemology be True?" *Philosophy and Phenomenological Research* 61, 2 (2000): 279, Mark Kaplan, "Austin's Way with Skepticism," in *The Oxford Handbook of Scepticism*, ed. John Greco (Oxford: Oxford University Press, 2011), John Greco, "Virtue, Luck and the Pyrrhonian Problematic," *Philosophical Studies* 130, 1 (2006): 9-34, Michael Bergmann, "Externalist

understand it, he would see that he was wrong to deny ordinary knowledge attributions, and he would see that his arguments were unsound. So John Greco argues against a sceptical use of Agrippa's Trilemma in this way:

> All knowledge, says the skeptic, must be grounded in good reasons. But not any reason is a good reason – one must have reasons for believing that one's reasons are true. But this, in turn, ensures that any attempt to ground knowledge in good reasons must be inadequate. For either (a) one's reasons will go on in an infinite regress, (b) they will come back in a circle, or (c) they will end arbitrarily. But none of these outcomes is satisfactory- none provides knowledge with grounding in good reasons. And therefore, the skeptic concludes, knowledge is impossible.... [however] knowledge is true belief resulting from a reliable process... put another way, the skeptic is just wrong to think that all knowledge producing processes are reasoning processes.[2,3]

In a discussion of Cartesian Scepticism, Greco first isolates the premises of a sceptical argument, and then criticizes the first one:

> But in fact premise (1) of [the sceptical argument] is false... (1) requires that our evidence discriminate the truth of our belief from every alternative possibility whatsoever. But it is questionable whether our ordinary concept of knowledge in fact requires that our evidence do this.[4]

Likewise, Kaplan writes:

> it seemed that the point of the argument was (a) to display a set of claims about the extent and nature of our knowledge to which we recognize ourselves to be committed and (b) to show that these claims jointly lead to disaster... Austin's response shows that the argument, at least to this extent, fails to accomplish the first of its two objectives: a crucial premise of the argument [the one about the meaning of "knowledge"!]... does not seem to be among our commitments, if ordinary practice is any guide.[5]

And DeRose's "solution" to the "sceptical paradox" is one in which it turns out that the sceptic fails to refute ordinary knowledge attributions:

> according to contextualists, the skeptic, in presenting her argument, manipulates

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Responses to Skepticism," in *The Oxford Handbook of Scepticism*, ed. John Greco (Oxford: Oxford University Press, 2011), 504-32, David Armstrong, *Belief, Truth and Knowledge* (Cambridge: Cambridge University Press, 1973), 157, Alvin Goldman, *Epistemology and Cognition* (Cambridge, Mass: Harvard Univ. Press, 1995).

[2] Greco. "Virtue, Luck and the Pyrrhonian Problematic," 9-10.

[3] Greco is not a straightforward Reliabilist, contra what this quotation might suggest. For his more detailed analysis, see Greco, "Putting Skeptics in their Place."

[4] Greco, "Putting Skeptics in their Place," 56.

[5] Kaplan, "Austin's Way with Skepticism," 353.

the semantic standards for knowledge, thereby creating a context in which she can truthfully say that we know nothing or very little. What we fail to realize, according to the contextualist solution, is that the skeptic's present denials that we know various things are perfectly compatible with our ordinary claims to know those very propositions.... Once we realize this, we can see how both the skeptic's denials of knowledge and our ordinary attributions of knowledge can be correct.[6]

For Externalist philosophers like Greco, the sceptic just flatly defines knowledge in a non-ordinary way. For Contextualists like DeRose, the sceptic fails to recognize that the standards for correctly saying "I know that P" change with context. On either view, the sceptic fails to show that ordinary assertions like "I know that P" are false, so Scepticism is undermined.[7]

The object of this paper is to present an alternative way of thinking about Scepticism against which this kind of criticism is of no force at all.[8] Let's say that the kind of Scepticism which denies ordinary knowledge attributions is "Semantic Scepticism." In this paper I distinguish an alternative conception of Scepticism to the semantic one. I call it Pessimistic Scepticism.[9]

Section 3 makes an analogy between the critics above and a similar critic of debates about the existence of God. Section 4 concerns the critic's argument that there is no philosophically significant alternative version of Scepticism to the knowledge-attribution conception of Scepticism. Sections 4-8 show that argument to be unconvincing, and develop my promised alternative conception of Scepticism. Section 9 canvasses a promising way in which debates about Scepticism may fruitfully proceed in my proposed direction.

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[6] DeRose, "Solving the Skeptical Problem," 4-5.

[7] For lucidity, I will continue to speak of this criticism as the idea that "the sceptic presupposes a non-ordinary definition of knowledge." That isn't quite an accurate characterization of Contextualist approaches to Scepticism, but those approaches do share the basic idea which I want to discuss.

[8] Have sceptics historically asserted anything like "when ordinary people say that they know things, what they are saying is false"? For my money, the answer to this question is plainly "no," but I won't defend that claim here.

[9] To be clear, I am not trying to criticize the work of the philosophers cited above. They might be right in their criticisms of Scepticism the way that they understand it. Here I try to see if there is any other way of understanding Scepticism which sidesteps the issues that these philosophers discuss.

2. An Analogy: Theism and The Problem of Evil

Suppose that Richard and Graham argue about whether or not there is a God.[10] They argue in the usual ways—Graham with the argument from evil; Richard with the fine-tuning argument. Now suppose I walk in and I tell Richard and Graham that the truth conditions of ordinary statements like "God exists" are just these: (i) religious practices and beliefs make a lot of people happy and (ii) lots of people believe very confidently that something created the universe. Suppose that I manage to convince Richard and Graham that that's what "God exists" *means*. I think if I interrupted the debate in this way, neither Richard nor Graham would have any idea what I was trying to do. They might come to agree with me that "God exists" means what I say that it does, for most ordinary English speakers, but they would surely insist that the point is of no significance at all for their debate. Moreover, Graham, even though he says things like "God does not exist" in his debate with Richard, would laugh at the suggestion that his view was false or his arguments unsound because of my quibbles about the *words* "God exists." Graham and Richard would unite in explaining to me:

> we have already agreed that we will mean by 'God exists' that there is some thing which is all good, all powerful, all knowing and created the universe. Never mind what other people mean. Don't get us wrong, its very interesting that other people mean something different, but it doesn't have anything to do with our debate.

Graham and Richard can properly lecture me in this way because their views are not about the *words* "God exists" as used in ordinary English. Richard is not saying that when ordinary people say "God exists," what they are saying is true, and Graham is not saying that when ordinary people say "God exists," what they are saying is false. Their views are about whether there is in reality a certain entity, defined in a specific way. Moreover, the interesting and philosophically significant positions that Graham and Richard could take are those about whether there is in reality an entity which meets their definitions. The parallel positions about whether ordinary people are speaking truly in saying "God exists" are just besides the point.

These same ideas are applicable in discussions of Scepticism. The sceptic gives an argument for the claim that nobody knows anything, or that nobody knows anything in a certain domain. The anti-sceptic replies that the sceptic is

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[10] Richard and Graham are of course Richard Swinburne and Graham Oppy. See Richard Swinburne, *The Existence of God* (New York: Oxford University Press, 2004) and Graham Oppy, *Arguing About Gods* (Cambridge: Cambridge University Press, 2006).

using the word "know" in a non-ordinary way, and so fails to refute ordinary knowledge attributions. The sceptic could surely reply:

> What I mean by 'knowledge,' never-mind what anyone else means, is belief that meets such-and-such conditions. I am interested in convincing you that you do not know anything, in my sense of that word. Don't get me wrong, it's very interesting that other people mean something different, but it doesn't have anything to do with what I am trying to do.

3. Is Scepticism Significant?

If the sceptic responds to his critics in the way I paint him as responding in the last section, then he cannot be a Semantic Sceptic. He must agree that when people say things like "I know that P," what they are saying is true. His only qualification will be that he wants to convince the anti-sceptic that he does not know anything, *in the sceptic's sense of "knows."*

If the sceptic makes that move, there will be critics who will say that his sceptical conclusion is therefore of no significance whatsoever. They will say that if the sceptic isn't using the word "know" in an ordinary way, then his arguments are uninteresting, and his position not worth philosophical attention. Barry Stroud endorses this way of thinking. In discussing a Cartesian argument against knowing anything about the physical world, Stroud comments:

> So it can easily look as if Descartes reaches his sceptical conclusion only by violating our ordinary standards and requirements for knowledge... the sceptical conclusion is... a misunderstanding or distortion of the meanings of the words in which it is expressed. It is at first astonishing to be told that no one can ever know anything about the world around us, but once we learn that the 'knowledge' in question is 'knowledge' that requires the fulfilment of a condition which is not in fact required for the everyday or scientific knowledge we are interested in, we will no longer be surprised or disturbed by that announcement. We do not insist that the dream-possibility must always be known not to obtain in order to know things in everyday or scientific life. When we find that Descartes's sceptical reasoning does insist on that requirement, we will find that his sceptical conclusion does not contradict anything we thought we knew at the outset. We might find it quite believable that there is no knowledge of the world fulfilling all the conditions of Descartes's special 're-definition' of knowledge. But properly understood, his conclusion would not deny what its peculiar linguistic form originally led us to suppose it denies, and it would pose no threat to our everyday knowledge and beliefs.[11]

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[11] Barry Stroud, *The Significance of Philosophical Scepticism* (New York: Oxford University Press, 2008), 40.

So far, all Stroud has done is point out that a sceptic who stipulates his own sense of "know" will not refute ordinary knowledge attributions. The sceptic can grant this, but he will insist that his view is not about ordinary knowledge attributions anyway. But, Stroud argues that if the sceptic makes this move, then:

> Any exhilaration or disquiet we might have felt on first encountering [the sceptical argument] must therefore have been due to nothing but illusion...If there were nothing more behind Descartes's sceptical conclusion... it would indeed be profoundly uninteresting... Descartes's assessment of his own position is thought to deviate so radically and so obviously from our familiar assessments that it cannot be expected to reveal anything of deep or lasting significance about the human knowledge we are interested in.[12,13]

If the sceptic is not denying ordinary knowledge attributions then he is, according to Stroud, not doing anything interesting or of deep and lasting significance. We may sum this up as an argument against the sceptic of this sort:

<div align="center">The Significance Argument</div>

1) Either the sceptic wants to deny ordinary knowledge attributions or he doesn't.

2) If he doesn't want to deny ordinary knowledge attributions, then his arguments are uninteresting and of no deep and lasting significance.

3) Therefore, either the sceptic denies ordinary knowledge attributions or his arguments are uninteresting and of no deep and lasting significance.

The difficulty with the argument is obviously (2). What reason is there to accept (2)? Stroud gives an analogy:

> Suppose someone makes the quite startling announcement that there are no physicians in the city of New York. That certainly seems to go against something we all thought we knew to be true. It would really be astonishing if there were no physicians at all in a city that size. When we ask how the remarkable discovery was made, and how long this deplorable state of affairs has obtained, suppose we find that the bearer of the startling news says it is true because, as he explains, what he means by 'physician' is a person who has a medical degree and can cure any conceivable illness in less than two minutes.1 We are no longer surprised by his announcement, nor do we find that it contradicts anything we all thought we knew to be true. We find it quite believable that there is no one in the whole city

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[12] Stroud, *The Significance of Philosophical Skepticism*, 40.

[13] Stroud (*The Significance of Philosophical Skepticism*, 40-82) defends the sceptical argument from this line of criticism in the end, by arguing that the sceptic does not use the word "know" in a non-ordinary way. Still, he does endorse (at least in this book) the idea that, if the sceptical conclusion is not at odds with ordinary knowledge attributions, then it is of no significance.

who fulfils all the conditions of that peculiar 're-definition' of 'physician.' Once we understand it as it was meant to be understood, there is nothing startling about the announcement except perhaps the form in which it was expressed. It does not deny what on first sight it might seem to deny, and it poses no threat to our original belief that there are thousands and thousands of physicians in New York.[14]

According to Stroud, a sceptical argument that is compatible with knowledge attributions is analogous to the argument about Physicians in New York. We find the sceptic's conclusion absurd, but then once he explains what he means to say, we find his conclusion boring because it does not contradict anything that we believe. That is why Stroud thinks that (2) is correct.

4. Does the Sceptic Contradict What I Believe?

I find Stroud's line of thought here baffling. There are two problems; one small and one serious. I begin with the smaller one and consider the serious one in the next section.

Stroud depicts the physician argument as being insignificant because it does not contradict our belief that there are physicians in New York, and he takes the sceptic's arguments to be much the same; insignificant because they fail to contradict anything that we believe. But, it is far from clear that the usual sceptical arguments do not contradict anything that I ordinarily believe. Stroud's argument hinges crucially on the claim that if the sceptic does not contradict our belief that we have knowledge of the world, then he does not contradict *anything* that we believe. This is a non-sequitur. It does not follow from the fact that the sceptic does not contradict one belief of mine, that he does not contradict any belief of mine.

Consider the sceptic who says that nobody knows anything about the physical world. It is logically possible that the sceptic's conclusion, although it does not contradict our belief that we know about the world, nevertheless contradicts *something else* that we ordinarily believe. Let X stand for the stipulated meaning of the sceptic's word "know," whatever that meaning is. When the sceptic says we cannot know about the world around us, he says:

4) We cannot X about the world around us.

Now, it is logically possible that I possess both of these distinct beliefs:

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[14] Stroud, *The Significance of Philosophical Skepticism*, 40.

5) I do know about the world around me.

6) I do X about the world around me.

We may suppose that the sense of "know" in (5) is the ordinary sense and that the sceptic's X is not that ordinary meaning. Then, although my belief which I would express by (5) does not contradict (4), I might have a different belief, (6), which *does* contradict (4). Furthermore, my belief that contradicts (5) need not be so explicit as (6). Suppose that I have another concept, "schnowledge" which is such that X is a necessary condition for "schnowing" something. Then I might hold both (5) and:

7) I do schnow about the world around me.

And (7) does contradict (4). Thus it is not true that if the sceptic's definition of "knowledge" is not the ordinary one, then his conclusion does not contradict anything that we ordinarily believe.

This is how the matter stands with respect to logic. An anonymous referee responded to this point claiming that, while it is correct, it makes no difference. Sure, the reply goes, we might have all sorts of beliefs which contradict (4), but the only relevant point is that (4) does not contradict our belief that we have knowledge.

I do not see why this must be the only relevant point at all. If the sceptic has a sound argument for a conclusion which contradicts something that we believe, I do not see why the fact that the contradicting belief is not the belief that we have knowledge should somehow undercut the significance of the sceptic's conclusion. After all, he would still refute something which we believe.

Certainly, if we are thinking of the sceptic as someone who aims solely to refute our belief that we have knowledge, the fact that his conclusion does not contradict that belief would be the only relevant point, but in this paper I am exploring whether there is any *other way* for the sceptic to develop his views, and so to insist that the only relevant point is whether the sceptic contradicts my belief that I have knowledge is to beg the question.

The sceptic might, for example, aim to refute our belief that our evidence favours ordinary beliefs about the world over the dream hypothesis, and if we really do have that belief, he will succeed in contradicting something that we believe. If the critique is to provide a solution to the problem of Scepticism, it must really be a solution to the problem. It won't do for the critique to show merely that Scepticism is not in conflict with beliefs about knowledge while conceding that it is in conflict with various other beliefs of ours. Such a "solution" is so in name only. It must be claimed that Scepticism does not contradict *any* belief of ours.

It might be countered, however, that we simply do not have the belief that our evidence favours ordinary beliefs about the world over the dream hypothesis. It might even be added that once I realize that my belief, (5), does not involve a sense of "know" a necessary condition for which is X, I will be able to see that I never really believed that I could do what the sceptic says I cannot. Once we clear aside the confusions which arise when we frame the issue in terms of "knowledge," perhaps it is just obvious that I never did believe that I could tell whether or not I was dreaming.

This last argument is incredibly presumptuous about what I do and do not believe. It presumes that nobody really ever believed that their evidence favoured that they are sitting at their desks over the hypothesis that they are dreaming, and that they were only tempted to suppose that they believed it because they got confused by the sceptic's use of the concept of "knowledge." I can only speak for myself in saying that I really think that I did believe, prior to considering the sceptic's argument, that my evidence favoured the view that I was not dreaming. This is why when I considered what Descartes had to say about there being no marks by which to tell whether or not one was awake or asleep, I was every bit as astonished as he was. If this is right, then the sceptic's conclusion does contradict something that I believe, and in such a case, the argument will be significant, pace Stroud, exactly because it does contradict something that I believe. I leave the reader to determine whether they too have this belief.

5. Pessimistic Scepticism

On to the serious problem with Stroud's line of thought. Even setting aside all of the sociological questions about what we already believe, the sceptic's conclusions might well be interesting and significant even if they are compatible with ordinary knowledge attributions and *even if they do not contradict something that we believe*. It isn't as though a necessary condition for an idea being interesting is that it contradicts something I already believe.

Suppose, for the sake of argument, that the ordinary meaning of "knowledge" turned out to be merely "belief held very confidently." The sceptic will not deny that there are such beliefs. If that were the meaning of "knowledge," would it follow that there is no interesting way for the sceptic to develop his position? Surely not. Surely he could argue that although ordinary knowledge attributions are true (because many people believe things confidently), it is still the case that nobody knows anything *in his sense of the word,* and, that what is ordinarily called "knowledge" is really quite pathetic. It is a mere charade of little to no value. Meeting ordinary standards of knowledge is not something which we

should be happy with, because it is a pretty low bar to begin with. This, at any rate, would be sceptic's pessimistic assessment of human cognitive achievements.

That, I suggest, is how the sceptic should develop his views against his critics. He should allow that perhaps ordinary knowledge attributions are all true, whatever they mean, but insist that the ideas that pass for "knowledge" ordinarily are a miserable achievement not worth taking authoritatively over alternative beliefs. He should insist this even if the ordinary meaning of "knowledge" is best understood in terms of, for example, reliability or if the standards for "knowledge" vary with context.

Call this pessimistic assessment, "Pessimistic Scepticism." It is important to note that Pessimistic Scepticism is not the denial of ordinary knowledge attributions at all. It is first and foremost a negative and dismissive attitude towards human cognitive achievements. The pessimistic sceptic is someone who thinks that nothing that we normally call "knowledge" is worthy of any respect or appellation. He thinks we haven't really achieved anything particularly valuable with any of our efforts to discover the truth. Maybe—just maybe—we have actually got the truth about some things, but even still, our methods of arriving at our views are feeble, full of doubts, fragility and insecurity, and the resulting views are not worthy of respect or authority over the alternatives. Equally, maybe—just maybe—we "know," in the ordinary sense of "know" various things, but the sceptic will insist that "knowing" in that sense is pretty thin soup. He will say that there isn't anything valuable or noteworthy about our "knowing" in the ordinary sense, because meeting those standards is meeting pitifully low standards.[15]

Pessimistic Scepticism would be an interesting and significant sceptical position, even if the sceptic conceded that ordinary knowledge attributions are true. Moreover, it would be interesting and significant even if it did not contradict something that we already believe—perhaps we have just never given much thought to how valuable our standards for knowledge are.

6. Jackson and Ordinary Concepts

Despite my insistence that Pessimistic Scepticism is a philosophically significant view, some philosophers will demur. Frank Jackson, discussing conceptual analysis, writes:

> If I say that what I mean—never mind what others mean—by a free action is one

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[15] We might disagree with the sceptic that the ordinary standards for knowledge are pitifully low, but this will be a further matter to be debated with the sceptic, not something which is self-evident or otherwise painfully obvious.

such that the agent would have done otherwise if he or she had chosen to, then the existence of free actions so conceived will be secured, and so will the compatibility of free action with determinism. If I say that what I mean—never mind what others mean—by 'belief' is any information-carrying state that causes subjects to utter sentences like 'I believe that snow is white,' the existence of beliefs so conceived will be safe from the eliminativists' arguments. But in neither case will I have much of an audience. I have turned interesting philosophical debates into easy exercises in deductions from stipulative definitions together with accepted facts. What then are the interesting philosophical questions that we are seeking to address when we debate the existence of free action and its compatibility with determinism, or about eliminativism concerning intentional psychology? What we are seeking to address is whether free action according to our ordinary conception, or something suitably close to our ordinary conception, exists and is compatible with determinism, and whether intentional states according to our ordinary conception, or something suitably close to it, will survive what cognitive science reveals about the operations of our brains.[16]

According to Jackson, the interesting philosophical questions are questions couched in terms of our ordinary concepts. Presumably what he says about "free action" and "belief" is the same sort of thing he would say about "knowledge." So, Jackson would argue that the interesting question about knowledge is whether or not we have "knowledge" in our ordinary sense of "knowledge." Thus, if the sceptic has an argument that we do not know anything, but his concept of knowledge is non-ordinary, his conclusion will not be that interesting. Why not? Jackson makes two points. The first is that using non-ordinary concepts in philosophical discussion turns interesting philosophical debates into 'easy exercises in deductions from stipulative definitions together with accepted facts.' The second is that if I frame my discussions using non-ordinary concepts I will not have much of an audience.

Consider Jackson's first point. A lot depends on whether the sceptic's argument is an "easy deduction," and that obviously depends on which argument is at stake. Consider a Cartesian sceptical argument for the conclusion that we do not know anything about the world around us. Such arguments typically depend on something like the assumption that I cannot tell—just by looking—that I am sitting at my desk and not deceived by an evil demon or having a grand hallucination. At least some philosophers have thought that I really can tell, or that if I can't, I can at least argue the claim that I am sitting at my desk from some more secure starting point. Now, I am not saying that these philosophers are right, but to take it that none of these accounts is viable and assume that if we grant the sceptic

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his definition of "knowledge," his Scepticism follows as an "easy deduction," is to take quite a controversial position without much argument.

Yet, let us grant that the sceptic's argument is an easy deduction from stipulated definitions. I am not sure what relevance this has to the question of whether or not the sceptic's argument is interesting, since there is no reason to suppose that an interesting argument must be very complex. I am inclined to think that a simple argument for an independently interesting conclusion is more interesting than a complex argument for the same, because a simpler argument seems less likely to contain a mistake.

Turn now to Jackson's second point that a sceptical argument would not have much of an audience if it deploys non-ordinary concepts. Is it true that people are generally not interested in concepts which are different to their own? Many people are interested in scientific theories, even though the concepts used in science are very different to the concepts used in everyday life. What strikes at least some people as interesting about philosophy is precisely that it gives one the opportunity to consider radically different ways of viewing the world. There is no reason to suppose that using non-ordinary concepts makes people less likely to take an interest in sceptical arguments.

7. Too High Standards

I suspect that behind all of the insistence that Pessimistic Scepticism is not philosophically significant and the insistence that Semantic Scepticism is the only kind worth discussing is the old thought that the sceptic presupposes absurdly high standards for knowledge. The idea is, not merely that the standards of the sceptic are non-ordinary, but that they are just unnecessarily high; so high that it doesn't really matter if we can't meet them. The sceptic, according to the critic, is just whining that our beliefs don't meet some very intense set of standards that only a philosopher like Descartes would fantasize about. That this does lie in the background is at least suggested by Stroud's comparison with the physician argument and by the DeRose quote from earlier:

> For skeptical arguments... threaten to show, not only that we fail to meet *very high requirements for knowledge of interest only to misguided philosophers seeking absolute certainty*, but that we don't meet even the truth conditions of ordinary, out-on-the-street knowledge attributions. They thus threaten to establish the startling result that we never, or almost never, truthfully ascribe knowledge to ourselves or to other mere mortals.[17,18]

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[17] DeRose, "Solving the Skeptical Problem," 4.

DeRose writes here as though there are only two options for the sceptic. Either he whines about our inability to reach absolute certainty, or he denies "out-on-the-street" knowledge attributions. If he takes the former line, we can object that absolute certainty is a ludicrously high demand of interest "only to misguided philosophers" and settle for lower standards. If he takes the latter line, he must use the word "knowledge" in its "out-on-the-street" sense and show that knowledge attributions are false. Yet, there are surely other options. The sceptic could set fairly modest standards for "knowledge," far short of absolute certainty, and yet take no interest at all in "out-on-the-street" knowledge attributions. That option is made quite attractive by the fact that contemporary arguments for Scepticism don't presuppose absolutely certain standards for knowledge. Their requirements are in fact fairly modest. Here is a brief overview of some of them.

A number of sceptical arguments work given the empiricist assumption that the only ways that humans have of knowing things about the world are sense perception and inferences from things learnt by sense perception.

There is the contemporary Cartesian Sceptical argument.[19] Say that two things, x and y, are perceptually indistinguishable to you only if x and y effect your visual system (they produces the same retinal image, or the same pattern of activity in the optic nerve), in the same way.[20] For example, Homer will be perceptually indistinguishable from a molecule for molecule replica of Homer just when the two would have the same effect on your visual system. The sceptic will maintain that if Homer is perceptually indistinguishable to you from his replica, then you do not know just by looking that who you are looking at is Homer, and the same for any x and y. Of course, my sitting at my desk (or any similar proposition about the physical world) is perceptually indistinguishable from classic sceptical hypotheses, like that I am a brain-in-a-vat having a massive hallucination. So I cannot know by perception that I am sitting at my desk. Or plainly, since I cannot tell just by looking that I am not in a sceptical scenario, I cannot know just by looking that I am sitting at my desk.[21] Combine that thought with the thought that, there is no good argument from anything I can know by sense perception to the claim that I am sitting at my desk, and we arrive at the conclusion that I cannot know that I am sitting there at all. The requirement for knowledge here isn't that, for any P, to know that P, S must be absolutely certain that P, but just that P can only be known

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[18] My emphasis.

[19] Jonathan Vogel. "Skeptical Arguments," *Philosophical Issues* 14, 1 (2004): 426–455.

[20] Jonathan Vogel, "Skepticism and Foundationalism: A Reply to Michael Williams," *Journal of Philosophical Research* 22 (1997): 16

[21] Vogel, "Skepticism and Foundationalism,"11-28.

either by perceptually distinguishing it from alternatives or by inferring it from things known by perception. The sceptic is free to say that any such inference could be merely probabilistic in character, so that his standards are a long way from a general requirement of certainty.

The idea that being able to perceptually distinguish x from y is a necessary condition for knowing by perception that x, will allow for the same sort of argument against our knowing that anyone else has feelings, emotions or thoughts. After all, a person with a mind has the same effect on my visual system as a mindless zombie who looks just like a person, and most philosophers think the usual arguments for the existence of other minds are painfully weak. Note again the requirement for "knowledge" here is not a general requirement of absolute certainty. The sceptic would be happy with either your being able to tell by perception that other people have minds, or with your giving an argument for it, and the argument need not be iron-clad; a good argument from analogy or inference to the best explanation would do the trick. Are there any other ways that humans can know things about the world beyond their own minds than by sense-perception and inferences from what we learn by perception? The dominant empiricist tradition in philosophy says otherwise, and no-one has yet defended any other source.[22]

There is a sceptical argument against knowing that anything ever exists when we aren't looking (or otherwise perceiving).[23] I suspect that the argument can be run on most of the definitions of "knowledge" which contemporary philosophers favour, but it works given the currently popular sort of definition where knowledge means something like "true belief produced by a reliable process." The basic idea is this. I can know that something about the physical world is the case only by perception or inference from things I know by sense perception. But I cannot know that anything exists when I'm not perceiving it *by perception*, because that is a contradiction. On the assumption that I cannot know by inference that things exist when I'm not looking—and no one has ever made the argument—then I cannot know it at all. The belief seems to be completely groundless in any sense you might care about. Lest you think that the belief that things exist when you aren't perceiving them is of little consequence, if I do not know such things, then I don't know that any of my friends or family exist when

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[22] Anil Gomes, "Skepticism about Other Minds," in *Skepticism: From Antiquity to the Present*, eds. Diego Machuca and Baron Reed (Bloomsbury Academic, 2018).

[23] Aaran Burns, "Can I Know that Anything Exists Unperceived?" *Logos and Episteme* 9, 3 (2018): 245-260.

I'm not around, nor that my kitchen exists when I'm not in it, nor even that the wall behind me exists when I'm not looking at it!

The point of Scepticism, as I am here thinking of it, is not that ordinary knowledge claims are false. It is, rather, that ordinary knowledge is *just not that impressive*. It might be that, in the ordinary sense of "knows," we know all sorts of things. But the sceptic, on the position I am here offering him, thinks that our system of beliefs or "knowledge" is depressingly doubtful. Ordinary human knowledge is in this sense a charade of little to no value.[24] The sceptic's assertion, "nobody knows anything" should be qualified to read, "nobody knows anything except, maybe, in a meagre, unimportant sense."

In the face of the sceptic's pessimism, its always open to the anti-sceptic to make the too high-standards objection whenever he feels under pressure. When the standards are set at certainty and the sceptic argues that nothing can meet them, the anti-sceptic complains of the sceptic's fantastically high standards, and lowers the standards to good but inconclusive reasons. When the sceptic argues that nothing can meet them either, the anti-sceptic complains again that the standards are pointlessly demanding, and abandons the demand for reasons all together, saying that it is enough when a belief is just caused in a reliable way. When the sceptic argues that a lot of the anti-sceptic's beliefs don't meet that standard either, the anti-sceptic might complain yet again, and retreat even further. How many of these moves are acceptable? When should we just admit that the sceptic's pessimistic attitude towards our belief system is a sensible one?

That is what the issue comes down to. Is the sceptic right to despair that we cannot meet his standards, or is he foolishly whining about our inability to meet fantastically high standards? The critic might say the latter, but if that is what is wrong with Scepticism, the problem *isn't* that his standards aren't the ordinary ones.

8. Conclusions

The pessimistic sceptic is defined by his pessimistic attitude towards human cognitive achievements, not by the denial of ordinary knowledge attributions. We need to argue with the sceptic on wholly different grounds when he is understood this way. The debate we must have is over whether his standards for knowledge are valuable, and whether there is any value in alternative standards. The question

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[24] Greco writes in this context that "even if there is some sense in which one does not really know without [meeting the sceptic's standards for knowledge], it does not follow… that knowledge in any ordinary sense requires that" (Greco, "Virtue, Luck and the Pyrrhonian Problematic," 31). That is indisputably true, but the pessimistic sceptic is not claiming otherwise.

is entirely evaluative, and there is no need to worry about which standards are "ordinary."

The sceptic will hold, of course, that his standards are important and that alternatives don't amount to very much, and that is why the sceptic will say that "nobody knows anything except maybe in a meagre, unimportant sense." The anti-sceptic will hold the contrary view. I haven't here given any arguments for thinking that the sceptic is right. I have only tried to distinguish it clearly from the currently more widely discussed Semantic Scepticism and to dispel the thought that anything other than Semantic Scepticism is insignificant. The arguments for pessimism await another occasion.

NUMBERS, EMPIRICISM AND
THE A PRIORI

Olga RAMÍREZ CALLE

ABSTRACT: The present paper deals with the ontological status of numbers and considers Frege´s proposal in *Grundlagen* upon the background of the Post-Kantian semantic turn in analytical philosophy. Through a more systematic study of his philosophical premises, it comes to unearth a first level paradox that would unset earlier still than it was exposed by Russell. It then studies an alternative path that, departing from Frege's initial premises, drives to a conception of numbers as synthetic a priori in a more Kantian sense. On this basis, it tentatively explores a possible derivation of basic logical rules on their behalf, suggesting a more rudimentary basis to inferential thinking, which supports reconsidering the difference between logical thinking and AI. Finally, it reflects upon the contributions of this approach to the problem of the *a priori.*

KEYWORDS: philosophy of mathematics, Logical Empiricism, Gottlob Frege, Bertrand Russell, a priori.

1. Introduction. The Historical Backstage

Logical Empiricists critically rehabilitated the Kantian epistemic project meant to set apart genuine knowledge of the external world from our own contributions to it—a task that, though distinctive of philosophy from its very origins, has over and again become swallowed up by the outgrowths of different forms of undifferentiated idealisms and re-enchantments. Their project, though, adopted the specific form of dispelling those confusions brought about by misleading grammatical appearances, which often deceive us into believing a surplus of phantom realities and the pursuit of pseudo problems. But their Verificationist Criterion of Meaning (VCM) aimed nevertheless, as did Kant, to separate out experientially based knowledge that could serve scientific progress from speculative metaphysics and the possible projection onto the world of human emotions and values, characteristic, they thought, of morality and aesthetics—aspects that, significantly, Kant did not understand in any experientially based mode either.

Kant distinguished, however, two different ways that human beings might contribute to external world knowledge. These contributions could be due to extra content or they could be due to form, to our own form of cognition. The first

characterised the excesses of transcendent metaphysics, illegitimately enhancing the world with further non-experientially based additions of our own. The second, though, constitutes his transcendental philosophy with the introduction of synthetic a priori judgments. These latter he found not only legitimate but absolutely essential if any knowledge of the world were to be possible at all. It is here where the Empiricists, getting rid of what they considered unnecessary, and misconceived, a priori conformations of experiential knowledge, most strongly departed from Kant. But, in doing away with the whole Kantian transcendental apparatus and his conception of synthetic a priori judgments in favour of just logic and language, they arguably arrived at much too restrictive criteria, which ended up making their own position untenable—since the removal left an explanatory lacuna when it came to giving an account of the constitution of the objects of experience from sensory data alone, the explanation of causality and other forms of necessity present in even in our most basic scientific laws. How successful their later attempts were to provide alternative accounts of these aspects by appeal to logic and language alone is still a troublesome issue. None of it obviates the important reasons that spoke against the Kantian position on this specific point—not just the revolutionary transformations brought about by Non-Euclidian Geometry, Einstein´s Relativity Theory and Quantum Mechanics into our scientific picture, but also the increased centrality gained by semantics owing to the writings of Bolzano and the later reception of Frege: the first appeared to directly contradict the Kantian theory; the second showed how well we could do without it. The perfect match between the difficulties of the theory and the incipient success of its abandonment, set the conditions for a paradigmatic overturn.[1]

1.1. The Resulting Epistemic Setting

Once the Empiricists had renounced any other source of knowledge from a provenance external to our own, *prima facie* less mysterious, logico-linguistic equipment, experience became the only ground on whose basis to derive and validate our knowledge claims, the ultimate and sole criterion of existence. To this end, Russell´s analysis of definite descriptions[2] opened up what can be considered

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[1] This paper is a contribution to the philosophy of mathematics from a non-technically trained philosophical perspective, as will become apparent for colleagues in the field. I do not pretend it to be otherwise. I, nevertheless, hope it can offer a valuable perspective on these problems. A first shorter version of this paper was presented at the conference *The Philosophy of Logical Atomism 1918-2018*, Complutense University Madrid, 28.01.2019.

[2] Bertrand Russell, "On Denoting," *Mind*, New Series 14, 56 (1905) 479-93.

the most consequent and properly empiricist line of existential analysis. If the grammatical surface can mislead us into believing in non-existent objects through deceitful singular terms, the way to expose it is precisely to lead them back to the 'tribunal of experience.' There we could see whether or not there was an individuum satisfying the descriptions associated with the term. From this perspective, the claim that because our specific theoretical postulates require the existence of such objects they must be taken to exist in some other way, could not be taken to hold, simply because there is no other way to exist. Actually, as Coffa[3] points out, Russell saw himself as thereby 'neutralizing the tendency to produce false abstractions.' The kind of things that exist is, of course, a complicated issue, but at least we had to be able to find some basis in experience that allows us to confirm or disconfirm existential claims or else show how our terms are related to it. Otherwise, the whole fuss about transcendent metaphysics would have seemed superfluous were we to end up postulating entities as we see fit. The importance of Russell´s theory of descriptions was celebrated by Ramsey[4]who, following its lead, proposed his famous 'Ramsey Sentence' with the purpose of dealing with theoretical scientific terms in a similar way, a proposal that was later elaborately developed by Carnap.[5] As in Russell´s case, scientific sentences with singular terms seeming to refer to some abstract entities had at least to be seen as conditional to corresponding existential sentences from whose truth the truth of the theories would depend. Following this string of thought, the Fregean proposal to introduce numbers as abstract objects referred to by the corresponding singular terms in mathematical sentences, could scarcely be accommodated.

But the problem in this case was that neither of the options available appeared to provide the resources needed to deal with the status of mathematical knowledge—those options being either 1) to reduce numbers to experience or 2) to provide an account of them through mere logic and language. In the first case, neither a direct reduction of Mill's empiricist type, nor one analogous to Ramsey and Carnap's treatment of theoretical scientific terms, showed any means of success; but neither did the possibilities opened up by the second—Conceptualism and Formalism—the preferred route of authors such as Schlick, Hahn or early Carnap.[6] Conceptualism, which was Russell´s option after the breakdown of

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[3] Alberto Coffa, *The Semantic Tradition from Kant to Carnap. To the Vienna Station* (Cambridge: Cambridge University Press, 1991), 109.

[4] According to Coffa (*The Semantic Tradition from Kant to Carnap*), Ramsey would have seen in it as one of the greatest achievements of the century.

[5] Rudolf Carnap, *Philosophical Foundations of Physics* (New York: Basic Books, 1966), Ch.26

[6] See, for example, Warren Goldfarb, "Philosophy of Mathematics in Early Positivism,"

Frege´s project,[7] was problematic mainly for two reasons: concepts, even if understood as conventions, could not be *mere* conventions on pain of being absolutely hollow and useless; but if they weren't, showing them to be meaningful required remitting them to their verification conditions (as required by the VCM) or, at least, showing through an explicit conceptual analysis their ultimate possible connection to experience. This implied that there had to be something that these concepts were *about.* It had to be possible to prove whether what was said through them was the case or not, and this brought us back to the initial problem. Understanding them as some kind of properties, as Russell did, thus made things no better, since it equally required either showing how exactly they were to be derived from experience or accepting them as some new kind of abstract objects, giving rise to the consequent problems again. Formalism, on the other hand, attempted to find a solution by assimilation of them with logic, believing that, at least for some concepts, the question of their 'aboutness' could be dealt with differently. The corresponding concepts would actually concern rules, having more to do with relations among objects than referring to any objects or properties. But, far from being wholly unproblematic, implicit in this option was the assumption that the status of logical laws and our peculiar 'a priori grasp' of their necessity was absolutely no issue. Not even the conventionalist account, which according to Coffa[8] would have provided the semantic tradition's solution to the problem of the a priori, can be considered to have given an appropriate response to this question. As Prior[9] exemplified with the case of *Tonk,* the fact that we should set a concept with its corresponding rules of use to run, and then appeal back to those very inferential rules to justify it can be seen as circular.[10] The source of necessity of logical laws was through such explanations in no way exhausted. Actually, much of what is at issue here, as we will see later on,[11] depends on this question. But, as an explanation of mathematical statements, Formalism could not give an account of their truth in any substantive manner. There are, of course, contemporary defences of Formalism of which I cannot pretend to give a proper account here, such as

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Minnesota Studies in The Philosophy of Science. Origins of Logical Empiricism, Minneapolis, London: University of Minnesota Press, 16 (1996): 213-231.

[7] Bertrand Russell, *Principles of Mathematics*, 2nd ed. (London, NY: Routledge, 1937).

[8] Coffa, *The Semantic Tradition from Kant to Carnap*, Ch. 14.

[9] Arthur Prior, "The Roundabout Ticket," *Analysis* 21, 2 (1960): 38-9.

[10] Questions of conservativeness and consistency might be appealed to here, but not even in this way is the matter clarified. There can be untruthful consistent systems, and non-conservative rules might be worth incorporating, forcing consistency to be rearranged.

[11] See 4.2 in this paper.

Field's,[12] that ascribe to themselves the capacity to adopt talk of truth. But as long as there is nothing independent of those very forms capable of deeming mathematical statements true, I do not see how it could be defended that such truth is significant in any sense or that it could be nothing more than correct uses made of pre-given rules.

The reasons why the attempt to reduce numbers to experience in accordance with the route of existential analysis opened up by Russell's Theory of Descriptions did not appear worthy of a try, might not be immediately obvious. So, I think it deserves at least a quick look, since it might bring out more clearly the starting point and motivations of Frege´s own account. Three possibilities can be distinguished here: 1) direct reduction; 2) existential conditionalized reduction; 3) functional conditionalized reduction.

The first can be taken to represent the position defended by Mill, for whom what we mean by natural numbers are compilations of objects. Natural number terms would be general terms obtained per induction from different sample groups. When we say there are 'Five apples on the table,' what we actually mean by 'five' is something to be found in the apples on the table, something they have in common with groups of five oranges or five peaches. The immediate problem, as it appears, is that the *fiveness* itself is nowhere to be experienced in the examples given.

The second corresponds to a parallel treatment of numbers to theoretical scientific terms. Here, we would discard claims containing numbers as singular terms, by way of conditionalizing such expressions upon some existential sentence no longer containing the term. The main difference with the previous option is that as with theoretical scientific terms, number statements would have conditions of application. If a track in a cloud chamber justifies claims about 'neutrino,' the existence of specific compilations of objects does so with corresponding number statements. In both cases we assume that the application conditions do not exhaust the cognitive content of the terms. This would imply the existence of something else, 'a surplus of content' as Carnap puts it, going beyond what the application basis justifies (the presumed entity *neutrino*, and the *number* in question). The problem now is that while we can know what it would take to prove the existence of the assumed entity in the scientific case, and so make the truth of the initial statements dependent upon it, no similar hope is available in the case of numbers. We are not able to go beyond compilations of objects to a more adequate candidate of existential substitution. The problem of numbers reveals itself, therefore, as

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[12] In accordance with Stewart Shapiro, *Thinking about Mathematics* (Oxford: Oxford University Press, 2000), 226.

being clearly of a quite different sort. What makes this option interesting to consider is, though, that contemporary critiques of the Neo-Logicist programme[13] argue along similar lines to prove its implausibility. They assimilate Frege's contextual introduction of number to a procedure aimed at introducing numbers as abstract objects on the basis of what might be seen as application conditions, but with no way to existentially legitimise the assumed further claim of the existence of numbers—no more than we could try to legitimise the existence of God through a conceptual introduction as in the Ontological Argument.

The third option, however, differs from this one and also from Russell´s own conceptual solution, coming from an empiricist perspective closer to Frege´s own proposal. This I call the 'Functional Conditionalization' option. The starting point would be the same, that is, the compilations of objects that would deliver the application conditions. It would provide the contact point with experience, but again would not exhaust the cognitive content of number claims. But now, instead of hoping for a hopeless existential candidate upon which to conditionalize the truth of such claims, we would make it dependent upon the existence of a recognisable and acceptable function (for pragmatic reasons acceptable, perhaps) that could justify the transition from application conditions (compilation of objects) to claims about numbers. This brings us into the vicinity of Frege´s own functional introduction of numbers, since we could imagine such a function in similar terms to Frege's '1-1 correlation' between the members of different compilations. But the point of the reconstruction from this reductionist perspective would, rather, be the opposite to Frege's: to deny the existence of numbers. Since the mediating function could just be a man-made one, not itself provided through experience, and since from its fulfilment the acceptability of number claims depends, the thereby legitimised claims can just be (however else understood) man-made products. The strategy could be seen as analogous to a similar treatment of thick moral concepts, which would justify the transition from behaviours to values through the fulfilment of a moral function;[14] the attribution of the one to the other being then implicitly registered in the concept. If the behaviour fulfils the function, we consider it good in the thereby defined moral sense. In our case, the transition from compilations of objects to 'numerical values,' so to speak, would be made possible by a number-building function. That would be the idea. Could a

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[13] Such as Hartry Field, "Platonism for Cheap? Crispin Wright on Frege's Context Principle," *Canadian Journal of Philosophy* 14, 4 (1984): 673-62. For a discussion of this point, see Crispin Wright and Bob Hale, *The Reason Proper Study* (Oxford: Clarendon Press, 2001), 160-164.

[14] See, for example, Olga Ramírez, "Beyond Witches, Angels and Unicorns. The Possibility of Expanding Russell's Existential Analysis," *E-Logos* 25, 1 (2018): 4-15.

response along these lines answer Field's[15] type of complaints of having extracted an abstract object from an insufficient basis? Here, our result would be obtained through a specific mediating operator that takes application conditions as input and obtains numbers as output; by each added member to the compilation a successive number. The answer to our previous question would be that there is an 'intermediate reason' and that we have to do with a product, not a discovery. But what could be said of the number term so obtained? Does it refer to anything? Can it be considered to be justified in any empiricist-satisfying terms through our contact point with experience via application conditions? Even if we were to say that we have a constructed referential object, what would be its character? In the moral case, we can say that what we obtain is a moral *value* (in the sense of being good for the purpose of the fixed moral standard). But what is it that we obtain here? Would it make sense to talk of 'numerical values,' as I did before (bringing, perhaps, the comparison to rely on the equal measurability of benefits, pains, lengths, weights or whatever, and arguing that actually the real 'value' is the number therein)? Would we not then again be required to give an account of their status? Or should we talk rather of 'a substitutive symbol' for such equivalences or maybe 'merely a term'? But even if we were to adopt a non-problematic position that reduces the obtained product to something like a 'shortage term' whenever the functional mediation is possible, the question is whether an interpretation along these lines is in fact available to our empiricist. As Frege´s approach makes clear, and for reasons we will see in a minute, the answer is that it is not.

From this perspective, we might be better able to see the very dimension of the solution that Frege proposes, since Frege, I believe, is the one who really makes an attempt to respond to the lacuna left by the Kantian synthetic a priori, not just in the philosophy of mathematics but as a whole.

2. Frege´s Motivations

Although Frege was not as moved as others by the discovery of Non-Euclidean Geometry to abandon the notion of the synthetic a priori as an explanation of geometrical knowledge—nor might he necessarily have been by discoveries in astrophysics—he had his own reasons to abandon the realm of spatial and temporal Intuition,[16] as he saw it, when it came to Arithmetic. It was the generality of

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[15] Field, "Platonism for Cheap."

[16] Since Intuition here is meant in a sense akin to the Kantian notion of 'pure Intuition,' I will use it with a capital I, to distinguish it from the idea of (non-sensible) 'intuition,' understood as some special undetermined faculty capable of acquiring knowledge beyond the realm of experience, that Kant himself criticizes.

arithmetical thinking, the certainty and necessity of its proofs, the fact that we could not, as he argued, question its basic principles without contradicting ourselves, that indicated an intimate connection with our own thinking processes. Arithmetical thinking was not simply a specific way of thinking but appeared to be our own way of thinking on itself. This would explain the fact that it would have such an overall embracing domain: 'to it belongs not only the actual, not only the intuitable but everything thinkable. Should not the laws of number, then, be connected very intimately to the laws of thought?'[17] Since it was logic that represented the laws of thought being equally general in character, it had to be possible to make this 'intimate connection' explicit and show how the concerns of arithmetic arose through pure logical thinking. It had to be possible to prove that the reason why arithmetical thinking applied with certainty and necessity[18] was because of its derivability from logical laws and definitions alone.

But what Frege had set himself to do in his reconstruction of the logical form of our discursive thinking about the world, had a much wider reach. It amounted to including in the formal laws of logic, and thereby in the analytical realm, the epistemic possibility of our knowledge of objects[19] and those further necessary structures through which we would think about them, capturable themselves, in his view, through potential new, creative, conceptual synthesis,[20] thereby reintroducing back into the field of logic, as I will come back to, essential features of the Kantian synthetic a priori.

Since his analysis of the logical form of linguistic discourse went beyond the mere reconstruction of its logical rules to include how such rules referred to objects, it was now possible too to reason about objects without the objects themselves; to do so in a universal and certain way about whatever objects we could possibly have to do with, and so to reason about the world without the

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[17] Gottlob Frege, *Grundlangen der Arithmethik* (Hamburg: Felix Meiner Verlag, 1884)/ *Foundations of Arithmetic* (first published 1884), 2nd ed., translated by J.L. Austin (New York: Harper & Brothers Harper Torchwoods, 1953), §.14, 21.

[18] Michael Dummett, *Frege: Philosophy of Mathematics* (Cambridge, Massachussetts: Harvard University Press, 1991).

[19] It is quite striking how far Kant's introduction of his idea of the recognition of transcendental objects of understanding is already in line with Frege's proposal: "What does one mean, then, if one speaks of an object corresponding to and therefore also distinct from the cognition? It is easy to see that this object must be thought of only as something in general = X, since outside of our cognition we have nothing that we could set over against this cognition as corresponding to it" (Immanuel Kant, *Critique of Pure Reason*, first published 1781, translated and edited by Paul Guyer and Allen Wood (Cambridge: Cambridge University Press, 1998), KrV A104, 231).

[20] See Dummett, *Frege: Philosophy of Mathematics*, 305-36.

world—precisely what would be needed in arithmetical thinking, whose objects had those very eternal and universal features too.

Frege´s *Begriffschrift* in this way brought logic much nearer to arithmetic. However, he thought that despite this communality, mathematics was not simply the same as logic but had a topic of its own, a topic it was *about*; something beyond the mere thinking procedures that made its statements true. Mathematical statements could be substantively true, and this was for him a non-negotiable idea. The task was, thus, to come to identify, through a similar logical procedure, the objects that made mathematical statements true, thereby giving our logical reasoning not just the capacity to think about objects but its own objects to think about. That this should be possible departing from mere logic and definitions, required somehow turning the forms of thinking, our very mechanism of objectual apprehension, upon themselves in such a way that we obtain a new form of second-order synthesis. Something along these lines is suggested by Dummett:[21] it would be synthetic in the sense of it being knowledge gained by encapsulating a content different from itself.

From Dummett's reading, what Frege attempted to do was a matter of dissecting some kind of second-order pattern 'within the expressed thoughts themselves'—the same procedure he would have taken himself to have used to come to his logical form in his *Begriffschrift* in recognising the hidden structure lying in our discursive thinking.[22] It would be possible not just to extract conceptual information about the objects we speak about, but to build new concepts in grasping the more complex patterns of inferential reasoning we were able to discern in our linguistic constructions. It is this very idea of creatively recognising new patterns whose justification would be independent of experience, that in my view very much resembles a form of synthetic a priori knowledge—the difference being, of course, the absence of reference to experience or Intuition. But I leave further discussion of this until later. However, in a parallel sense Frege would see it as possible to extract a pattern to arithmetical reasoning that would

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[21] Dummett, *Frege: Philosophy of Mathematics*, Ch. 4.

[22] The example he gives to illustrate this possibility is how it would be possible to 'dissect' a complex pattern from the proposition 'Either Jupiter is larger than Neptune and Neptune is larger than Mars, or Mars is larger than Neptune and Neptune is larger than Jupiter,' into the pattern 'Either Jupiter is larger than x and x is larger than Mars, or Mars is larger than x and x is larger than Jupiter.' Which then can be captured with the concept 'Intermediate in size between Jupiter and Mars.' It would require understanding the whole proposition, and not as a derived result from its components, to obtain the pattern (Dummett, *Frege: Philosophy of Mathematics*, 40-41).

lay open what it is, we are referring to in talking about numbers. So, how is this to be understood?

Frege´s contextual introduction of the concept of number in *Grundlagen* attempts to explain the identity of what is referred to by the concept of number through an equivalence relation. The concept of being 'equinumerous' between two concepts is explained via an identity relation with a 1-1 correlation between the members of each concept.

> The number of Fs = the number of Gs if and only if there is a one – one correlation between the Fs and the Gs

The question is, therefore, what exactly is being done here? Dummett would say that Frege is attempting to explain the concept of number in terms of a new synthesis exercised upon the correlation 1-1 between the concepts on the right-hand side. That is, what this new synthesis records with the concept of 'equinumerous' is a pattern found in the established correlation on the right-hand side. There is i) the correlation 1-1- and there is ii) the recording of the pattern, being thereby created through a new concept: the concept of the specific number. This is supposedly the idea. But, first of all, what is the pattern supposed to be a pattern of? The fact that we establish a 1-1 correlation is, in principle, just the fact that we do so, even if we capture it with a new concept. What would be the difference between the concept 'correlation 1-1 between the individuals of the two conceptual extensions' and the concept of 'equinumerosity'?[23] Unless we are ready to say that the first delivers the application conditions[24] and operational resources (via the correlation) on whose basis something else is to be proved (as in the case of conditionalizing upon functions in the third empiricist option before),[25] I fail to see

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[23] This is a possibility that even Wright (Wright and Hale, *The Reason Proper Study*, 164) considers as a possible counterargument, putting it in terms of whether the ontological commitments would be the same. He does give an answer to it, but I must say I am not sure what to make of it.

[24] Although I derive this from the proposed functional conditionalization offered before, Dummett comes to suggest too that Frege might be appealing to the truth conditions of the numerical terms, though not suggesting what I go on to say about a construction of number terms through the operational procedure.

[25] It is the possibility of seeing such a procedure as opening a gap, through which the introduction per identity of all kind of imagined figures, that forms the core of Field´s arguments against Logicism, focusing specifically on the Ontological Argument of the existence of God. However, I do think that there is a difference here, to the extent that while in the Ontological Argument we require that through mere definition of existential possibility (by way of comparison with what there is) there must be such a Being, in the numerical case we are assumed to be able to grasp it in what is given to us. In this line also Wright (Wright and Hale,

how a difference in what each concept is supposed to refer to could be found. If we do follow such an explanation, we would say that the 1-1 correlation acts as a functional *operator* (similar to a multiplying one, for example) allowing us to derive a new product. But while a standard mathematical operation explains how in using it, we come to something new, here we would be doing something different. We are tracing a 1-1- correlation among the members of the extension of different concepts and are expected by virtue of it to grasp something new *there,* capturable through a new concept. But, in what sense is this extracting a higher order pattern *within thought*, as Dummett says, and not something more similar to the way a concept is extracted from a reality by finding something in common between two instances? It is usually explained that, if such a correlation holds, the new numerical concept acts as a second-order conceptual function applied to the first concepts (F and G). The new synthesis thereby created in each case is said to be the same 'number.' But I am not sure whether with it we really become aware of what is happening here and how the pattern is ultimately obtained. We are supposed to do this in view of the correlation on the right-hand side. So, let us try to be more specific. One could say, in accordance with Frege[26] that each of the members of the extension is turned into such through the concept that encloses it. It is through the concept of an apple that we sort out the unities of such. That is, the unities have been conceptually defined as such. So, it is upon two sets of such conceptually conformed unities, resulting, that is, out of a previous work of conceptualisation, that we are to find the correlation. The 1-1 correlation marks the conditions determining where attention should be directed. What he would be asking us to grasp is the common pattern in such groups of individuated conceptual apprehensions through a new conceptual synthesis. So just as a concept applied to a reality sorts out a unity, the concept of the group sorts out one too, a new unity upon already conceptualised ones, which would be the number.

   Connecting now to the reasons why the third functional empiricist option does not work for the empiricist, it becomes most clear what would be wrong. We have proceeded as though talking of a compilation of objects as a starting point would be no problem. We took a group of five apples or oranges as our point of

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The Reason Proper Study), according to which there would be in the numerical case, besides it, true instances of the application of the term. This brings us back, however, to the problem of what exactly are the application conditions here. If we adopt the functional-operational explanation, we would be assuming that there is a justifiable procedure that drives from one to the other. But defending this without the procedure delivering sufficient reasons, seems to me problematic on second thoughts.

[26] Frege, *Grundlangen der Arithmethik/Foundations of Arithmetic*, §46.

contact with experience. But as Frege was well aware of, we are in no way appealing to the experientable apples or oranges themselves. They can be quite different—big, small, red, green or with different shades of orange—but what we expect someone to grasp in this context is the fact that they are a given set 'of unities' of a sort; what they have been turned into by virtue of conceptual differentiation. That is what is relevant in this case. Their 'being unities' is something that we can just recognise as a result of conceptual work but not as something experientable or abstractable in itself.[27] This act of individuation is what makes it possible that no matter whether it is apples or half apples that we sort out, they can be equal when tracing a 1-1- correlation among the unities of corresponding extensions. Whatever we say about them in inferring from their being such a number of unities is necessary and certain independently of experience. About this, Frege was, of course, right.

The empiricist ambition, therefore, to get rid of abstract objects by reducing them to compilations of objects (when we actually mean their unities), starting from which we could then reconstruct functionally upwards what our numerical concepts refer to, is a fraud. But the importance this understanding of unities has in the whole Fregean enterprise is in my view greater than is commonly acknowledged, since it is upon this basis that the second-step proof for recognising the identity of numbers is built. While specific singular numbers would be based on this previous conceptual work, the notion of a unity allows two possible interpretations: a) the very idea of something being separated out through a concept, the content of it (the resulting unity); or, more in line with the procedure used with the singular numbers, b) the very act of synthesis done through the concept. If we follow Dummett in understanding how Frege's idea of analytic unities is to be understood, we should take it that he believes that here we must also do with a second-order synthesis upon concepts, since it would be not the conceptual synthesis itself but the second-order realisation of what is done in this

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[27] It is important to understand this properly, since the idea is not that the concept makes up the reality, in some version of the idea that reality itself should be seen as conceptual. The distinctions in reality must previously have been there and recognisable for us first, in order to introduce the concept. The point would actually hold up if we were to adopt a version of non-conceptual contents, since it would be the act of distinguishing (the making of a synthesis upon) whatever aspect (even if we should not be talking yet of an intersubjective linguistic normative concept) it is that already separates out a unity. Actually, this possibility, that we should have this prior capacity, is what Frege would be appealing to when asking us to be able to grasp a pattern there, since, even if we should be grasping in a second-order synthesis the result of our own conceptualisations, its recognition requires exactly the same capacity as that in the first-order one.

process. Therefore, what Frege would be grasping through the concept of a unity would be what different acts of conceptual synthesis have in common, thus option b), the very conceptual unification. This makes sense, since this is what we would recognise when 'turning logical form upon itself' in a second order synthesis, while remaining in the realm of logic. The repercussions this will have for Frege´s project goes, in my view, to the very heart of his difficulties.

When asked, then, to recognise in a second-order synthesis the pattern in the 1-1 correlation at the right-hand side of the equivalence, we would be capturing such conceptual unifications (in b) in a new all-embracing one. Representing thereby the common pattern between both sides of the correlation.

Frege is known to have found this procedure unsatisfactory as is expressed in what has become known as the Julius Caesar Problem, because if you come to try to introduce a new entity per identity, how can you know what it is that you are finding in common with the other side of the equivalence if you cannot already presuppose that it is the unities that you mean? The fact of there being a correlation might be a pure casualty and what is meant is something else present there. Frege's solution to the difficulty was to opt to provide us directly in an already explicit form that it is the extensions that were meant. A concept G would be equinumeral to a concept F if its extensions were equivalent: $(x)F(x) \rightarrow G(x)$. This change of terms struck me like a sudden jump achieving its goal by departing from the careful epistemic derivation he had accustomed us to, to deliver a ready-made product without an explanation of how we came to it. As Wright argues, he seems to have thought that since classes were already part of logic, this was legitimate.[28] But wouldn´t there be an issue too regarding how we came to such classes of individuals with their own identity as unities in the first place? However that might be, this proposal, known as Basic Law V, delivered Frege's final understanding of numbers as class extensions. These class extensions would have, nevertheless, been constituted by equal numbers of unities understood in the sense of b) above.

3. Unities and How They Interweave with Frege´s Difficulties

The problem that arises through the understanding of unities is entrenched in the very issues Frege[29] arrived at in his *Begriffschrift* with the discovery of variables as

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[28] Crispin Wright, *Frege's Conception of Numbers as Objects*, Scots Philosophical Monographs 2 (Aberdeen: Aberdeen University Press, 1983). Whether classes are part of logic is a disputed matter. See, for example, Wright, *Frege's Conception of Numbers as Objects*, 111.

[29] Gotlob Frege, *Begriffschrift und andere Aufsätze*, first published 1879 (Zurich: G. Olms Verlag, 2nd ed., 1964).

formal 'conditions' for objects. The idea was to be able to reconstruct the common formal structure of our thinking and talking about objects. Concepts were understood as incomplete functional expressions to which different (numbers of) objects could be assigned. This would allow, as he thought, a parallel treatment of numbers. Frege[30] considered his most significant insight to be the idea that, as he said, it was only relative to a concept that we can count—just if you consider 'Books on the table' you can say there are (a, b, c) (if we are to represent each book) or if you consider the 'Moons of Jupiter' (a, b, c, d, n). Therefore, he concludes that in attributing numbers what we are doing is ascribing a given set of *unities* to a concept. These unities, thus, are not the apples or oranges we are experiencing but rather what makes them unities of the sort independently of what they exactly are. When he describes what is being done through this process Frege tells us

> In the sentence: 'Jupiter has four Moons' the unity is 'Jupiter-Moon.' Under this concept fall the I as well as the II as well as the III, as well as the IV. That is why we can say: that the unity referred to by I is the same as the unity referred to by II, and so on. Here we have the Sameness. But when what we assert is the divergence of the unities, what we understand is that of the counted things.[31]

In using the Roman numerals, he marks the distinction between the objects and the unities, thereby stressing that it is only through the concept that we can come to consider the different objects falling under it as equal in their being unities, that, as such, we can count. That is, when we say that Jupiter has four Moons, what we ascribe is the same as we ascribe when we say that there are four Russian armies in Stalingrad; however different the armies or the Moons are, what we are ascribing is a given amount to the respective concepts.

We can express this, following Frege, in representing the Moons of Jupiter through corresponding unities—not the objects, of course, but placeholders of them, such as in 'Moons of Jupiter' $(()_1, ()_2, ()_3, ()_4, ()_n)$. Each would be individualised by the conceptual application and not independently of it. Frege dedicated some sections in *Grundlagen* to argue against others who claimed to obtain unities directly through an abstractionist process for getting rid of the

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[30] Frege, *Grundlanglen der Arithmethik/Foundations of Arithmetic*.

[31] Frege, *Grundlanglen der Arithmethik/Foundations of Arithmetic*, §42. I use a more literal translation of the original German edition, even if it might sound a bit awkward since I find more clear the way Frege expresses this thought there; marking the distinction between the objects and the unities in starker form than might be apparent in the English version. Of course, the English translation attempts to say the same and you can readit that way too. I just think the original one makes this relevant contrast for the point I want to stress more apparent.

particularities of an object. Through such a process, he argued, we would not end up with an abstract notion of unity, since being a unity is not something that we can somehow grasp in experience too (without the conceptual work) and then keep stripped of all other properties. It is in this sense that he rejected seeing numbers as sets of unities obtained per abstraction from reality. If we got rid of the experienced particularities of the reality, nothing would actually be left. Rather, in attributing unities to a concept we would be representing how many such conceptual individuations we can separate out. But, in this last sense, we do refer to what is common to them as such conceptual individuations, as explained in the previous section. If I can subsequently draw a correlation with some other concepts' unities, it will be precisely because as conceptually individuated ones they are the same.

The idea in *Begriffschrift* of representing the mere possibility of objects falling under a concept through conditions (again something like placeholders)[32] would allow us to make general claims. These conditions would be turned into realities when saturated by any real, corresponding individual. However, these placeholders (variables, in normal terms) were actually to count as 'numerical' (one place) unities differentiated through a concept too. The quantifiers, as their name implies, would then help to specify *how many* of such unities we are referring to, whether all of them or at least one, or whether we could talk about two of them falling under a concept without having to specify which determinate one it was. But here too we are talking about unities. We could say that three men crossed the road, without having to specify which particular ones they were. To the concept 'men crossing the road' three individuals could be assigned. Then we could try to see whether this was true, by finding as many corresponding particulars satisfying the predicate (no matter those originally meant or others, since in either case the claim would be true). The same goes for 'Jupiter has nine Moons.' I need not know which Moons these are to understand it, and if I happen to distinguish a corresponding number of them (even if completely different ones[33]) the claim would be true. Actually, this versatility is very important.

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[32] We can see an allusion to this in the following quotation: "This seen, we can also see the following possibility. Instead of linking our chain of deduction to any matter of fact, we can leave the fact where it is, while adopting its content in the form of a condition. By substituting in this way conditions for facts, throughout the whole of a train of reasoning we shall finally reduce it to a form in which a certain result is made dependent on a certain series of conditions…It is not impossible that the laws of number are of this sort. This would make them analytic judgments despite the fact that they would not be discovered by thought alone" (Frege, *Grundlangen der Arithmethik/Foundations of Arithmetic*, §23).

[33] Which is not unlikely, since they go up to as many as 79 now, according to NASA.

However, Frege´s final goal, as he himself says, was to set the conditions for referring to numbers,[34] to quantify over numbers. So, precisely those numbers that were going to end up being understood in Frege's work as classes of such (conceptually sorted out) unities, (capable of representing equally apples, oranges or whatever 'equinumeral' sets) were supposed to end up being seen as objects saturating those spaces. If we were to represent this to aid visualisation, we could picture it as follows:

1st We obtain possibilities of objects, variables, falling under a concept. Let us represent them as this

2nd These, we said, could be turned into realities when satisfied by experiential objects, such as here

3rd But, the idea was to come to see numbers (that are going to be understood as classes of unities, as explained) as objects saturating the spaces[35] of the variables. This is better seen if we first consider the separated *numerical* unities saturating the space of the variables, that amounts to something like this
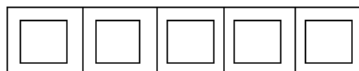
What this does, therefore, is to allow us to take 'possibilities of objects' (since that is what numerical unities[36] are when considering the pattern obtained through the concept in the sense of b), sec. 2, turned into objects themselves, as saturating the very same possibilities of objects represented by the variables; that

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[34] For example, here: "As I remarked at the beginning, arithmetic was the point of departure for the train of thought that led me to my ideography. And that is why I intend to apply it first of all to that science" (Frege, *Begriffsschrift und andere Aufsätze*, 6).

[35] This can be said of each numerical unity separately as well as for whole classes of them.

[36] These numerical unities can already be considered equivalent to number one, since they represent the pattern obtained by sorting out what is common to two such conceptualized unifications. Figure 1. below refers to their obtention.

is, to take possibilities as realities—what we might call a Parmenidean monstrosity!—since possibilities define themselves by not-being. Therefore, if we treat possibilities as realities, as real objects, what we get is that Not-Being Is! A paradox thus arises, right at the moment that we are feeding, so to speak, the space of a possible object with itself.

That is, since variables are actually sets of one possible unity, if you saturate them with themselves (a unity of the same empty sort, with no determinate reality) you are already saying that a corresponding set belongs to itself and taking this path you go directly to Russell´s paradox. I will expound this point some more later.

If we try to follow Frege's original line of thought, as developed in section 2., it required giving the obtained unities derived through conceptualization, and assigned to the corresponding concept, an identity as specific singular numbers. This, it seems to me, demands that we first establish an identity among unities in isolation,[37] a process through which we would obtain a synthesis of the conceptual unification in terms of b) above. This would be needed if we are to be able to identify what is it that is meant by the 1-1 correlation, since identifying the whole set of unities presupposes being able to identify the individual ones. Recognition of the whole would then be achieved through the equivalence relation between unities, through which one obtains corresponding empty entities (possibilities of classes of unities) that could be represented then shortened into the entity 5; since if the unities are conceptual, so is the unity of unities that the numerical concept introduces.

Figure 1.

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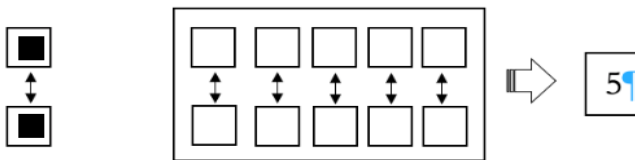
[37] This would seem to cohere with Frege's own thought process, which considered essential the process of identity to talk of numerical unities, as expressed for example here: "Is the dog conscious however dimly in that common element in two situations which we express by the word 'one'?" (Frege, *Grundlangen der Arithmethik/Foundations of Arithmetic*, §42).

On the left has been represented the identity of two conceptually sorted out unities, on the right the identity of the whole group. The patterns thereby obtained correspond in the case of the unity to the conceptual unification (in the sense of (b) sec.2.). The alternative possibility of having captured the content of the unity, as suggested in option (a) sec.2., is marked through the small black square. In the case of the identity of the group, the common pattern to both sides of the correlation, embracing all such previously identified conceptual unities, is represented in figure 2.

Figure 2

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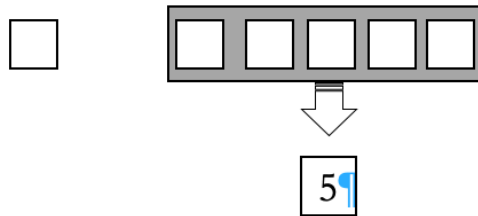
Now, repeating the procedure of taking singular numbers as objects saturating our variables, we are allowing the new corresponding entity, the specific singular number (in darker grey), to take the place of this possibility of class as though it were a reality

Figure 3
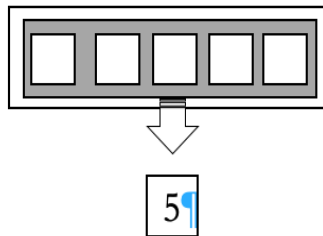
If this is finally understood, as Frege does through his Basic Law V, as a class of equinumeral extensions, and we repeat the procedure of taking singular numbers as objects saturating our variables, we come to obtain Russell´s paradox.

There are several things to comment here. To start with, it should be noticed that in this process the paradox actually presents itself twice: first at the level of

unities, as explained before—what we might call a 'First-Level Paradox;' and then, at the level of class extensions—a 'Second-Level Paradox.' Another aspect I wish to comment on is the step that I have included in Frege´s process requiring an identity for a unity first. It seems to me that the lack of one is related to the difficulties Frege found in Hume´s principle expressed in his Julius Caesar Problem. Maybe Frege initially believed it sufficient to have determined what ascriptions of unities to concepts amount to. But, identifying their 'unicity,' what is meant by such, as expressed in a second-order pattern dissected within the very conceptual synthesis issued by the specific concepts, seems to me not to be done with it yet.[38] Once we have already sorted out the pattern of the unity, it could have been argued that it is not the object or something else we mean, since in obtaining the pattern of the whole we are already working upon the extracted unity patterns.

In general terms, this reconstruction seems to me to allow one to see more clearly a deeper source of difficulties in Frege´s project than that usually considered to have doomed the project to failure from early on. But we have so far given no consideration to what a proper reconstruction would have to look like, and that is what I want to try next.

4. Flipping Things Around

Understanding the difficulties that Frege arrived at requires understanding the whole scale of what he had attempted to do, his whole understanding of logical form and how he thought the grasping of it took place. As we saw, Frege included in his understanding of logical form the conceptual conformation of objects and the necessary relations through which we reason about them in our linguistic discourse, equally capturable through a second-order conceptual synthesis. So, if at a first level we were to conceptualise the world linguistically and establish such connections, at a second one we would capture the structure of this very work of conceptualisation, the formal unification introduced by our concepts and the implications traced among them, by way of new concepts. Remaining all the way through within the limits of the formal domain. This last step is to be differentiated from the idea of capturing, through a higher-order concept, an already conceptually understood reality. It is more like capturing the very act of capturing, the unifying synthesis thereby taking place.

The way Frege understands what is achieved through this second-order conceptualisation is essential to his project. Therefore, I wish to consider first, how

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[38] Consider the previous footnote.

the route of interpretation he took affects the understanding of the objects he was after, and where the alternative would have led. And secondly, I will focus on the understanding of the logical connections at the first-order level and the idea of a second-order conceptualisation of their patterns.

4.1 The Objects We Were After

Frege's central idea—that it is relative to concepts that we can count—together with his realisation that we could represent the role of objects through corresponding 'conditions,' were the two important moves that connected his 'objectual' reconstruction of logical form with the arithmetical understanding of number. But, although what he finally represented as the numerical unities was the very form of the unification done through the concept, and thus the concept of the unity, his intention was to differentiate between the logical unities (the variables) and the rightly so-called numerical ones. The second were supposed to be the objects of the former (as also, later, the specific numbers). Therefore, I think that while, among the options drawn at the end of section 2, he was actually aiming at a), the resulting content of the unification, what he ended up representing was b) the concept of the unity. This was not without reasons, of course; it was all that he could allow himself, if he was to remain within the boundaries of the formal framework, he had set himself. He thought he could do well enough with the formality of the unity, since the alternative in a) went beyond the logico-linguistic realm representing the result of the synthesis done by the concept into something *other*. Probably a key aspect of it was that he was paying less attention to the notion of the unity he was arriving at and more to the possibility of obtaining the pattern of the whole conceptual extension through a second-order synthesis, which he hoped could achieve his aim. But since this second synthesis was done on behalf of what were already conceptual ones, what he obtained was again of a conceptual, formal nature.

What would the alternative have looked like? Where would route a) have taken us? What are the numbers we would have arrived at and how are they to be understood? I think that the metaphor of 'figuring out' what it is that we are doing when separating out a unity through a concept into some background extension, into *something other* to itself, into a virtual representation of reality itself, is more than a metaphor. I think the only way to make sense of this is through a representation that requires both sides: what the concept does, and its counterpart, what it does it upon. In this way we can gain not just the concept of a unity but what it is a concept of. This way of putting things brings us back to something akin to Kantian transcendental Intuition.

Actually, if we are to properly comprehend what it is that we are doing through conceptual application we have to represent not just the concept but the whole background extension of conceptual activity, since differentiating something from something else always requires some extension where the severed out remains in a different position at some level other than that from which it detaches itself. An illustration of the process might be helpful to see better,

Figure 4.

Although conceptual demarcations normally take place in reality, pre-existent differences being necessary for us to discriminate conceptually upon, to understand what is it that we do in conceptual use we must represent it to ourselves, such as in Figure 4. Since we cannot really be said to *perceive* the result of our own conceptual labour, such representation enables us to get a proper grasp of it. This way we can separate out what is taking the place of reality, the extension in the horizontal line, and what is done through conceptualisation in separating out a unity by way of using a concept. On the basis of such a representation, we can then better understand what we mean by talk of 'being' (being real), 'being something,' being 'an object' from the perspective of a concept; we can differentiate what becomes a real unity (symbolised by the 1 in Figure 4.), a Moon of Jupiter or whatever, and what is not (symbolised by the 0).

The importance of representing the conceptualisation in its context, is that through it we can also realise that what remains outside the conceptualized, 0, has its own part in the process, since on the one hand it makes 1 actually possible as a unity (otherwise both would be the same undifferentiated whole). However, at the same time it is cut off as 'not being,' that is, not being there to be counted in the Fregean conceptual sense, and is, therefore, literally 0. Now, through this process 0 becomes 'some kind of unity' too, through being separated out, but one that 'is-not' from a countable perspective. Thus, it exists, but not in any linguistically accountable sense: it is nothing. Mystifying as it might sound, this represents I believe, the *noumenons*, *non-being*s, *limits of language* and *silences* of our literature.

If this interpretation is right, then maybe the insight to be gained is that 0 is not to be defined as the class of all objects that are not identical to themselves, as Frege said, but rather as 'what remains outside of any class,'[39] since if counting, as well as in the representation we can make of it through Imagination,[40] is always counting from a perspective (virtually *ad infinitum*) 0 is always what can never be included. Also, if we were to count all there is, *being*, since this requires a perspective too, it must be conceptually detached, leaving something outside—the consequence thereof being, which can hardly be news, that no account of what there is can ever be complete.

A further consequence of this perspective is that it would allow to answer problems such as those relative to the truth of negative sentences that interested Russell. Since in any complete account of the individuals existent in a world, given by the conjunctive set of conceptually individuated ones, 0 would always have to be taken to exist. Therefore, negative sentences would be true, because now we must say that there is something, 0, that is somehow there too. But how do we explain a sentence such as "there is no *rhinoceros* in the room"? Since that doesn´t just say that there is nothing in the room, but what kind of individual there-is-not. In a sense, it is absurd to say that "that nothing" that exists in the room is of one kind rather than another. But we do want to speak that way, that there is nothing relative to a specific concept and in that sense to speak of *possibilities* of this and that, that do not exist in this world. So maybe we could also represent such non-existence, but we have to differentiate such representations from those of existence. So, on the one hand they would be forms of 0, but relative to a concept, giving rise to corresponding unities of such sort, they would 'exist' in the negative way of 0, we could represent them as the result of conceptualising in that realm. That is, they would correspond to the negative numbers. This are, of course, tentative approaches, but I think, they are worth considering.

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[39] Someone might wonder how we then explain the existence of empty classes, if we cannot say that there are classes with 0 members. This doubt was expressed by Peter Simons at a Conference on *The Philosophy of Logical Atomism 1918-2018* at the Complutense University in Madrid on the 28.01.2019. My answer would be the following: saying that an empty set has one peculiar member not identical with itself which happens not to exist is not accurate, since it would be like saying its content is a Meinongian figure. By this account, on the contrary, we are saying precisely that what corresponds to such a class is non-existence (not as a mere modal issue). It captures no reality. We can write 0, but we need not say that it is a peculiar impossible entity.

[40] Since 'Imagination' is meant in a sense akin to the Kantian notion of Pure Transcendental Imagination I will use it with a capital 'I' to make this more explicit.

4.2 The Logical Rules

Going now back to the understanding of logical rules in Frege's picture, and actually in Logical Empiricism more generally, we distinguished at the first level conceptualisations and inferential relations whose patterns were likely to be reconstructed conceptually again at a higher, second-order level. A first question, then, is how the inferential relations themselves, at the first level, are justified in this picture. Frege explains how we might recognise and conceptualise new patterns in our linguistic discourse, but how these are introduced in the first place, why we put them forward with inferential necessity, is not explained. It is also insufficiently explained in the conventionalist picture, as we briefly saw in discussing Formalism. As Coffa[41] argues, the solution given by Wittgenstein and Carnap to the problem of the a priori was to turn things around: instead of saying that we grasp the meaning of logical constants or geometrical undefinable terms through some form of intuition (or Intuition in the geometrical case), deriving then from them further axioms and a priori truths, it is the methods of measuring and those axioms themselves, or the logical rules in the case of the constants, that determine meaning, this being the reason why their truth struck us as necessary. But this just delays the question, since our problem now is how we come to those rules that determine the constitution of meaning, how we derive their necessity if it is not to be seen as conventional. We have singled out two models: a) meaning determines rules and requires intuition (or Intuition); b) rules determine meaning and the necessity of rules is therefore seen as unquestionably presumed, unless we reject meaning as a whole. This, Coffa argues, was initially shown for the field of Geometry by Poincaré and Gilbert, for whom the measurements and axioms of Geometry defined primitive notions such as 'distance,' and not the other way around. But then we are driven back to the question of the origin of those rules that define the semantic primitives.[42]

My proposal now is to see how this problem turns out from the perspective we have adopted, if we were to accept that understanding the very idea of unities, and therefore the very notion of measurement units, requires representation of what it is to obtain a unity (in general) through conceptualisation in some virtual representation of the extended context upon which it takes place. Then, whatever conclusions we might derive in thinking about them would apply to whichever

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[41] Coffa, *The Semantic Tradition from Kant to Carnap*, Ch.14.

[42] According to Coffa, Wittgenstein was aware of this problem and thought that 'grammar' itself could not be regarded as conventional; there was a way grammar should be, but no justification could be given not requiring a justification itself in an infinite regress.

units and conceptual applications we were talking about. Again, visualising might be helpful,

Figure 5.
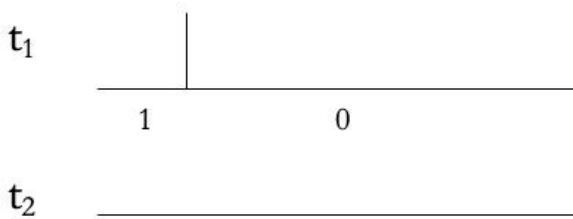
If we depart from Figure 4 above (t_1 in Figure 5), once we have this representation of what severing a unity from its background amounts to, we can come to a few further conclusions. We can, for example, come to realise that if the conceptual detachment did not take place, the unity, 1, gained thereby would not be a unity at all. This necessary conclusion is not gained through some mysterious capacity of intuition, but simply because in modifying things through Imagination in some extension akin to Kantian Intuition, we can come to *see* it, as pictured in the transition from t_1 to t_2 in Figure 5 where the vertical line is taken away. We literally see that the very existence of the generated unity is only the case through its being differentiated from what remains outside of it; were it not so, it doesn´t exist as a unity. Since this represents any unity whatsoever, as we said, it applies generally and therefore necessarily in all cases. In other words, this allows the *introduction* of a necessary connection $Not\ 0\ \rightarrow\ Not\ 1$. Starting from here, we can also come to conclude that for this unity, 1, to be itself, the unity we might identify with the singular term '1,' it cannot be whatever other (conceptualised or not) it leaves outside, call it 'Not 1,' in this case 0. Otherwise, as Frege in his own context puts it, we go back to an undifferentiated whole. We see, then, how through this very rudimentary process we can come to a first law of identity, expressing exactly that $'1' \rightarrow Not\ ('Not\ 1')$, equivalent in more standard terms to this other $\alpha \rightarrow \neg(\neg\alpha)$.[43] These simple relations hold from the very fact of something being differentiated as a unity and, since this is what characterises any object

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[43] Notice that while with the numbers 1 and 0 we move at the ontological level of unities, by using names for them we are identifying them linguistically. The same goes for the representation in terms of α and $\neg\alpha$.

conformation whatsoever, it will necessarily apply to any relation among objects we happen to consider.

However simple this reconstruction might seem, I think it stands, and coheres, for example, with some counterfactual reconstructions of necessity. The idea is that the capacity to Imaginatively represent general epistemic contexts in 'Intuition' in order to keep them fixed and allow them to be modified easily (while keeping the different moments of the transition present, moving back and forth between them)[44] allows us to understand how the most basic logical rules themselves come to stand. Showing that they are not primitive and, in that sense, not a priori given in our cognitive equipment but developed—although we should be taking them, once acquired, as a priori justified for further uses. We can study what is or is not possible by virtue of what actually happens when introducing such modifications and advance on that basis what is *necessary*. Since here what we are reconstructing, as explained, is the *constitution* of inferential relations affecting any unity on the grounds of being such, we have to do with a general idea. That is, it is not because other unities should be similar to this one that we are allowed to make a generalising inference, requiring an explanation of a supposedly pre-existent capacity of so inferring. It is rather that the represented unity is instantiated in any occurrent one; and so, the question—how do you know that you can infer from this case to all others?—does not pose itself. It would be tantamount to asking: how do you know that what happens to this, happens to this? Notice too, that advancing the first inference, its necessity, is simply a matter of acknowledging, as a matter of fact, that this 'unity' ceases to be what it is if some conditions are removed. Since, again, we have to do with a general claim, this will be so in all cases, and thus we can advance an inferential claim.

Returning now to the previous conventionalist idea, by picturing the basics of arithmetic in these terms we would be explaining the constitution of those very measurement units and the further necessary conclusions we advance on their basis whereupon meanings can then be said to be built. We would thus be delivering a deeper *constitutive* account of the a priori. This would not deliver a justification of the laws then requiring, in an infinite regress, a further justification

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[44] This is an essential feature in Kant (*Critique of Pure Reason*, KrV A101-102, 230) and I think an essential one in any reconstruction we are to give of such a background extension. As he points out in the paragraph 'On the Synthesis of Reproduction in Imagination:' "Now it is obvious that if I draw a line in thought or think of the time from one noon to the next, or even want to represent a certain number to myself, I must necessarily first grasp one of these manifold representations after another in my thoughts. But if I were always to lose the preceding representations (…) then no whole representation (…) could ever arise."

for it (as Wittgenstein argued), but rather simply depicts their very constitution. The difference with the Kantian picture is that here the constitution of logic too comes out as synthetic a priori. That is, if it is right, as argued, that representation in 'Intuition' is required in order to arrive at the notion of necessity and the necessity of identity, then the conclusion to be drawn is that not just mathematics but also logic is synthetic a priori. Furthermore, since inferential connections emerge as *derived* from more basic distinctions (Figure 4), we appear to come closer to a computational picture of human's most rudimentary cognitive capacities.

The contrast between this picture and a traditional reconstruction of the undefinable concepts of geometry, for example, as synthetic a priori, on the other hand, is that in putting arithmetic and thereupon derived logical laws as more basic first, they are no longer primitives, but could, nevertheless, be explained further in the conventionalist way. All these results are put down with care for the weight I know they carry, but I want to put them down for further reflection. What is clear is that whatever this might otherwise imply, this way of looking at things definitively turns things around for Frege´s project.

One last point I must return to is the idea I have been putting forward that Frege actually reintroduces the idea of the synthetic a priori even if he claims to do away with any recourse to anything like experience or Intuition. This relates to the role of the second-order synthesis of deductive patterns 'within thought,' as Dummett puts it. From Frege´s perspective, since it all takes place in the logic-linguistic realm they can safely be regarded as analytic. The idea of there being such second-order patterns of reasoning procedures seems to me perfectly fine. But the question is, whether understanding the necessity of such reasoning procedures, grasping their pattern, is at all possible without going all the way down to their application in the first-order realm? Whether we could make any sense of them without figuring out, as we have argued, what their application in some counterpart extension amounts to? Think of it this way. Take the three models of *a priori* necessity considered: 1) meaning determines rules; that is why, in knowing the meaning of a term, we can immediately (a priori) see that the predicate belongs to the understanding of the subject; 2) conventionalist, model, we said that it is actually because the rules themselves determine meaning that this is the case. Frege would actually say something along these second lines, since it is in grasping new patterns of such rules that we come to a new synthesis. But if we now, 3) constitutive model, ask ourselves how such rules themselves come to stand, we conclude that it is through such a representation of what being an object[45] amounts

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[45] Of course, the idea of an object is meant here in much more general sense than the usual one.

to that we can derive some further necessary relations with other objects. So, we end up building the different models upon each other (in inverse order), with the ultimate one giving sense to the necessity of the others. And this, in a way that cannot be dismissed as being merely the triggering origin that says nothing to an a priori knowledge we would grasp as having been always necessarily there,[46] since it is the very grounds for why such rules are introduced as certain at all. Would we not then have rather to say that the (supposedly analytic) synthesis built thereupon, is ultimately understandable on this basis? Is Frege then, inadvertedly, ultimately appealing to that which he had wanted to ban, that is, the synthetic a priori in a more traditional sense? On the other hand, there is also an issue with the idea itself of a second synthesis obtaining *a pattern* required for conceptualisation, since the very act of doing so and not deriving it from experience is precisely what Kant characterises as a synthesis a priori. From the Kantian perspective it would not be that we grasp it, but that we can get it because we ourselves *do* it.

5. But Then, What Are Numbers?

Well, before we make ourselves a picture of the singular numbers, if doing so requires prior identification of the unities that conform their extensions, we must first ask what unities are. The answer would not be the concepts of unities, as it ended up being for Frege, but rather what we can represent as the result of applying concepts; something like shadowy representations of real unities through Imagination. This is what Frege himself realised at the end of his life and what Kant said. But these are no realities, for reality requires still something further: corresponding unifications in experience, which result from the application of specific concepts to specific experience.

Frege´s difficulties arise if we understand his proposal in terms of b), as the concept of the unity, and then understand them as objects saturating variables. Doing so gave us the First-Level Paradox, a one-possible unity inside another, representable through a unitary set with another unitary subset as its object, such as this: $U = ((x))$. The initial unity now contains the numerical (conceptual) unity, *and* its occupiable space as its subset. The new empty unity in the subset could be saturated again and again by another and another one-place unity, giving rise to unities with increasing members, and the question is whether this recursive process is not paradoxical in itself. However, I believe the most important problem

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[46] Frege argues that the fact that experience (or here Intuition) might be the source is irrelevant for the justification of the claims. This is true once the claims are already obtained, of course.

is that it raises a version of Russell´s paradox, which we might call the Paradox of the Unity (or unitary set) since the question would be whether the set of all (unitary) should or should not include itself as a member. If it includes itself as a member, can it still be considered the set of all? Since as a member it is not the set of all anymore unless it includes itself again (and then again recursively), it cannot really include itself without ceasing to be what it is; however, if it does not include itself, it is no longer the set of all. If we have a set with one possible individual as a member, and we allow number one (understood as a set of one possible member) to be a possible substitution instance, we are doing exactly what we have described: allowing our set to include itself, with the consequence of sabotaging its very possibility of being the set of all possible one individuals anymore, since as an instance it is not. But if we do not allow number one as an instance, it could not be the set of all possible one individuum either (since number one is to be considered one such).

By considering numerical unities now as Imaginative representations of the real individuals obtained by conceptual application, we are simply representing the results of conceptualisation. So, the situation is a different one: numerical unities are then being used as mere (non-saturable) *representatives* of individuated realities in order to figure out things about relations among them. Being representatives not of *factive* reality but of what individual reality *is like*, they therefore need not constrain themselves to a real number of individuals but can exceed this with ease (ad infinitum). Since such representatives are not sets, no paradox applies.

One last question I wish to address is the status of such Imaginative representations. I have not made it clear whether I am referring to a mental representation or whether we are talking of possible intersubjective representations. In Kant´s own transcendental philosophy, the point was that such synthetic a priori knowledge is not obtained per exceptional capacities of direct intuition of something going beyond the realm of experience. This view amounts to reopening the door to transcendent metaphysics with all its potentially intuitable creatures. Rather, it is when we try to reconstruct how it is possible that we arrive at something not derived from experience, if not by such means that we try to represent to ourselves how this must take place for things to be how they are for us. I think this is right in the sense that, as previously said, we cannot claim to epistemically perceive our own conceptual conformation of unities, for example, but rather come to grasp what is it that *we do* when representing this very performance to ourselves. But the question is how this exactly takes place. In the Kantian model, it is actually in abstracting the experiential (in the sense of

stripping it away) from our recognition of already singled-out objects that we come to realise that there is something that we are taking for granted about them, but which cannot be said to have been experienced or grasped by special faculties. In this way too, in Kant's view, we would come to the consciousness of pure spatiotemporal Intuition. It too, is understood ultimately as a form of a priori synthesis (remitting to the a priori synthesis of apperception). We would realise that in our very ordering of impressions in our keeping of their transition present, there is also something, not in itself experientable but which is necessary for our knowledge of experience to be possible at all. The contemporary critic would see a problem in the mentalistic aspect of such a reconstruction, but things could be put the other way around. We can come to represent in intersubjective sharing ways, virtual computer representations, architectonic schemes or pictures, formal universal aspects of our reality, because we must be capable of schematising them in some such form. We can all agree in view of such shared depicted representations, but it must be because we can recognise something there, the same structure that enables us to come personally to depict it in such a way in the first place. I am afraid there is much too discuss on this point, so I will have to leave it here.

There are many other issues open for further thought. One, for example, is to what extent the proposal put forward is coincident with positions such as those of Brouwer or Dummett, who defend understanding numbers as synthetic a priori too. This is something I still have to think about. I have intentionally remained relatively neutral about how the background extension is to be understood, in order to reserve myself the right to think further on the extent to which I share the Kantian picture as a whole and make up my mind on such matters. Another issue is whether the position outlined implies that numbers are constructed or not, or whether the possibility of such representations reproducing themselves ad infinitum is enough to consider them given—which actually seems a plausible option, since Wittgenstein´s 'Rules as Rails' picture, which in the standard conceptual case fails (for the need of human assessment to determine further conceptual application), appears perfectly unproblematic in this one. Finally, the initial question of how we are to come to the singular number from here, although I think it enables a reconstruction in some such representative terms, is left for a further occasion too.

CASULLO ON EXPERIENTIAL JUSTIFICATION

R.M. FARLEY

ABSTRACT: In *A Priori Justification,* Albert Casullo argues that extant attempts to explicate experiential justification—by stipulation, introspection, conceptual analysis, thought experimentation, and/or appeal to intuitions about hypothetical cases—are unsuccessful. He draws the following conclusion: "armchair methods" such as these are inadequate to the task. Instead, empirical methods should be used to investigate the distinction between experiential and non-experiential justification and to address questions concerning the nature, extent, and existence of the a priori. In this essay, I show that Casullo has not refuted armchair explications of experiential justification, in particular those that appeal to introspectively accessible phenomenology. I do this by presenting a phenomenal theory of experiential justification that (a) has a significant degree of initial plausibility and (b) survives Casullo's general attack on such theories. As a result, a premise in the central argument for Casullo's signature proposal concerning the a priori is undermined.

KEYWORDS: rationalism, a priori justification, experiential justification, armchair philosophy, conceptual analysis, phenomenal properties, Albert Casullo

1. Introduction

Are any beliefs justified a priori? This central epistemological question is often thought to reduce to the question of whether any beliefs are non-experientially justified. To answer the latter question, however, it seems that we must be able to distinguish non-experiential justification from its experiential counterpart. And to do that, we need an explication of experiential justification.

In the important book *A Priori Justification*, Albert Casullo argues that extant attempts to explicate experiential justification—whether by stipulation, introspection, conceptual analysis, thought experimentation, and/or appeal to intuitions about hypothetical cases—are unsuccessful.[1] He draws the following pessimistic conclusion: these methods (hereafter "armchair methods") just aren't suitable for the task. Instead, Casullo claims, *empirical* methods should be used to investigate the distinction between experiential and non-experiential justification

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[1] Albert Casullo, *A Priori Justification* (Oxford: Oxford University Press, 2003), 147-185.

and to address questions concerning the nature, extent, and/or existence of a priori justification. He writes:

> I argue that [no extant proposals for articulating the relevant concept of experience by use of armchair methods] succeeds, and I propose viewing "experience" as a putative natural kind term whose extension is fixed by reference to the cognitive processes associated with the five senses. Whether those processes have important common properties, and, if so, what they are, are questions to be settled by empirical studies of human cognition. In short, uncovering the nature of experience is a matter for empirical, rather than a priori, investigation.[2]

It follows that, since the a priori/a posteriori distinction rests upon an account of the nature of experience (in the relevant sense), if the latter is a matter for largely or exclusively empirical investigation, then the same should hold true for the a priori/a posteriori distinction.[3]

The preceding argument can be reconstructed as an instance of modus ponens:

> (1) If armchair methods cannot provide an adequate account of experiential justification, then we should largely or exclusively employ empirical methods to investigate the a priori/a posteriori distinction.
>
> (2) Armchair methods cannot provide an adequate account of experiential justification.
>
> (3) Thus, we should largely or exclusively employ empirical methods to investigate the a priori/a posteriori distinction.

In what follows, I rebut premise (2) of Casullo's argument. I do this by showing that he has not decisively refuted explications of experiential justification that appeal to introspectively accessible phenomenology (hereafter "phenomenal theories"). It should be emphasized that Casullo's signature contribution to contemporary literature on the a priori is his development and defense of (3). Thus,

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[2] Casullo, *A Priori Justification*, 148.

[3] Casullo's writings suggest, at times, that he thinks the a priori, construed as a general research topic, should *only* be investigated using empirical methods. But I'm not sure whether the writings that I discuss here imply methodological exclusivism. This depends, in part, on (a) how the a priori *qua* research topic is understood and (b) how empirical methods are distinguished from armchair methods. And this brief essay is no place to take up these difficult questions. For this reason, I formulate Casullo's first premise using the disjunction "largely or exclusively." Note, too, that even the non-exclusivist conclusion that we should *largely* employ empirical methods to investigate the a priori is both novel and provocative.

if, as I contend, the primary argument Casullo deploys in support of (3) has a false premise, this is a significant result.

I proceed by three steps. First, I propose a *prima facie* plausible phenomenal theory. Second, I introduce Casullo's critique of phenomenal theories and show that the theory I propose survives it. Third, I consider an alternative way of interpreting Casullo's critique of phenomenal theories. I then argue that it too fails. As a result, a central plank of Casullo's platform in *A Priori Justification*—namely, a premise of the primary argument for his most original and provocative claim about the a priori—is undermined.

2. A Phenomenal Theory of Experiential Justification

In this section, I put forward an account of experiential justification that is based upon the role that introspectively accessible phenomenological properties play in securing justified belief. This account has a significant degree of initial plausibility insofar as it (a) captures and articulates a notion of experiential justification that is common to epistemological discourse and (b) yields a promising distinction between a priori and a posteriori justifications that correctly classifies most of the paradigm cases of each. Although I do not claim that my account is beyond correction, I am confident that it is not vulnerable to Casullo's attack on phenomenal theories of experiential justification.

I begin with a stipulative definition of experience:

(EXP): For any mental state *m*, *m* is an experiential state *iff m* has phenomenal character.[4]

David Chalmers describes the phenomenal character of an experience as "what it is like to have that experience."[5] He continues: "Two perceptual experiences share their phenomenal character if what it is like to have one is the same as what it is like to have the other. We can say that in such a case, the

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[4] Although (EXP) is introduced as a stipulative definition, it can be justified by reflection on paradigm experiential states, such as those associated with the five standard sensory modalities. For the sake of expository economy, I leave this task to the reader. I should note, in addition, that something close to this definition is *de rigor* among philosophers who work on knowledge, mind, and perception. For example, in *Stanford Encyclopedia of Philosophy*, Siegel writes "It is definitional of experiences… that they have some phenomenal character, or more briefly, some phenomenology." See: Susanna Siegel, "The Contents of Perception," in *Stanford Encyclopedia of Philosophy*, ed. Edward N. Zalta, URL = https://plato.stanford.edu/entries/ perception-contents/, 2016.

[5] David Chalmers, "Perception and the Fall from Eden," in *Perceptual Experience*, ed. Tamar Gendler and John Hawthorne (Oxford: Oxford University Press, 2006), 50.

experiences instantiate the same phenomenal properties."[6] Phenomenal properties are, in turn, qualitative properties like redness, painfulness, and sweetness. They are constitutive of what it's like to be in a particular mental state.[7]

Although it's tempting to characterize experiential justification as simply "justification that is provided by experiential states," this proposal should be rejected. For there can be cases in which an experiential state justifies a belief but does so independently of its phenomenal character. Suppose, for instance, that whenever you grasp a basic arithmetical truth, the mental state through which you grasp this truth presents it to you as a sentence token constructed from bright green characters. You are thus aware of an accompanying phenomenal greenness whenever you grasp that $28 + 12 = 40$. From your own first-person point of view, your grasp of the arithmetical proposition is concurrent with your apprehension of phenomenal greenness; indeed, both are constitutive of a single mental state. Since the state in question has phenomenal character, it's an experiential state. Even so, since its color phenomenology is unrelated to arithmetical truth, it's quite unlikely that this state provides *experiential* justification for your belief that $28 + 12 = 40$.[8]

Given the preceding example, we should conclude that an experience *e*'s phenomenal character must play a role in its justifying belief *b* when the justification it provides (with respect to *b*) is experiential. Here is a more precise rendering of this proposal:

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[6] Chalmers, "Perception," 50.

[7] There are multiple mental state types that have phenomenal character, including visual states, auditory states, tactile states, gustatory states, olfactory states, emotional states, memorial states, and imagination states. Some philosophers have proposed that other common mental state types, such as occurrent beliefs and desires, have phenomenal character. See, for instance: David Chalmers, *The Conscious Mind*, (Oxford: Oxford University Press, 1996); H.H. Price, "Some Considerations about Belief," *Proceedings of the Aristotelian Society* 35 (1934–1935): 229-252; Galen Strawson, *Mental Reality (second edition)* (Cambridge, MA: MIT Press, 2010). Michael Tye and other representationalists deny that the states in question have phenomenal character; see, for reference, Tye's *Ten Problems of Consciousness*, (Cambridge, MA: MIT Press, 1995). I take no definite stance on the matter. Nevertheless, it is clear enough that (EXP) need not yield the dubious classificatory result that *all* mental states are experiences. Unconscious, subconscious, and dispositional mental states are, presumably, bereft of phenomenal character and thus are not classified as experiential states by (EXP).

[8] In his memoir, Daniel Tammet, a synesthetic mathematical savant, reports "seeing numbers as shapes, colors, textures, and motions." This gives my example some purchase. Since Tammet reports that the number five sounds like a thunder clap, his thoughts about that number are, presumably, experiences. Yet, surely, when he thinks about the sum of five and five the accompanying phenomenal character does not play a role in justifying his belief that it is ten. For reference see: Daniel Tammet, *Born on a Blue Day* (New York: The Free Press, 2006), 2.

(EJ): For any mental state *m* and any belief *b*, *m* is an experiential justifier for *b* *iff* *m* is an experiential state that justifies *b* in virtue of its phenomenal character. If *m* is an experiential justifier for *b*, then *m* can provide experiential justification for *b*.

To illustrate, suppose Sven has a visual experience as of a black cat on the banister. His experience (a) instantiates the property of phenomenal blackness and in virtue of doing so (b) provides *prima facie* experiential justification for the belief that something black is on the banister. For if something black were on the banister, then, were Sven to look toward the banister, he'd have an experience with just that sort of phenomenal character.

The in-virtue-of relation that is central to (EJ) can be unpacked in multiple ways. The task of specifying and endorsing any particular way—i.e., the project of developing a general theory of experiential justification—is beyond the scope of this essay.[9] I therefore leave the in-virtue-of relation at the intuitive level. For present purposes, the key point is that *m* provides experiential justification for *b* *iff* *m*'s power to justify *b* depends upon its phenomenal character.

One strong reason to endorse (EJ) is that it yields, as should any satisfactory account of experiential justification, an illuminating and sensible characterization of the a priori/a posteriori distinction. To see this, consider the following rather uncontroversial definitions of a posteriori and a priori justification:

A Posteriori Justification: For any belief *b*, *b* is justified a posteriori *iff* (i) *b* is immediately experientially justified or (ii) *b* is justifiably inferred from beliefs at least one of which is immediately experientially justified.[10]

A Priori Justification: For any belief *b*, *b* is justified a priori *iff* (i) *b* is immediately but non-experientially justified or (ii) *b* is justifiably inferred from beliefs all of which are immediately but non-experientially justified.

When combined with (EJ), the above definitions yield:

A Posteriori Justification₂: For any belief *b* and justifier *j*, *b* is justified a posteriori *iff* (i) *b* is immediately justified by *j* in virtue of *j*'s phenomenal character or (ii)

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[9] To be clear, the project of developing a general theory of experiential justification includes the project of explaining, in detail, *how* perception justifies belief. This is one of epistemology's central explanatory aims. It should go without saying that I cannot offer a comprehensive discussion of this topic in a short essay with a rather different aim; i.e., developing and defending an account of what experiential justification *is* (rather than explaining how and why experiential justifiers are justificatory.)

[10] The concept of immediate justification is here understood as justification that is non-inferential, where non-inferential justification is justification that does not derive from a subject's doxastic states.

can be justifiably inferred from at least one belief that is so justified.

A Priori Justification₂: For any belief *b* and justifier *j*, *b* is justified a priori *iff* (i) *b* is immediately justified by *j* in virtue of something other than *j*'s phenomenal character or (ii) is justifiably inferred from beliefs all of which are so justified.

Now, suppose it visually appears to Bill that there's red bird in the tree. It's reasonable to think that (a) this visual appearance justifies Bill in believing that there's a red bird in the tree and (b) the justification it provides, in this instance, is a posteriori. Indeed, this appears to be a paradigm case of a posteriori justification. Given (a), **A Posteriori Justification₂** predicts and explains (b). This is because Bill's visual appearance wouldn't be able to justify the relevant belief if it didn't instantiate the property of phenomenal redness. Thus, the visual appearance's capacity to justify Bill's belief depends upon its phenomenal character. The justification it provides is thereby a posteriori.

Now suppose Abby has the intuition that everything that has a shape has a size. It's reasonable to think that (a) this intuition justifies Abby in believing that everything that has a shape has a size and (b) the justification it provides in this instance is a priori.[11] Indeed, this appears to be a paradigm case of a priori justification. Given (a), **A Priori Justification₂** predicts and explains (b). This is because, even if we assume Abby's intuition has phenomenal character (including the kind of phenomenal character that would enable her to introspectively identify it as an intuition) it would still justify the belief in question if it had a *different* phenomenal character (or no phenomenal character at all).[12] Indeed, we can vary the intuition's phenomenal character without modifying its content and, by extension, its justificatory powers.

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[11] I hold that intuitions are intellectual seemings. One influential characterization of intellectual seemings is found in: George Bealer, "A Theory of the A Priori," *Pacific Philosophical Quarterly* 81 (2000):1–30. Another is found in: Michael Huemer, *Ethical Intuitionism* (New York: Palgrave Macmillan, 2005). A more recent account is found in Elijah Chudnoff, *Intuition* (Oxford: Oxford University Press, 2013). For present purposes, I withhold judgment on the matter of which characterization of intellectual seemings is the most promising.

[12] Assume that intuitions have a certain kind of phenomenal character that enables a subject to introspectively identify them as intuitions. Even so, in ordinary circumstances, one suspects that intuitions justify beliefs without being introspectively identified as intuitions. For instance, Abby's intuition justifies the belief in question even when she doesn't engage in any active reflection of the sort that would yield judgments like "I am having the intuition that δ" or "This thought about δ is an intuition." For accounts of intuitions that make similar assumptions about their phenomenal character, see: Laurence Bonjour, *In Defense of Pure Reason* (Cambridge: Cambridge University Press, 1998), 100-110 and Alvin Plantinga, *Warrant and Proper Function* (Oxford: Oxford University Press, 1993), 105-106.

To see this, suppose that when Abby has the intuition in question she also becomes aware of a buzzing sound. Indeed, her awareness of the sound is partly constitutive of her intuition that everything that has a shape has a size. Even so, if the auditory properties of Abby's intuition were altogether absent, it would still have the power to justify her belief that everything that has a shape has a size. In this instance, then, the intuition's capacity to justify Abby's belief is independent of its phenomenal character. The justification it provides is therefore a priori. This result, together with the one in the paragraph before last, shows that (EJ) can be used to fix the a priori/a posteriori distinction in a way that is consistent with paradigm cases of a priori and a posteriori justified belief. And, contra Casullo, (EJ) is the product of and is supported by armchair methods; e.g., thought experimentation and reflection on hypothetical cases.

3. Casullo's Challenge to Phenomenal Theories

Casullo maintains that phenomenal theories of experiential justification are uniformly inadequate. He writes:

> … for the [distinction between experiential and non-experiential states/justifiers] to be marked at the phenomenological level, there must be some general phenomenological feature that is (a) exemplified in the phenomenological states associated with all the various types of sense experience, and (b) is also exemplified in the phenomenological states associated with all the other forms of experience alleged to be incompatible with a priori justification. It is dubious that either condition obtains.[13]

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[13] Casullo, *A Priori Justification*, 150. In both this passage and the chapter from which it is drawn, Casullo primarily focuses on critiquing extant accounts of *experience* rather than accounts of experiential justification. Even so, his ultimate focus is squarely on the distinction between experiential and non-experiential justification. Indeed, Casullo begins the chapter in question by claiming (*op. cit.*, 148) that if "that distinction is not coherent, the traditional debate over the a priori is rooted in conceptual confusion. Hence, we now turn to the question of whether there is a coherent concept of non-experiential justification." Given this focus, it is not surprising that Casullo moves back and forth between discussing experiential states and experiential justification. And it is not surprising that he proposes *epistemic* conditions on the concept of experience, such as the second condition in the passage quoted above. Ultimately, Casullo is committed to the view that if we cannot distinguish between experiential and non-experiential states, then we cannot draw a distinction between experiential and non-experiential justification. But as (EJ) shows, this view is unfounded. Even if *all* occurrent mental states are experiences, in the sense that they have phenomenal character, (EJ) can be used to differentiate between experiential and non-experiential justification. Thus, (EJ) provides the sort of distinction between experiential and non-experiential justification that Casullo takes to be a necessary pre-condition for making sense of the a priori.

In this passage, Casullo proposes two adequacy conditions on phenomenal theories. First, a generality condition: to identify states m_1 and m_2 as (potential) experiential justifiers, we must be able to identify a general phenomenological feature they share. Second, the phenomenological feature in question must be possessed by justifiers other than sense experiences (e.g. testimonial and memorial justifiers) that are widely thought to provide a posteriori (rather than a priori) justification.[14] He then argues that these conditions are not satisfied by extant phenomenal theories and unlikely to be satisfied by any successor theories.

Casullo notes, correctly, that there are no general phenomenal properties common to all sense-experiential states (and, by extension, sense-experiential justifiers). For instance, the phenomenal character of visual experiences is quite different from the phenomenal character of auditory experiences. Indeed, there do not appear to be any phenomenal properties that the two kinds of sensory states share (with each other or any other kind of sense-experiential state). As a result of this, Casullo infers that no phenomenal theory will meet his generality condition.[15]

This conclusion is premature. (EJ) is a phenomenal theory that marks the distinction between experiential and non-experiential justification, but it does not do so by appeal to any common phenomenal property shared by all paradigm experiential states. Instead, (EJ) appeals to the bare instantiation of justification-enabling phenomenal properties. The theory presupposes that experiential states possess the second-order property of *having phenomenal character* in addition to instantiating the particular first-order phenomenal properties that enable

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[14] This condition is motivated by remarks from Plantinga, *Warrant*, 91. Plantinga claims that memorial and testimonial justification cannot be a priori; this position is no doubt shared by other friends of the a priori.

[15] See, for instance, Casullo, *A Priori Justification*, 150: "[concerning] experiences associated with the five senses… we readily distinguish between, say, auditory and visual experiences on the basis of differences in their phenomenological character. The fact that these different forms of experience (in the broad sense) have a unique phenomenological character is not sufficient to ensure that the difference between experiential (in the narrow sense) and non-experiential states can be marked in terms of differences in their phenomenological character." He continues (*op. cit.*), "Roderick Chisholm, for example, characterizes the states associated with the five senses in terms of *sensible characteristics.* Sensible characteristics, in turn, comprise the "proper objects," which are unique to each of the senses, along with the "common sensibles," which are common to all the senses. Chisholm illustrates the proper objects of each of the senses by providing examples of visual characteristics, auditory characteristics, and so on. The common sensibles are also illustrated by examples such as rest, number, figure, and magnitude. Hence, in the final analysis, Chisholm fails to provide a general characterization of the concept of a sensible characteristic. He fails to identify some general phenomenological feature common to sense experience in its various forms."

individual state tokens to serve as experiential justifications for specific beliefs. Possession of the second-order property is the general phenomenological feature these states have in common such that they are experiences. It's what facilitates their falling within a single classificatory category. Moreover, instantiation of the relevant first-order properties is what enables token experiential states to provide experiential justification. Thus, the general phenomenological feature that experiential justifiers share is the second-order property of *having justificatory powers in virtue of their phenomenal character*. In view of the above, it is safe to say that (EJ) is a phenomenal theory that meets Casullo's (well-motivated) first condition.

Casullo's second condition, by contrast, is not well motivated. This is because, aside from the core condition of being non-experiential in character, there aren't any necessary conditions on a priori justifiers—conditions of the sort that would decisively prohibit memorial and testimonial justifiers from being a priori—that have been uniformly endorsed by advocates of the a priori. Indeed, a review of the recent literature reveals that a variety of different and sometimes incompatible conditions on a priori justification have been proposed.[16]

For instance, Kitcher claims that if *b* is justified a priori, then its justification is *infallible*.[17] Swinburne claims that if *b* is justified a priori, then it is *necessarily true*.[18] Ewing claims that if *b* is justified a priori, then it is *self-evident*.[19] By contrast, Kaplan calls attention to the belief that *I am here now*, arguing that it can be justified a priori despite the fact that its content is contingent and its provenance is introspection.[20] And Burge contends that testimony can provide a priori entitlement.[21] What the views of these philosophers have in common is a conception of a priori justification as experience-independent justification. Beyond this minimalist conception of a priori justification, however, there appears to be little consensus about whether there are additional conditions on the a priori and, if so, how they should be articulated.

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[16] In support of this point see Casullo's own review of the literature in *A Priori Justification*, 9-32.

[17] Philip Kitcher, *The Nature of Mathematical Knowledge* (Oxford: Oxford University Press, 1985).

[18] R.G. Swinburne, "Analyticity, Necessity, and Apriority," *Mind* 84, 334 (1975): 225–243.

[19] A. C. Ewing, *The Fundamental Questions of Philosophy* (London: Routledge, 1951), 26-52.

[20] David Kaplan, "Demonstratives: An Essay on the Semantics, Logic, Metaphysics, and Epistemology of Demonstratives and Other Indexicals," in *Themes from Kaplan*, eds. Joseph Almog, John Perry, and Howard Wettstein (New York: Oxford University Press, 1989), 481–614.

[21] Tyler Burge, "Content Preservation," *The Philosophical Review* 102, 4 (1993): 457-488.

Given this rather significant absence of consensus, it's hard to see why we should follow Casullo in presupposing that an acceptable phenomenal theory must classify as experiential those justifiers—in particular testimonial and memorial justifiers—that *some* philosophers have "alleged to be incompatible with a priori justification." Perhaps the most sensible way of drawing the distinction between experiential and non-experiential justification just cannot accommodate the complete set of views held by some of the leading champions of the a priori. So much the worse, then, for them!

In conjunction with this point, it should be acknowledged that beliefs about color incompatibilities might be cited as counterexamples to (EJ). This because many philosophers think that the belief that *nothing can be red all over and green all over at the same time* is a paradigm case of a priori justified belief.[22] Since, however, this belief concerns phenomenal properties like red and green, one might conclude that any mental state that justifies it will only do so in virtue of its phenomenal character. If that's the case, then, given the classificatory scheme that results from (EJ), the belief that nothing can be red all over and green all over at the same time will, surprisingly, be justified a posteriori. And this outcome might be thought to undermine the claim that (EJ) fixes the a priori/a posteriori distinction in an acceptable way.

Although the force of this objection should be recognized, there is a straightforward *tu quoque* defense that rests on the observation that color incompatibility claims are very odd. They are alleged to be necessary truths.[23] Yet they appear to be expressed by synthetic sentences and to concern relations between phenomenal properties, acquaintance with which results from particular visual experiences of contingent empirical entities and events.[24] This quizzical conjunction of attributes, however, is precisely why color incompatibilities are philosophically interesting: they are not very easily captured by a number of otherwise promising classificatory schemas. Indeed, Dale Jacquette makes the provocative claim that Wittgenstein's abandonment of the semantic program of the *Tractatus* was the result of his "dissatisfaction with its…implications for the color incompatibility problem."[25] Jerrold Katz makes the even more provocative claim

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[22] See, for instance: Bonjour, *In Defense of Pure Reason*, 2; Quassim Cassam, *The Possibility of Knowledge* (Oxford: Oxford University Press, 2007), 196-210.

[23] One popular source for this allegation is: D. F. Pears, "Incompatibilities of Colours," in *Logic and Language (second series)*, ed. Antony Flew (Oxford: Blackwell, 1953), 112-122.

[24] For an early defense of the syntheticity of color incompatibility claims see: Arthur Pap, "Are All Necessary Propositions Analytic?" *Philosophical Review* 50, 4 (1949): 299-320.

[25] Dale Jacquette, *Wittgenstein's Thought in Transition* (West Lafayette, IN: Purdue University Press, 1998), 185.

that "The three movements in which most analytic philosophy of this century has been done, Wittgenstein's late philosophy, Rudolf Carnap's neo-Humean empiricism, and W.V. Quine's neo-Millian empiricism, were each, in large measure, responses to the [color incompatibility] problem."[26] Unsurprisingly, then, there has emerged a large body of literature that attempts to make sense of the star-crossed semantic, epistemic, and modal properties of color incompatibility claims.[27] And given the significant classificatory challenges found in that literature, I should think that if an otherwise plausible theory of the a priori assigns a posteriori status to beliefs about color incompatibilities, this hardly suffices for outright rejection of the theory.[28]

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[26] Jerrold Katz, "The Problem in Twentieth Century Philosophy," *The Journal of Philosophy* 95, 11 (1998): 549

[27] For a strong bibliography and an overview of many significant twentieth century papers on color incompatibilities, see: R. G. A. Dolby, "Philosophy and the *Incompatibility* of *Colours*," *Analysis* 34 (1973): 8-16.

[28] I should also note that there is at least some basis for thinking that claims about color incompatibilities are analytic. For a classic defense of this view, see: Hilary Putnam, "Reds, Greens, and Logical Analysis," *The Philosophical Review* 65, 2 (1956): 206-217. For a qualified defense, see: Katz, "The Problem," 574-575. More recently, Brian Kierland has argued, with some force, that *nothing can be red all over and green all over at the same time* is either analytic or contingent (contra the view that it is necessary but not analytic). For reference see: Brian Kierland, "Necessity and Color Incompatibility," *Disputatio* 31, 4 (2011): 235-237. If these philosophers are correct and color incompatibility claims *are* (in some sense) analytic, then it's unlikely that beliefs about them are justified by appeal to the phenomenal character of the justifying mental state. I don't purport to offer an account of how beliefs with contents expressed by analytic sentences are justified. But presumably such beliefs are about logical relations between concepts and/or meanings. And neither concepts nor meanings are colored. Thus, it's hard to see how color phenomenology would have any direct role to play in justifying beliefs concerning logical relations between concepts and/or meanings.

Now, suppose that claims about color incompatibilities are synthetic. If so, then there's some basis for thinking that they are about universals; i.e., that *nothing can be red all over and green all over at the same time* ultimately makes a claim about the incompatibility of the properties redness and greenness *qua* universals. Although it's tempting to think claims about redness and greenness are going to be justified in virtue of the phenomenal character of their justifiers, this temptation can (and perhaps should) be resisted. For one thing, if we say that, for instance, redness is itself a red entity, we invite Plato's Third Man to lecture us about the danger of explanatory regresses. If, however, we deny that redness is a red entity, then it's rather difficult to see how red color phenomenology would play a direct role in justifying beliefs about redness (though having experiences with red phenomenal character is perhaps a necessary precondition for *forming* beliefs about redness). Alternatively, if we insist that redness is red, despite the potential explanatory regress, it's still difficult to see how red color phenomenology would play a direct role in justifying beliefs about redness *qua* universal. This is because any mental state that

Suppose that, despite the concerns expressed above, we accept Casullo's second condition. The point of the second condition is, it seems, to prevent testimonial and memorial justifiers from being categorized as a priori. Thus, it should be noted that there is a reasonable basis for thinking that (EJ) classifies (most familiar) testimonial and memorial justifiers as experiential. Assume, with a number of prominent philosophers, that memory and testimony are *preservative* sources of justification; i.e., they maintain and transmit previously acquired justification rather than generating new justification.[29] On this view, when a testimonial or memorial justifier is anchored by an experiential generative justifier, the justification it provides is also experiential. We should therefore expect a priori and a posteriori status to track generative justifiers rather than preservative justifiers. If that's correct, then (EJ) will classify as a posteriori any testimonial and memorial justifiers that transmit (or preserve) experiential justification, even if their justificatory powers are independent of their immediate phenomenal character. This, in turn, should moderate the concern that a phenomenal theory such as (EJ) would yield untenable classificatory results.[30]

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has redness as part of its phenomenal character will instantiate a specific shade (or shades) of red. But the obviousness and immediacy of *nothing can be red all over and green all over at the same time* suggests it's not justified by considering the incompatibility of this particular shade of red with that particular shade of green and then making an inductive inference. Perhaps, instead, we immediately grasp, however inchoately, that *nothing can be red all over and green all over at the same time* is an instance of the more general claim that any two determinates of a determinable exclude one another (e.g., that being ten pounds excludes being twelve pounds). This proposal is hinted at, though not fully developed in: W.D. Hart, *The Evolution of Logic* (Cambridge: Cambridge University Press, 2010), 40. In a related context, Mares notes that "we do sometimes see that certain concepts have logical relations to one another and this does not require further propositional thought…just an ability (that is innate or learned) to see certain logical connections." For reference see: Edwin Mares, *A Priori* (Montreal & Kingston: McGill-Queen's University Press, 2011), 48-49. If the above is correct, then maybe color incompatibility beliefs are justified in virtue of our grasping that their contents exemplify a relation between determinates and determinables *in general* rather than a relation between color properties in particular. It would then seem, however, that color phenomenology is epistemically superfluous with respect to the positive epistemic status of *nothing can be red all over and green all over at the same time*.

[29] See, for instance: Robert Audi, *Epistemology: a Contemporary Introduction to the Theory of Knowledge*, (New York: Routledge, 2011), 131-153; Michael Dummett, "Testimony and Memory," in *Knowing from Words*, eds. Bimal Matilal and Arindam Chakrabarti (Dordrecht: Kluwer Academic Publishers, 1994), 251-272; Plantinga, *Warrant*, 65-88.

[30] It should be emphasized that there is a striking lack of consensus about how to even begin to model testimonial and memorial justification. For that reason, it hardly seems appropriate to demand, well in advance of any such consensus, that testimonial and memorial justifiers be

4. The Concept of Phenomenal Properties

It may be that Casullo intends to present a deeper challenge to phenomenal theories of experiential justification. This is suggested when Casullo claims, while critiquing Roderick Chisholm's analysis of experience, that Chisholm "fails to provide a general characterization of the concept of a sensible characteristic" and "… fails to identify some general phenomenological feature common to sense experience in its various forms."[31] The second sentence here suggests that Casullo wants Chisholm to identify a feature or property common to all sense-experiential states (and thus to all experiential justifiers). As I demonstrated in the previous section, (EJ) satisfies this demand. The first sentence, however, suggests that Casullo may also want an explication of the concept of a phenomenal property.

Notice, then, that Casullo takes Chisholm to task for failing to provide a general account of the concept of a sensible characteristic. Chisholm stands accused, rather like Euthyphro, of giving mere *examples* of φ (in this case colors, odors, shapes, and so forth) where what's needed is a theoretical definition or conceptual analysis of φ. Although Chisholm is focused on sensible characteristics (which he thinks of as the *objects* of sense-experiential states and thus as properties of external entities) rather than phenomenal properties, one might expect Casullo to level similar accusations at the advocates of (EJ). For if (a) what experiential justifiers have in common is that their justificatory powers depend upon their phenomenal character and (b) phenomenal character is understood in terms of instantiating phenomenal properties, one might sensibly wonder what makes the properties in question phenomenal. Perhaps, then, Casullo intends to question whether the concept of a phenomenal property (and the related concept of phenomenal character) can be given an intelligible explication. If not, then the second-order properties that fix (EJ) cannot be clearly articulated. No doubt this would be cause for concern. For if there is no basis for distinguishing phenomenal

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classified as a posteriori. Even so, aside from the preservative model, there are other reasonable positions on testimonial and memorial justification that do not, given (EJ), yield the result that (most) memorial and testimonial justifiers are a priori. For instance, one might conclude, after adopting the Humean position that testimonial justification is reducible to a conjunction of perceptual, memorial, and inferential justification, that testimonial justification is always inferential. One might argue, similarly, that memorial justification is inferential insofar as the memory that *p* is a premise, along with the belief that one's memory is reliable, in an argument that can be used to justify the belief that *p*. In each case, there will usually be empirical premises at work in the generation of testimonial and memorial justification.

[31] Casullo, *A Priori Justification*, 150.

properties from other kinds of properties, then, by extension, there's no basis for distinguishing experiential justifiers from other kinds of justifiers.[32]

Unfortunately, there's no widely accepted explication, criterion, or set of jointly necessary and sufficient conditions for the concept of a phenomenal property. Instead, when philosophers introduce the terms "phenomenal property" and "phenomenal character," they usually define them by ostentation. For instance, here is how Chalmers introduces the concept of a phenomenal property:

> Consciousness involves the instantiation of phenomenal properties. These properties characterize aspects of what it is like to be a subject (what it is like to be me right now, for example, or what it is like to be a bat) or what it is like to be in a mental state (what it is like to see a certain shade of green, for example, or what it is like to feel a certain sharp pain). Whenever there is something it is like to be in a mental state, that state has specific phenomenal properties.[33]

Chalmers points to various first-order "feelings" and claims that what they have in common is that there is something that it is like to have them. Definitions of this kind pervade the philosophical literature and are widely taken to render the concept of a phenomenal property intelligible. While Chalmers' definition may not be as illuminating as we'd like, it does pick out a feature of numerous mental states that we are prepared to grant *prima facie* recognition; namely, that they have properties that somehow give rise to something-it-is-likeness.

When we reflect on a token pain state, we can discern that it has among its various properties both painfulness and being-indexed-to-time-*t*. It's doubtful that there is "something it is like" when a mental state instantiates the latter property. By contrast, the former property is a paradigm of something-it-is-like-ness. Thus, there is a seemingly intelligible distinction between the phenomenal and non-phenomenal properties of mental states. Of course, absent some further account of "something-it-is-like-ness," this approach may well be hopeless; it offers only to exchange one insufficiently clear term for another. Does this point undercut (EJ)?

The proper response here is to note that even among philosophers who think phenomenal properties themselves are superfluous, explicable in terms of representational content, or otherwise able to be explained away, there is a near consensus that the concept of a phenomenal property is intelligible.[34] There is

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[32] It wouldn't be hard to miss this point in Casullo's discussion of phenomenal theories, since the discussion goes by very quickly. Indeed, it lasts for only three paragraphs.

[33] David Chalmers, "The Representational Character of Experience," in *The Future for Philosophy*, ed. Brian Leiter (Oxford: Oxford University Press, 2004), 154-155.

[34] For (EJ) to be intelligible, what's needed, at a minimum, is a coherent *conceptual* distinction between phenomenal properties and other kinds of mental state properties. We don't necessarily

agreement that, for instance, there is something it's like to hit one's thumb with a hammer. And none of us would like to be hit with a hammer precisely because we have a sense of what it would be like. If we knew how to theoretically account for something-it-is-likeness, we might then be in a position to deliver a satisfying explication of the concept. Still, a consensus prevails despite our philosophical failings. We are able to sensibly use the concept of a phenomenal property even though we cannot provide necessary and sufficient conditions for its extension. And we are not required to give a final analysis of a concept (or distinction) before putting it to use. Indeed, as Michael Huemer contends, "no generally accepted analysis of any philosophically interesting term has yet been devised."[35] If he's correct, then such a constraint would render much of philosophy (and ordinary conversation) impossible; ergo, the constraint is untenable. Moreover, given the rather poor track record to which Huemer points, even those of us who aren't quite as pessimistic about conceptual analysis should nevertheless be wary of any proposal to link the intelligibility or theoretical *bona fides* of a concept to our grasp of its final analysis.

According to (EJ), when mental states that possess phenomenal character justify beliefs in virtue of that character, the kind of justification they provide is experiential. If Casullo is prepared to deny the very intelligibility of the concept of a phenomenal property, he can then deny that (EJ) is an intelligible account of experiential justification.[36] But this would be a high price to pay for a rather limited philosophical victory. For the concept of a phenomenal property is intelligible, even if the best we can do to limn the borders of its extension is to

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need this conceptual distinction to track a fundamental metaphysical difference. It could be that representationalism is correct; i.e., it could be that phenomenal character supervenes on (or consists in) representational content and that phenomenal properties are not *sui generis* properties but supervene upon (or are a species of) representational properties. Even so, if we can conceptually differentiate the phenomenal-seeming representational properties from other representational properties, a view that advocates of representationalism tend to endorse, then that's all we need to ensure that (EJ) makes sense. The same point applies, *mutatis mutandis*, to other reductionist approaches to the metaphysics of phenomenal properties. For more on representationalism see: Alex Byrne, "Intentionalism Defended," *The Philosophical Review* 110, 2 (2001): 199-240; Fred Dretske, *Naturalizing the Mind* (Cambridge, Massachusetts: The MIT Press), 1995; Tye, *Ten Problems of Consciousness*, 1995.

[35] Michael Huemer, "The Failure of Analysis and the Nature of Concepts," in *The Palgrave Handbook of Philosophical Methods*, ed. Chris Daly (New York: Palgrave Macmillan, 2015), 52.

[36] For some evidence that Casullo does not think the concept of a phenomenal property is unintelligible, see: Albert Casullo, "Phenomenal Properties," *Australasian Journal of Philosophy* 60, 2 (1982):167-169.

point to various mental states and note that they feel some kind of way.[37] Indeed, even if the concept of a phenomenal property is *sui generis*, we can still sensibly employ it in our account of experiential justification.

5. Implications for Casullo's Project

Casullo claims that "[armchair] arguments both for and against the existence of a priori knowledge are largely inconclusive" and, as a result, we should take "a different approach to addressing the issue of the existence of a priori knowledge: one that appeals to empirical evidence."[38] The proposal that we use largely or exclusively empirical methods to determine whether there is a priori knowledge or justification (and, if there is, its nature and scope) is Casullo's signature contribution to the literature on the a priori. Its credibility rests upon the claim that armchair methods cannot yield an adequate account of experiential justification; i.e., premise (2) in my reconstruction of his argument for empirical investigation.

In preceding sections, I presented a *prima facie* plausible armchair-based phenomenal theory of experiential justification—(EJ)—which says, roughly, that *m* provides experiential justification for *b iff m*'s power to justify *b* depends upon its phenomenal character. I then showed that (EJ) is not susceptible to Casullo's attempts to refute phenomenal theories. This result undermines premise (2) of Casullo's argument for empirical investigation. Thus, I conclude that armchair methods can and should play a substantial role in our ongoing investigation of the a priori.[39]

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[37] Perhaps there will be intractable disagreements, then, about what properties are correctly classified as phenomenal. That result is perfectly consistent with the claim that there is a category of properties that are what they are because there is something it is like for them to be instantiated.

[38] Albert Casullo, "Response to my Critics: Chris Pincock, Lisa Warenski and Jonathan Weinberg," *Philosophical Studies* 173, 6 (2016):1706.

[39] I would like to thank Walter Edelberg, Alejandro Vazquez del Mercado, and an anonymous referee for helpful feedback on this paper and/or the views it expresses.

JUSTIFIED BY THOUGHT ALONE

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ABSTRACT: The new rationalists – BonJour and Bealer – have characterized one type of *a priori* justification as based on intellectual intuitions or seemings. I argue that they are mistaken in thinking that intellectual intuitions can provide *a priori* justification. Suppose that the proposition that a surface cannot be red and green all over strikes you as true. When you carefully consider it, you couldn't but realize that no surface could be both red and green all over. Ascertaining the truth of what you believe (when you believe that a surface cannot be red and green all over) requires conscious experiences of thinking. The character of such experiences (propositions' striking you as true, and the sense of incoherence you would experience were they to be false) is what justifies your belief. It should follow that the justification for such propositions (and your believing them) is *a posteriori*, i.e., based on conscious experience. Your cognitive phenomenology plays a constitutive role in justifying your belief. Hence your belief is not *a priori* justified, *contra* the new rationalists.

KEYWORDS: *a priori* justification, cognitive phenomenology, intuitions, intellectual seemings, rationalism

1. Introduction: The Problem

Let's start with a well-known example. Suppose you believe that a surface cannot be (wholly) red and (wholly) green all over. What justifies your belief? All it takes is for you to *understand* the proposition you believe, in order for your belief to be justified. When you *carefully consider* what it is for a surface to be red, and what it is for a surface to be green, you *couldn't but realize* that no surface could be both red and green all over. So, in *ascertaining* why you believe this, all you need to do is aptly use concepts you possess. The proposition that a surface cannot be red and green all over then *strikes you as true*. Your belief is *a priori* justified. Or so the thought goes.

Crucially, *a priori* justifications are independent from experience.[1] But the phrases in italics just used sound very much as if one undergoes *conscious experiences of thinking*.[2] You come to understand a proposition: it dawns on you,

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[1] This, I believe, is in tune with how almost everyone uses the terms "justification" and "experience." But see the next section for some controversy.

[2] I remain neutral about whether conscious thinking presupposes that what is experienced are conscious thoughts, or if, on the contrary, the imagery underpinning what it is like to undergo

you now fully grasp it. You carefully consider what the proposition says, comparing it to your own conceptions of red and green, weighing if there is anything that might make you doubt it in the least. You ascertain, grasp, or apprehend the truth of the proposition, 'holding it before your mind's eye.' You *perceive* its truth.[3]

This poses a *straightforward problem:* if justifying your belief (that a surface cannot be both red and green all over) depends on your having certain cognitive experiences – of grasping concepts, considering what your conceptions are, weighing alternatives to them, pondering how your concepts fit together, etc. – then a justification had on the basis of such experiences cannot be *a priori*; it has to be *a posteriori*: following, and due to the having of, those cognitive experiences.[4] In argument-form:

1) Intuitions are experiences.
2) So, any justification based on intuitions is based on experiences.
3) No *a priori* justification is based on experiences.
4) So, no *a priori* justification is based on intuitions.

The argument is valid.[5] (2) follows from (1): if all intuitions are experiences, then anything based on intuitions is based on experiences; so any justification based on intuitions is based on experiences. Premise (3) is definitional: not being

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conscious thoughts is primarily sensory; cf. Michael Tye, "*Mental Reality* by Galen Strawson [Review]" *Journal of Philosophy* 93 (1996): 421-424. I also remain neutral about whether we can neatly carve out what it is like to think into what it is like to have a certain propositional attitude, and what it is like to be related to a proposition as a content of that attitude, cf. David Pitt, "The Phenomenology of Cognition, or, What is It Like to Think That *P*?" *Philosophy and Phenomenological Research* 69 (2004): 1-36.

[3] You may, of course, be wrong about what justifies what. Perhaps surfaces *can* be red and green all over. Or perhaps your conceptions of red and green don't rule this out. Or both. Or perhaps you can't clearly grasp the conceptions of red and green you possess. Or, grasping them, you have trouble applying them in imagination when considering whether there can be a surface red and green all over. Mishaps are everyday occurrences. But if philosophical trouble looms even when everything *goes well*, mishaps are by the by.

[4] As I use the word "experience" in what follows, experiences are always conscious, in the sense that there is something it is like to undergo them. I don't assume that experiences are conscious in any other sense, e.g., as objects of one's attention – though this isn't excluded either, obviously.

[5] If formalized as follows, where "B" denotes the basing relation, "J" denotes justifications, "I" intuitions, "E" experiences, and "A" aprioricity:

  1. $(\forall x)(Ix \rightarrow Ex)$
  2. $(\forall x)(Jx \rightarrow ((\exists y)(Iy \& Bxy) \rightarrow (\exists y)(Ey \& Bxy)))$
  3. $\sim(\exists x)(Jx \& Ax \& (\exists y)(Ey \& Bxy))$
  4. $\sim(\exists x)(Jx \& Ax \& (\exists y)(Iy \& Bxy))$

based on experiences simply *is* what it is for a justification to be *a priori*. So: if (1) is true, (4) is true. (1) might seem overly demanding; I will return to why it isn't.

This argument spells trouble for the view articulated by *the new rationalists*.[6] BonJour[7] writes:

> It is common to refer to the intellectual act in which the necessity of such a proposition [that a surface cannot be red and green all over] is seen or grasped or apprehended as an act of *rational insight* or *rational intuition* (or, sometimes, *a priori* insight or intuition), where these phrases are mainly a way of stressing that such an act is seemingly (a) direct or immediate, non-discursive, and yet also (b) intellectual or reason-governed, anything but arbitrary or brute in character...Since this justification or evidence apparently depends on nothing beyond an understanding of the propositional content itself, a proposition whose necessity is apprehended in this way... may be correlatively characterized as *rationally self-evident*: its very content provides, for one who grasps it properly, an immediately accessible reason for thinking that it is true.[8]

For Bealer,[9] *a priori* justification obtains when intellectual seemings are a source of evidence for beliefs. Grant Bealer that beliefs are justified, and ask: what are those intuitions?[10] He answers:

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[6] In what follows I mainly discuss BonJour, Bealer and Peacocke. But similar remarks may well apply much more widely. Thus, Chalmers writes: "A sentence S is *a priori* relative to a speaker if the sentence as used by that speaker expresses a thought that can be justified independently of experience, on ideal rational reflection." (David Chalmers, "On Sense and Intension," *Philosophical Perspectives* 16 (2002): 135-82).Why ideal rational reflection should be devoid of conscious character – or shouldn't even in part be constituted by conscious cognitive experience, Chalmers doesn't say. A different way of expanding the scope of the problem I raise considers intuitions not as conscious experiences but as inclinations to believe, or (another option) as propensities to undergo such conscious experiences. Both options are considered by Ernest Sosa, "Intuitions", in *Virtue Epistemology* (Oxford: Oxford University Press, 2007), 44-69. And we may raise analogues of the problem I point to in the text with respect to each of these. If intuitions as conscious experiences make the justification they contribute to count as *a posteriori*, it is only natural to think that a similar effect is obtained by inclinations to so believe, or propensities to so experience. After all, such inclinations or propensities have justificatory weight only when realized in intuitions (or in the beliefs such intuitions would support, were we to come to acquire them). I refrain from considering related issues (which remarks made by Chalmers and Sosa illustrate) for reasons of space and to keep the discussion fairly contained.

[7] Laurence BonJour, "A Moderate Rationalism," in *In Defense of Pure Reason* (Cambridge: Cambridge University Press, 2001), 98-129.

[8] BonJour, "A Moderate Rationalism," 102. I have elided a qualification BonJour makes that I will return to later in the text.

[9] George Bealer, "A Theory of the A Priori," *Pacific Philosophical Quarterly* 81 (2000): 1-30.

[10] Elijah Chudnoff, "Is Intuition Based On Understanding?" *Philosophy and Phenomenological Research* 86 (2013): 42-67, offers a convincing criticism of the idea that intuitions – as

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> For you to have an intuition that A is just for it to *seem* to you that A. Here 'seems' is understood, not as a cautionary or 'hedging' term, but in its use as a term for a genuine kind of conscious episode. For example, when you first consider one of de Morgan's laws, often it neither seems to be true nor seems false; after a moment's reflection, however, something new happens: suddenly it just *seems* true. Of course, this kind of seeming is intellectual, not sensory or introspective (or imaginative). For this reason, intuitions are counted as 'data of reason' not 'data of experience.'[11]

The obvious reply to Bealer has to be that once you admit the existence of the relevant "conscious episodes" (intellectual seemings), then, whatever else "data of reason" might be, they *must* be data of experience too – since they are procured in experiences of thought.

BonJour and Bealer wish to *both* ground our *a priori* knowledge in intellectual seemings, or intuitions (per 1),[12] *and* claim that the resulting justifications are *a priori* notwithstanding their intuitive source (contra 4). The argument from (1) to (4) shows that can't be done.[13]

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intellectual experiences – fully justify the conceptual understanding they manifest. My project is different: grant any justification being claimed, and conclude that any such justification – *if* it succeeded – would have to be *a posteriori*, rationalist claims to the contrary notwithstanding.

[11] Bealer, "A Theory of the A Priori," 3.

[12] I will indiscriminately speak of intuitions, insights, intellectual seemings, conscious experiences of apprehension, grasping, thinking appearances, and the like. Each may be quite different from the others, but their minute experienced differences matter little for the epistemological point I'm interested in. Just to illustrate here, BonJour seems to use "insight" and "intuition" interchangeably. For a nice distinction between them, see Rachel Henley, "Distinguishing Insight from Intuition," *Journal of Consciousness Studies* 6 (1999): 1-8. I myself construe Henley's differences as follows: in intellectual intuitions, we exploit an understanding we already possess, whereas, in insights, we come to understand something new. Michael Lynch, "Understanding and Coming to Understand," in *Making Sense of the World: New Essays on the Philosophy of Understanding*, ed. Stephen Grimm (Oxford: Oxford University Press, 2017) highlights the connection between insights and conceptual creativity, while E. M. Bowden, M. Jung-Beeman, J. Fleck, and J. Kounios, "New Approaches to Demystifying Insight," *Trends in Cognitive Sciences* 9 (2005): 322-328, explore the role of insightful experiences in problem-solving.

[13] The challenge is wider in scope than traditional *synthetic a priori* justifications. If we identify cardinal numbers with a representative sequence of sets and then prove counterparts of Peano's axioms in set theory, the justification is traditionally thought to be analytic (modulo set-theoretic axioms), but our problem is there. We need to *keep track* of assumptions throughout, *represent to ourselves* what a solution to the deductive problem should look like, *make sure* we haven't misapplied any rules or axioms. Problem-solving phenomenology (e.g., Bowden et al., "New Approaches to Demystifying Insight") is rich, varied, and primarily cognitive – even when what is proven turns out to be an analytic statement.

What replies can rationalists make? Three, as far as I can see. They can claim that (1) is false. Or they can claim that (3) is false. Or they can qualify (3) in a way that makes (1) irrelevant to how they construe *a priori* justifications. These defensive moves are, I believe, ultimately unsatisfactory. Each of the next three sections explores one such defensive move.

2. Intuition without Experience?

"Intuition" is said in many ways. Perhaps a belief is intuitive when it doesn't require justification at all, or when the way one arrived at the belief isn't also the way to justify it. For experimental philosophers, intuitions are verbal reports by philosophically naive but linguistically competent speakers of English, French, etc. An intuitive belief might be a belief one is *inclined* (or disposed) to hold, perhaps because one has the cognitive skills and expertise requisite to produce the belief in question.[14] Understood in any of these ways, intuitions aren't conscious experiences, so (1) would be false.

But none of these meanings of "intuition" is at play in the new rationalism. What *is* at play is a kind of intellectual seeing, a "quasi-perceptual" model of intuitions, per (1). If intuiting is much like seeing, only of matters intellectual, then justification on the basis of intuitions can be thought of along the lines of *perceptual* justification.[15] New rationalists exploit this – while insisting the resulting justification is *a priori* notwithstanding. But one can't have one's cake and eat it too. Perhaps (1) is false; but, given their epistemological project, it's not open to rationalists to deny (1).

To illustrate: BonJour says[16] that his use of "intuition" differs from Kant's, perhaps also meaning to suggest that intuitions are, for him, non-experiential. Yet BonJour also describes my intuiting that nothing can be red and green all over by saying "I am able to see or grasp or apprehend in a seemingly direct and

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And, even when problem-solving phenomenology seems absent, we should beware. J. Nakamura and M. Csikszentmihalyi, "The Concept of Flow," in *Handbook of Positive Psychology*, eds. C. R. Snyder and S. J. Lopez (Oxford: Oxford University Press, 2002), 89-105, theorize experiences of flow, where subjects are simply *absorbed* by the problems they are solving, their attention fully focused, not minding anything else – and not minding what they themselves might be experiencing in solving problems. This is consistent with undergoing incredibly rich conscious episodes that one simply fails to *attend to.*

[14] Ernest Sosa, "Intuitions", in *Virtue Epistemology* (Oxford: Oxford University Press, 2007), 44-69.

[15] Paul Boghossian, "Virtuous Intuitions: Comments on Lecture 3 of Ernest Sosa's *A Virtue Epistemology,*" *Philosophical Studies* 144 (2009): 111-119.

[16] BonJour, "A Moderate Rationalism," 102, footnote 7.

unmediated way that the claim in question cannot fail to be true."[17] How could a *direct* and *immediate* grasp *fail* to be experiential? Immediacy and directness are, presumably, properties of one's conscious experience, so that nothing is *felt* to come in-between the thinker and her thoughts.[18]

Where does this leave the first defensive move? It was, recall, that intuitions aren't experiences, contra (1). In reply, I have distinguished several senses of "intuition" and have argued that, in the sense relevant to the new rationalists and how they construe *a priori* justification, intuitions *are* experiences. Many may balk at (1) – experimental philosophers, for instance. But, then again, they wouldn't contemplate cashing out *a priori* justification in terms of intuitions either.

3. Cognitive and Perceptual Experiences

The second reply rationalists could make would be to say that (3) is false. This may sound awkward. (3), recall, is the claim that "No *a priori* justification is based on experiences." How could anyone deny this? By changing the definition of "*a priori*." Thus, A.C. Ewing writes:

> Most of our knowledge we obtain by observation of the external world (sense-perception) and of ourselves (introspection). This is called empirical knowledge. But some knowledge we can obtain by simply thinking. This kind of knowledge is *a priori*.[19]

Bealer may also be implying a shift from experience-in-general to sensory (and introspective) experience when he says intellectual intuitions are not "data of experience." And, along the same lines, BonJour writes:

> the relevant notion of experience should be understood to include any sort of process that is perceptual in the broad sense of (a) being a causally conditioned response to particular contingent features of the world and (b) yielding doxastic states that have as their content putative information concerning such particular, contingent features of the actual world as contrasted with other possible worlds... [And] mathematical intuition, even though it undoubtedly counts as experience

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[17] BonJour, "A Moderate Rationalism," 101.

[18] Bealer also formulates a principle of moderate rationalism by saying: "A person's phenomenal experiences and intuitions comprise the person's basic evidence" ("A Theory of the A Priori," 7). Relevance considerations strongly suggest he thinks intuitions are not phenomenal experiences. But in a quote given earlier, Bealer admitted that an intuition (= an intellectual seeming) is a "conscious episode." To phenomenally experience something simply *is* to undergo (= experience) a "conscious episode." So, unless Bealer (idiosyncratically) restricted the phrase "phenomenal experience" to refer to perceptual experiences alone, his position seems dialectically unstable.

[19] A.C. Ewing, "In Defense of *A Priori* Knowledge," in *The Theory of Knowledge: Classic and Contemporary Readings,* ed. Louis P. Pojman (Wadsworth, 2003), 385.

in the sense of consciously undergoing a mental process, would not count as experience in this more specific sense so long... as its deliverances consist solely of (putatively) necessary truths.[20]

Let's look at this passage for a moment. BonJour seems to be suggesting that experience, in the intended sense, the sense relevant to *a priori* justification and from which such a justification should be free, is that which roughly fits perceptual experiences.[21] It is, after all, perception which seizes upon the natural world we inhabit whose features are largely contingent. And it is perception which yields beliefs about such contingent states of affair, discriminating the actual circumstances from among conterfactual circumstances. In contrast, mathematical intuition clearly fails to meet both criteria for what counts as genuine experience.

BonJour seems to wish to *derive* the result that intuitions aren't experiences, *contra* (1) – and hence that justifications reliant on intuitions don't rely on experiences, *contra* (3) – from the two criteria, (a) and (b), he proposes. But it is hard to see why such criteria aren't simply question-begging. Consider intuitionists like Brouwer[22] who ground the foundations of mathematics – set theory, natural and real arithmetic – in pure intuition. And pure intuition, for Brouwer, *was* both conscious and cognitive. Why should it matter that this intuition doesn't concern matters contingent? No rationale has been given. BonJour's two criteria only push the problem one floor up. Yes, his notion of "experience" excludes intuitions on the basis of criteria (a) and (b). But these criteria themselves were made to fit, arbitrarily excluding conscious episodes like Brouwer's from consideration.

In general, one could hardly quarrel with a stipulation. But such a *re-definition* of "*a priori*" as Ewing, BonJour and Bealer operate *isn't* standard. Boghossian and Peacocke start their anthology by writing: "An *a priori* proposition is one which can be known to be true without any justification from the character of the subject's experience."[23] Later on, Peacocke repeats it, talking about "[p]ropositions that can be known in a way which is justificationally independent of experience – propositions knowable in a way which is *a priori*, as I will say for brevity."[24] Notice "experience" isn't qualified in any way, as it should be if mention were made of only certain *kinds* of experiences (sensory and introspective).[25]

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[20] BonJour, *In Defense of Pure Reason*, 8.

[21] BonJour mentions introspection, memory etc. as well but I focus on perception for clarity. The remarks to follow apply, *mutatis mutandis*, to these as well.

[22] L.E.J. Brouwer, "Intuitionism and Formalism," in *Philosophy of Mathematics*, eds. Hilary Putnam and Paul Benacerraf (Cambridge: Cambridge University Press, 2nd ed. 1983), 77-89.

[23] Paul Boghossian and Christopher Peacocke, "Introduction," in *New Essays on the A Priori* (Oxford: Clarendon, 2000), 1.

[24] Christopher Peacocke, "Explaining the *A Priori*: The Programme of Moderate Rationalism," in

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What to make of all this? One might think the issue here is merely terminological: on one characterization of "experience" the problem I put to the new rationalists gets a bite, on another definition it doesn't. But the issue is far from *merely* terminological. Recall Brouwer. The bulk of our intellectual intuitions in logic, mathematics and philosophy are, on one characterization of "experience," simply ignored. An arbitrary distinction is set up between experiences of contingencies and intuitions of necessity – arbitrary because it hasn't been shown what, *in point of conscious character*, separates them, and why such a putative distinction *should* matter when it comes to matters of justification.

The Ewing-style re-characterization of apriority and experience sets things up in a way that suits the new rationalism. But it itself lacks motivation. And it is, as far as I can tell, the only reason one might have to deny (3). Criticizing this reason undermines the rejection of (3).

4. Enabling and Justifying

The more logically minded might think the argument I have given in Section 1 is not so much wrong as it is misguided. Premises and conclusion are true alike, only premise (1) is irrelevant to both premise (3) and the conclusion. This is because the proper role of intellectual intuitions – admitted to be experiences for the sake of argument – is not to *justify* the beliefs they trigger, but to *enable* one to justify one's beliefs.[26] Much like breathing is a prerequisite for thinking anything at all, so would *conscious grasp in thinking* be a condition to access what, quite independently of the grasp, would justify one's belief. For instance, suppose you believe that a surface cannot be both red and green all over; and suppose it also seems to you that things are so. Things seeming to be so to you wouldn't justify your belief; the seeming would merely enable you to access the *conceptual*

New Essays on the A Priori, eds. Paul Boghossian and Christopher Peacoke (Oxford: Clarendon, 2000), 256.

[25] One might have thought that this is mere ellipsis that can only now be questioned, in light of debates about cognitive phenomenology. See, e.g., Tim Bayne and Michelle Montague, "Introduction," in *Cognitive Phenomenology* (Oxford: Oxford University Press, 2011). But characterizing *a priori* justification in terms of experience-in-general persists as late as Bruce Russell, "*A Priori* Justification and Knowledge," in *The Stanford Encyclopedia of Philosophy,* ed. Edward N. Zalta (Revised 2014, accessed April 20, 2020 from https://plato.stanford.edu/ entries/apriori/)*.* For Russell: "*A priori* justification is a type of epistemic justification that is, in some sense, independent of experience." (Parenthetically, note that Russell's is also a *Stanford Encyclopedia of Philosophy* article, presumably capturing a conception of *a priori* justification in wide currency nowadays.)

[26] Bruce Russell, "*A Priori* Justification and Knowledge," Section 4.1.

knowledge of redness, greenness, and surfaces that you possess quite independently of any insights you might have into it.[27]

Before looking into how the enabling/justifying distinction[28] might save the rationalist, it is important to ponder on just which ones the enabling experiences are supposed to be. Peacocke writes:

> When you come to know a logical truth by way of your having a proof of it, you may need to perceive the inscription of the proof, and you may need various perceptual capacities to appreciate that it is a proof. But the justification for your belief in the logical truth is the proof itself. Perceptual experience gives access to the proof, which provides an experience-independent justification for accepting its conclusion. By contrast, if you come to believe 'That's Mikhail Gorbachev,' when you see him at the airport, what entitles you to your belief is (in part) the perceptual experience by which you recognize Gorbachev. Your perceptual experience is not a mere means which gives you access to some experience-independent entitlement to believe 'That's Gorbachev.' This classical rationalist distinction between experience-dependent and experience-independent justifications or entitlements has been controverted, and objections to it raised and (in my own view) answered.[29]

Peacocke, here, makes a transition that is fairly standard, and which illustrates the problem I fleshed out earlier. He rightly starts from the fact that

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[27] Is this what rationalists have in mind? It would seem so; Bealer writes: "The analysis of concept-possession has further explanatory pay-offs. To begin with, in so far as *a priori* knowledge is a product, directly or indirectly, of *a priori* intuitions, the analysis of concept-possession serves as a cornerstone of a unified account of *a priori* knowledge. On the one hand, the correctness property provides the basis of an explanation of the *reliability* of *a priori* intuition and, in turn, *a priori* knowledge itself. On the other hand, the completeness property provides the basis of an explanation of the *scope* of *a priori* intuition and, in turn, *a priori* knowledge" (Bealer, "A Theory of the *A Priori*," 22). For Bealer, concepts have correctness conditions that ensure their reliability – if they are possessed at all. Concepts also have completeness conditions, which ensure that concepts are defined for all possible circumstances we evaluate propositions at. Intuitions, in turn, source not from some mysterious faculty of insight, but from our conceptual knowledge. So intuitions inherit their modal reliability from the conceptual knowledge *they* are based on. Intuitions, on this view, *transmit* justification but don't *generate* it.

[28] Sometimes the distinction is made as one between enabling and warranting. For instance, see Jonathan Jenkins Ichikawa and Benjamin Jarvis, "A Theory of the *A Priori*," in *The Rules of Thought* (Oxford: Oxford University Press, 2013), 161-178. With them, I agree that the distinction doesn't presuppose any form of epistemic internalism. However, I demur from thinking that the justificatory power of, say, intellectual intuitions, needs to be warranting in order to carry justificatory weight at all.

[29] Peacocke, "Explaining the *A Priori*," 255.

consciously perceiving the proof (that justifies your belief that a proposition is a theorem) merely *enables* you to access the justification, rather than constitute the justification itself. Peacocke then seems to infer what doesn't in fact follow, namely, that *no* conscious experience constitutes your justification. But consider this example. Producing a proof of a theorem is a problem facing everyday reasoners – sometimes a quite difficult problem. Solving it requires *careful* thinking. Reasoners undergo conscious experiences that at least *seem* to them to be cognitive through and through; such experiences go far beyond perceiving an already written-out proof. The effort involved in constructing a proof, the constant double-checking, reflectively considering and rejecting ways of challenging the proof: these are as many ways in which problem-solving differs from merely cognitively ingesting a ready-made proof on the basis of mere visual inspection.[30] In drawing the distinction between experiences that enable and experiences that constitute justifications, we should leave behind prejudice against cognitive phenomenology – at least for the purpose of argument.[31]

Can the enabling/justifying distinction save BonJour and Bealer's rationalism of intuitions? The resulting rationalist picture is, I think, implausible. I next point to two theoretical problems and two intuitive cases where the picture seems inadequate.[32] Let me preface those problems with a challenge. As far as I can tell,

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[30] For the intricate ways in which reliable problem-solving, conscious cognitive experiences, and our reflective abilities as epistemic agents might relate to each other, also see Andrei Mărăşoiu, "Understanding, Problem-Solving, and Conscious Reflection," *Acta Analytica* 34 (2019): 71-81.

[31] It turns out to be surprisingly tricky to draw the enabling/justifying distinction in a way that could serve the new rationalism. In their recent defense of (their version of) rationalism, Ichikawa and Jarvis do draw the distinction appropriately. But they explicitly mobilize it in defense of their own version of rationalism, which they oppose to "experiential rationalism" as typified by BonJour and Bealer. And, without delving into details, one aspect is certainly crucial: Ichikawa and Jarvis' rationalism concerns propositional rather than doxastic justification. When we consider, however, examples motivating both philosophers and mathematicians (Descartes and Brouwer come to mind), what matters is that we are able to *apprehend* necessary truths in conscious thought. No mere propositional justification is going to meet that demand. Only consciously appraised justification does justice to our being *struck* by truths we can't conceive to be otherwise. That is why doxastic justification is envisaged throughout this text.

[32] Before moving to what I take to be the problems and counterintuitive verdicts that rationalism delivers, let me briefly distinguish my approach from Timothy Williamson's "Knowledge of Metaphysical Modality," in *The Philosophy of Philosophy* (Oxford: Blackwell, 2007), 165-169. His main targets are *modal* beliefs and their justification; whereas I discuss the role of cognitive phenomenology in justification *tout court*. He considers roles experiences might play, in general, in justifying beliefs; I focus on cognitive experiences. He contemplates the possibility of experiences which might be more than enablers, yet less than constitutive to justification; I don't. On the contrary, I think the distinction itself – while useful on other grounds – need not

the new rationalist has to say that intellectual experiences enable, rather than justify, the beliefs they give rise to. But I have been unable to locate *an argument* for why such experiences should be thought to play the limited role of enablers.[33]

5. Theoretical Problems and Counterintuitive Verdicts

I'll now briefly raise two theoretical problems which a rationalist view would have to face if it provided that cognitive experiences were mere enablers for *a priori* justified beliefs. I then go on to sketch two commonplace cases where the same brand of rationalism would deliver counter-intuitive verdicts. The joint effect of the theoretical problems and counter-intuitive verdicts is, I submit, that a rationalism relegating intuitions to the status of mere justificatory enablers is deeply implausible.

First problem: If intuitions are only justificatory middlemen who never generate justification on their own, why invoke them *at all*? When following deductive proofs, for instance, justification may rely on automatic "blind reasoning"[34] rather than be enabled by conscious insights. And, when you seek to justify your belief that a surface cannot be both red and green all over, it might be *enough* for your long-term, dispositional conceptions of redness, greenness, and surface to jointly *entail* this.[35] Few would be willing to accept that *a priori* justification is a matter of "blind reasoning;" and few would base justification on dispositions alone.[36] This should suggest that intuitions are more than mere enablers of justification.

Second problem: To say intuitions enable justification seems to imply that they are prerequisite for justifying beliefs. This, in turn, seems to imply that

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be invoked to see the point I make about cognitive experiences and *a priori* justification.

[33] One may, of course, insist, in hindsight, that the resulting beliefs must be *a priori*, hence experiences cannot justify them. Such hindsight simply begs the question; why else think that the resulting beliefs are *a priori*?

[34] Paul Boghossian, "Blind Reasoning," *Proceedings of the Aristotelian Society, Supplementary Volume* 77 (2003): 225–248.

[35] According to this latter suggestion, no cognitive activity need *occur* at all. Mere possession of the requisite concepts, and their associate conceptions, suffices to justify the belief. Both what justifies and what is justified are *dispositional* entities. One is the disposition to use one's concepts (red, green, surface) in the right circumstances. The other is the disposition to occurrently think that a surface cannot be both red and green all over.

[36] BonJour, for one, would not. He thinks intuitions are needed for the following reason: "If one never in fact grasps any necessary connections between anything, it is difficult to see what reasoning could possibly amount to" ("A Moderate Rationalism," 110).

particular intellectual seemings are necessary for justification. To avoid the implication, BonJour includes the parenthetical:

> a proposition whose necessity is apprehended in this way (or, sometimes, whose necessity is *capable* of being apprehended in this way) may be correlatively characterized as *rationally self-evident*.[37]

BonJour demurs from saying that an individual intellectual intuition – as it phenomenally is – is necessary to justify one's belief. Rather, *the capacity* to have such intuitions, *with the right conceptual content*, is said to be necessary, no matter how intuitions realizing that capacity are presented in conscious experience ("it hits," "it dawns," "it slowly emerges" etc.). But BonJour's capacity line is unconvincing. He states that intellectual seemings give "internal clarity and firmness" to one's rational believing.[38] And no mere capacity can give that – only individual experiences can.

I now move to why the rationalist relegation of intuitions to the status of enablers of justification doesn't do justice to our everyday experiences. Here are two cases.

Suppose that, as good high school students tend to, you routinely apply mathematical induction over finite domains. And then you take an introductory course in logic, and it *strikes you* that you can do the same over infinitely denumerable domains (like the domain of natural numbers). "Aha!" you might think to yourself, maybe there is something to extending finite techniques to apply to infinities too. The rationalist would have to insist that what *doesn't* constitute your justification for believing this (i.e., that you can extend mathematical induction from finite to infinitely denumerable domains) is *precisely* what drove you to think it in the first place, namely, your conscious insight that it might work. That has the ring of implausibility.[39]

Or return to colors. You believe no surface can be both red and green all over. How do you go about justifying it? You try to imagine what it might be for a surface to be both red and green all over. You consider cooked-up lighting conditions. You consider intermediary nuances and what effect they might have on your (imagined) *experience.* You consider if, spelling out your conceptions of what red and green are like, *as far as* your experiences and the testimony of others go, would lead you to think no such surface can exist. And so on. A vivid imagination comes in handy when seeking to apply your concepts in thought.

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[37] BonJour, "A Moderate Rationalism," 101.
[38] BonJour, "A Moderate Rationalism," 120.
[39] True, when you explain induction to *me,* mentioning *your* experiences won't help – but that is only because I need to undergo experiences of understanding of my own.

Rationalists would have to insist that all that fancying is, strictly speaking, a gateway to something entirely foreign to it, namely, a conceptual knowledge disrobed of any phenomenal quality.

To tailor reports of rich cognitive experiences only to fit the Procrustean bed of intuitions-enabled a priori justification seems too high a price to pay: it saves the letter of rationalism at the cost of its plausibility. If the distinction between enabling and constituting doesn't make (1) irrelevant to (3) and (4), and if (1) is true, then (4) is true – and that undermines the rationalism of intuitions.

6. Conclusion

What to make of all this? It might, perhaps, be tempting to conclude that the discussion is merely terminological. Use "experience" to refer to sensory experiences *alone*, and the traditional definition of *a priori* justification as justification independent of experience can remain unchanged. Or: define "*a priori*" so as to refer to justifications independent of sensory experiences *alone*, and propositions traditionally deemed to be justified *a priori* preserve their status. Or: insist that intellectual seemings play *exclusively* an enabling role, and justifications thereby enabled still qualify as *a priori*.

I take none of these routes. I let "experience" refer to sensory and cognitive experiences alike. And I find no *motivated* distinction between enabling and justifying that can rescue the *a priori* character of beliefs formed on the basis of intellectual seemings, or intuitions. I conclude that one road to rationalism is closed: thinking that beliefs can be *a priori* justified by appeal to intellectual intuitions.

There is an upside: Once we divorce it from the tradition of *a priori* justification, we can start a *fresh* assessment of the epistemic standing with which conscious experiences of thinking may endow the thoughts experienced therein.[40] And, once we divorce *a priori* justification from the epistemic standing of intellectual intuitions, we may seek for *purer a priori* standards, with *no* hindsight to which of our beliefs should qualify as such.

Oddly enough, Peacocke anticipates much of the argument I just proposed, when writing that:

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[40] For instance, Earl Conee, "Seeming Evidence," in *Seemings and Justification: New Essays on Dogmatism and Phenomenal Conservatism, ed.* Chris Tucker (Oxford: Oxford University Press, 2013), 52-69, explores the sense in which intuitive experiences may provide justification for the beliefs formed or entertained on their basis. I remain neutral about how to further articulate the justificatory import of cognitive intuitive conscious experiences, beyond the point that the resulting justification, if it obtains, has to be *a posteriori*.

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> Faculties conceived by analogy with perception, far from helping to explain the possibility of rational intuition and *a priori* knowledge, are actually incompatible with the *a priori* status of the beliefs they deliver.[41]

Peacocke is right: if we conceive of intuitions as being akin to perception, the resulting justification will be *a posteriori*. But what makes intuition be like perception is not its being sourced in a special and mysterious faculty, akin to the senses. What makes intuition resemble perception, when each occur, is that they are both conscious experiences. We *grasp* the contents of such experiences, they are presented to us in experience. And experiences of grasp are experiences *no matter* if they manifest a special (extra-)sensory faculty *or* if they manifest our mastery of a general-purpose conceptual repertoire. It is hard to see what *else* grasping might be, if not a kind of conscious experience.[42] So it begins to look as though Peacocke's view falls prey to his own objection. Peacocke framed his objection as one against faculty-based views of intellectual intuitions; he might better have framed it against intuition-based[43] views of *a priori* justification.[44]

THE APORIA OF OMNISCIENCE

Daniel RÖNNEDAL

ABSTRACT This paper introduces a new aporia, the aporia of omniscience. The puzzle consists of three propositions: (1) It is possible that there is someone who is necessarily omniscient and infallible, (2) It is necessary that all beliefs are historically settled, and (3) It is possible that the future is open. Every sentence in this set is intuitively reasonable and there are prima facie plausible arguments for each of them. However, the whole set {(1), (2), (3)} is inconsistent. Therefore, it seems to be that case that at least one of the propositions in this set must be false. I discuss some possible solutions to the problem and consider some arguments for and against these solutions.

KEYWORDS: aporia, omniscience, infallibility, historically settled beliefs, the open future

1. Introduction

In this paper, I will discuss a new aporia, the aporia of omniscience. This puzzle includes ideas about omniscience, infallibility, the modal status of beliefs and the open future. So, maybe a more comprehensive name would be 'the aporia of omniscience, infallibility, historically settled beliefs and the open future.' But this is too long. So, I will call the puzzle 'the aporia of omniscience,' since the concept of omniscience plays an essential role in the problem. The aporia of omniscience consists of the following three propositions:

(1) It is possible that there is someone who is necessarily omniscient and infallible.

(2) It is necessary that all beliefs are historically settled.

(3) It is possible that the future is open. That is, it is possible that there is a proposition A such that it will (some time in the future) be the case that A even though it is not historically settled that it will (some time in the future) be the case that A.

Each proposition in {(1), (2), (3)} is intuitively plausible, but the whole set entails a contradiction. Hence, together these sentences constitute an aporia. The aporia of omniscience should be interesting to anyone who is concerned about such topics as the nature and possibility of omniscience and infallibility, the modal status of beliefs, and the nature of the future (whether it is open or not).[1]

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[1] The concept of omniscience has been discussed in the philosophy of religion for a long time. See, for example, George I. Mavrodes, "Omniscience," in *A Companion to Philosophy of*

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I have asserted that {(1), (2), (3)} is inconsistent but that the propositions in this set are intuitively plausible. To justify this claim, I will first say a few more words about (1)–(3) and express these sentences in symbols. '◆A' reads as 'It is possible that A'; '■A' reads as 'It is necessary that A'; '□A' reads as 'It is historically settled (necessary) that A'; '◇A' reads as 'It is historically possible that A'; 'Bc' reads as 'Individual c believes that A'; 'Kc' reads as 'Individual c knows that A'; 'ꟻA' reads as 'It will some time in the future be the case that A'; 'Oc' reads as 'Individual c is omniscient' ('O' is a predicate) and 'Ic' reads as 'Individual c is infallible' ('I' is a predicate). All other symbols are interpreted as usual. (1)–(3) can now be symbolized in the following way:

(1) ◆∃x(■Ox∧Ix). It is possible that there is some (individual) x such that it is necessary that x is omniscient and x is infallible.

Note that (1) only says that it is *possible* that there is an individual of a certain kind. (1) is consistent with the proposition that there are no (existing) individuals of this sort. If there is an (existing) individual that is necessarily omniscient and infallible, then obviously it is possible that there is an individual of this kind. But the converse does not necessarily hold. We shall define the concepts of omniscience and infallibility in the following way:

(O) ■∀x(Ox↔∀A(A→KxA)). It is necessary that for every (individual) x: x is omniscient if and only if (iff) for every (proposition) A, if A (is true) then x knows that A.

(I) ■∀x(Ix↔■∀A(BxA→A)). It is necessary that for every (individual) x: x is

infallible iff it is necessary that for every (proposition) A, if x believes that A then
A.

(O) is a definition of what we mean by 'omniscient,' and (I) is a definition of what we mean by 'infallible.' Obviously, (O) [(I)] in itself does not entail that there is anyone who is omniscient [infallible]. The first quantifier in (O) [I] varies over individuals and the second over propositions or sentences. Note that (I) entails that it is *necessary* that no infallible individual has any false beliefs, while (O) does not entail that an omniscient individual necessarily knows every truth.

> (2) ■∀x∀A(BxA→□BxA). It is necessary that for every (individual) x and for every (proposition) A, if x believes that A, then it is historically settled that x believes that A.

Note that '□BxA' in (2) does not assert that it is *necessary* that x believes that A; it says that it is *historically settled* that x believes that A. This is prima facie plausible. Facts about what someone believes seem to be historically settled; if someone believes something, it appears to be historically impossible for her not to believe this thing. Suppose that (2) is false. Then it is possible that there is someone who believes something even though it is not historically settled that she believes it. This is counterintuitive. Again, note that the first quantifier in (2) varies over individuals while the second varies over propositions or sentences.

> (3) ◆∃A(FA∧¬□FA). It is possible that there is a proposition A such that it will (some time in the future) be the case that A even though it is not historically necessary that A.

(3) is one way of expressing the idea that the future can be open. Suppose that (3) is not true. Then ■∀A(FA→□FA) is true; that is, then it is necessary that for every (proposition) A, if it will be the case that A then it is historically settled that it will be the case that A. If (3) is false, then it is necessary that nothing that will happen is such that it is historically possible that it will not happen. In other words, then it is necessary that the future is not open. The idea that the future is open can be symbolized in the following way: ∃A(FA∧¬□FA). Note that (3) only says that it is *possible* that the future is open. (3) is consistent both with the proposition that the future is *not* open and with the proposition that the future *is* open.

'■' and '◆' are used as 'absolute' S5-operators in this paper, and '□' and '◇' are used as 'relative' S5-operators. ■A is true in a possible world at a moment in time iff A is true in every possible world at every moment in time. ◆A is true in a possible world at a moment in time iff A is true in some possible world at some moment in time. □A is true in a possible world w at a moment in time t iff A is true in every possible world that is still historically accessible from w at t. ◇A is

true in a possible world w at a moment in time t iff A is true in some possible world that is still historically accessible from w at t. Intuitively, this means that □A is true in a possible world at a moment in time iff A is true no matter how the future turns out, and ◇A is true in a possible world at a moment in time iff there is still some way in which the future might evolve that would lead to the truth of A. Alternatively, we can say that A is historically possible in a possible world w at a certain moment in time t iff A is still possible at t given the history of w and the laws of nature that hold in w, and it is historically necessary that A in w at t iff A is true at t in every possible world that is still possible at t given the history of w and the laws of nature that hold in w. ■A is stronger than □A and ◆A is weaker than ◇A.[2] KcA is true in w at t iff A is true in all possible worlds that are epistemically accessible from w at t for the individual c. BcA is true in w at t iff A is true in all possible worlds that are doxastically accessible from w at t for the individual c.[3] The truth-conditions for the other formulas are standard.[4]

The main argument

Before I turn to the main argument and show that {(1), (2), (3)} is inconsistent, I will establish a lemma called '(O′).' Intuitively, (O′) says that every necessarily omniscient individual necessarily believes every truth. '¬■E,' '¬∀E,' '■E,' '∀E,' 'MP' ('Modus Ponens'), '◆E,' '∃E,' '¬□E' etc. in the derivations below represent standard derivation rules in propositional logic, predicate logic and modal logic. 'E' is an abbreviation of 'elimination.' 'PL' means that the step follows by standard propositional principles.

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[2] For more on modal logic, see, for example, Patrick Blackburn, Maarten De Rijke, Yde Venema, *Modal Logic* (Cambridge: Cambridge University Press, 2001), Brian F. Chellas, *Modal Logic: An Introduction* (Cambridge: Cambridge University Press, 1980) and George Edward Hughes and Max John Cresswell, *An Introduction to Modal Logic* (London: Routledge, 1968 (Reprinted 1990)).

[3] For more on epistemic and doxastic logic, see, for example, Roland Fagin, Joseph Y. Halpern, Yoram Moses, and Moshe Y. Vardi, *Reasoning About Knowledge* (Cambridge, Mass., London, England: The MIT Press, 1995) and John-Jules Ch. Meyer, and Wiebe van der Hoek, *Epistemic Logic for AI and Computer Science* (Cambridge University Press, 1995).

[4] Many references to the relevant literature on temporal logic can be found in John P. Burgess, "Basic Tense Logic," in *Handbook of Philosophical Logic*, Vol. 2, eds. Dov M. Gabbay and Franz Guenthner (Dordrecht: Reidel, 1984), 89–133 and Peter Øhrstrøm and Per Frederik Vilhelm Hasle, *Temporal Logic: From Ancient Ideas to Artificial Intelligence* (Dordrecht/Boston/London: Kluwer Academic Publishers, 1995). For some ideas about how to combine modal logic and tense logic, see Richmond H. Thomason, "Combinations of Tense and Modality," in *Handbook of Philosophical Logic*, vol. 2, eds. Dov M. Gabbay and Franz Guenthner (Dordrecht: Reidel, 2002), 135–165, (2nd edition 7, 2002, 205–234).

(O′) ■∀x(■Ox→■∀A(A→BxA)). It is necessary that every (individual) x who is necessarily omniscient is such that it is necessary that for every (proposition) A if A is the case then x believes that A.

In the derivation of (O′), I will use a basic assumption (KB). In a slogan, (KB) says that knowledge entails belief. Most epistemologists accept this proposition.

(KB) ■∀x∀A(KxA→BxA). It is necessary that for every (individual) x and for every (proposition) A, if x knows that A then x believes that A.

Let us now show that (O) and (KB) entail (O′). To establish this we assume that (O) and (KB) are true in some possible world w_0 at some moment in time t_0 and that (O′) is false in w_0 at t_0. This leads to a contradiction. Hence, (O′) follows from (O) and (KB).

(1) ¬■∀x(■Ox→■∀A(A→BxA)), w_0t_0	[Assumption]
(2) ¬∀x(■Ox→■∀A(A→BxA)), w_1t_1	[1, ¬■E]
(3) ¬(■Oc→■∀A(A→BcA)), w_1t_1	[2, ¬∀E]
(4) ■Oc, w_1t_1	[3, PL]
(5) ¬■∀A(A→BcA), w_1t_1	[3, PL]
(6) ¬∀A(A→BcA), w_2t_2	[5, ¬■E]
(7) ¬(X→BcX), w_2t_2	[6, ¬∀E]
(8) X, w_2t_2	[7, PL]
(9) ¬BcX, w_2t_2	[7, PL]
(10) ∀x(Ox↔∀A(A→KxA)), w_2t_2	[(O), ■E]
(11) Oc↔∀A(A→KcA), w_2t_2	[10, ∀E]
(12) Oc, w_2t_2	[4, ■E]
(13) ∀A(A→KcA), w_2t_2	[11, 12, PL]
(14) X→KcX, w_2t_2	[13, ∀E]
(15) KcX, w_2t_2	[8, 14, MP]
(16) ∀x∀A(KxA→BxA), w_2t_2	[(KB), ■E]
(17) ∀A(KcA→BcA), w_2t_2	[16, ∀E]
(18) KcX→BcX, w_2t_2	[17, ∀E]
(19) BcX, w_2t_2	[15, 18, MP]
(20) BcX∧¬BcX, w_2t_2	[19, 9, PL]

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We are now in a position to establish that $\{(1), (2), (3)\}$ is inconsistent. I will assume that all sentences in $\{(1), (2), (3)\}$ as well as (I) and (O) are true in a possible world w_0 at a moment in time t_0 and derive a contradiction. Intuitively, '$w_3 \equiv_{t_1} w_1$' in the derivation below reads as 'the possible world w_3 is historically accessible from the possible world w_1 at the time t_1,' and 'A, $w_0 t_0$' reads as 'A is true in the possible world w_0 at the time t_0,' etc. The following deduction shows that $\{(1), (2), (3)\}$ is inconsistent. Let us call this argument 'the main argument.'

(1) $\exists A(\mathsf{F}A \wedge \neg \Box \mathsf{F}A)$, $w_1 t_1$ [(3), ◆E]

(2) $\mathsf{F}X \wedge \neg \Box \mathsf{F}X$, $w_1 t_1$ [1, ∃E]

(3) $\mathsf{F}X$, $w_1 t_1$ [3, PL]

(4) $\neg \Box \mathsf{F}X$, $w_1 t_1$ [3, PL]

(5) $\exists x(\blacksquare Ox \wedge Ix)$, $w_2 t_2$ [(1), ◆E]

(6) $\blacksquare Oc \wedge Ic$, $w_2 t_2$ [5, ∃E]

(7) $\blacksquare Oc$, $w_2 t_2$ [6, PL]

(8) Ic, $w_2 t_2$ [6, PL]

(9) $\forall x(Ix \leftrightarrow \blacksquare \forall A(BxA \rightarrow A))$, $w_2 t_2$ [(I), $\blacksquare$E]

(10) $Ic \leftrightarrow \blacksquare \forall A(BcA \rightarrow A)$, $w_2 t_2$ [9, ∀E]

(11) $\blacksquare \forall A(BcA \rightarrow A)$, $w_2 t_2$ [8, 10, PL]

(12) $\forall x(\blacksquare Ox \rightarrow \blacksquare \forall A(A \rightarrow BxA))$, $w_2 t_2$ [(O'), $\blacksquare$E]

(13) $\blacksquare Oc \rightarrow \blacksquare \forall A(A \rightarrow BcA)$, $w_2 t_2$ [12, ∀E]

(14) $\blacksquare \forall A(A \rightarrow BcA)$, $w_2 t_2$ [7, 13, MP]

(15) $\forall A(A \rightarrow BxA)$, $w_1 t_1$ [14, $\blacksquare$E]

(16) $\mathsf{F}X \rightarrow Bx\mathsf{F}X$, $w_1 t_1$ [15, ∀E]

(17) $Bx\mathsf{F}X$, $w_1 t_1$ [3, 16, MP]

(18) $\forall x \forall A(BxA \rightarrow \Box BxA)$, $w_1 t_1$ [(2), $\blacksquare$E]

(19) $\forall A(BcA \rightarrow \Box BcA)$, $w_1 t_1$ [18, ∀E]

(20) $Bc\mathsf{F}X \rightarrow \Box Bc\mathsf{F}X$, $w_1 t_1$ [19, ∀E]

(21) $\Box Bc\mathsf{F}X$, $w_1 t_1$ [17, 20, MP]

(22) $w_3 \equiv_{t_1} w_1$ [4, $\neg\Box$E]

(23) $\neg \mathsf{F}X$, $w_3 t_1$ [4, $\neg\Box$E]

(24) $Bc\mathsf{F}X$, $w_3 t_1$ [21, 22, $\Box$E]

(25) $\forall A(BcA \rightarrow A)$, $w_3 t_1$ [11, ■E]

(26) $Bc\mathsf{F}X \rightarrow \mathsf{F}X$, $w_3 t_1$ [25, $\forall$E]

(27) $\mathsf{F}X$, $w_3 t_1$ [24, 26, MP]

(28) $\mathsf{F}X \wedge \neg \mathsf{F}X$, $w_3 t_1$ [27, 23, PL]

Obviously, (28) is a contradiction. Accordingly, {(1), (2), (3)} entails a contradiction. Therefore {(1), (2), (3)} is inconsistent. It follows that at least one sentence in {(1), (2), (3)} must be false. This fact justifies the assertion that {(1), (2), (3)} is an aporia.

2. Arguments for the Sentences in the Aporia

I have said that the sentences in the aporia of omniscience are intuitively reasonable. However, a sceptic might argue that we cannot rely simply on our intuitions. To take the puzzle seriously we also need some independent reasons for the sentences in {(1), (2), (3)}. Therefore, I will consider some arguments for the sentences in the aporia in this section.

<u>Arguments for (1): It is possible that there is someone who is necessarily omniscient and infallible</u>

Is it possible that there is someone who is necessarily omniscient and infallible? I will consider two arguments for this proposition, which I will call the 'the argument from conceivability' and 'the argument from doxastic consistency.'

The argument from conceivability

> (CP) It is conceivable that there is someone who is necessarily omniscient and infallible.

Hence,

> (1) It is possible that there is someone who is necessarily omniscient and infallible.

If conceivability implies possibility, as some seem to think, then the argument from conceivability is valid. Furthermore, if (CP) is true, as it certainly seems to be, then the argument is also sound and we must conclude that (1) is true. However, the conceivability implies possibility thesis is controversial. It is not obvious that everything that is conceivable is possible. Nevertheless, it appears to be reasonable to say that conceivability 'indicates' possibility. If something is conceivable, then we have a prima facie reason to believe that it is possible, even if it should turn out

to be the case that there are no necessary connections between conceivability and possibility. Accordingly, the argument from conceivability seems to give some support to (1).[5] The following scenario underpins (CP). Imagine an individual called 'The All-knowing One.' Suppose that The All-knowing One knows every truth in every possible world and that it is necessary that everything The All-knowing One believes is true. Then The All-knowing One is both necessarily omniscient and infallible. Such a being is conceivable. Hence, it is conceivable that there is an individual who is necessarily omniscient and infallible. Furthermore, this scenario does not seem to entail any contradiction. Therefore, (CP) is true. It follows that we have a prima facie reason to believe that (1) is true.

The argument from doxastic consistency

Here is a different argument for (1), which we will call 'the argument from doxastic consistency.' (1) follows from the proposition that it is possible that there is someone who is necessarily omniscient and necessarily doxastically consistent; in symbols, $\blacklozenge\exists x(\blacksquare Ox \land Ix)$ follows from $(1') = \blacklozenge\exists x(\blacksquare Ox \land \blacksquare Dx)$. An individual x is doxastically consistent iff there is no (proposition) A such that x believes that A and x believes that not-A. In other words, doxastic consistency is defined in the following way:

> (DC) $\blacksquare\forall x(Dx \leftrightarrow \neg\exists A(BxA \land Bx\neg A))$. It is necessary that an individual x is doxastically consistent iff there is no A such that x believes that A and x believes that not-A.

This argument is interesting since doxastic consistency seems to be a weaker property than infallibility. If someone is infallible, she is doxastically consistent, but someone can be (necessarily) doxastically consistent without being infallible. So, the fact that $\blacklozenge\exists x(\blacksquare Ox \land \blacksquare Dx)$ entails $\blacklozenge\exists x(\blacksquare Ox \land Ix)$ is noteworthy. To prove this proposition, we will first establish that everyone who is necessarily omniscient and necessarily doxastically consistent is infallible given that knowledge entails belief. Let us call this lemma (ODI).

> (ODI) $\blacksquare\forall x((\blacksquare Ox \land \blacksquare Dx) \to Ix)$. It is necessary that every (individual) x who is necessarily omniscient and necessarily doxastically consistent is infallible.

Here is the proof of (ODI). (The proof is a reductio argument where we assume the negation of (ODI) and derive a contradiction. This establishes our result.)

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[5] For more on the conceivability implies possibility thesis and for some arguments for and against this principle, see, for example, Tamar Szabó Gendler and John Hawthorne (eds.), *Conceivability and Possibility* (Oxford: Clarendon Press, 2002).

(1) $\neg\blacksquare\forall x((\blacksquare Ox \wedge \blacksquare Dx)\rightarrow Ix)$, w_0t_0 [Assumption]

(2) $\neg\forall x((\blacksquare Ox \wedge \blacksquare Dx)\rightarrow Ix)$, w_1t_1 [1, $\neg\blacksquare$E]

(3) $\neg((\blacksquare Oc \wedge \blacksquare Dc)\rightarrow Ic)$, w_1t_1 [2, $\neg\forall$E]

(4) $\blacksquare Oc \wedge \blacksquare Dc$, w_1t_1 [3, PL]

(5) $\neg Ic$, w_1t_1 [3, PL]

(6) $\blacksquare Oc$, w_1t_1 [4, PL]

(7) $\blacksquare Dc$, w_1t_1 [4, PL]

(8) $\forall x(Ix \leftrightarrow \blacksquare\forall A(BxA\rightarrow A))$, w_1t_1 [(I), $\blacksquare$E]

(9) $Ic \leftrightarrow \blacksquare\forall A(BcA\rightarrow A)$, w_1t_1 [8, $\forall$E]

(10) $\neg\blacksquare\forall A(BcA\rightarrow A)$, w_1t_1 [5, 9, PL]

(11) $\neg\forall A(BcA\rightarrow A)$, w_2t_2 [10, $\neg\blacksquare$E]

(12) $\neg(BcX\rightarrow X)$, w_2t_2 [11, $\neg\forall$E]

(13) BcX, w_2t_2 [12, PL]

(14) $\neg X$, w_2t_2 [12, PL]

(15) $\forall x(Ox \leftrightarrow \forall A(A\rightarrow KxA))$, w_2t_2 [(O), $\blacksquare$E]

(16) $Oc \leftrightarrow \forall A(A\rightarrow KcA)$, w_2t_2 [15, $\forall$E]

(17) Oc, w_2t_2 [6, $\blacksquare$E]

(18) $\forall A(A\rightarrow KcA)$, w_2t_2 [16, 17, PL]

(19) $\neg X\rightarrow Kc\neg X$, w_2t_2 [18, $\forall$E]

(20) $\forall x(Dx \leftrightarrow \neg\exists A(BxA\wedge Bx\neg A))$, w_2t_2 [(DC), $\blacksquare$E]

(21) $Dc \leftrightarrow \neg\exists A(BcA\wedge Bc\neg A)$, w_2t_2 [20, $\forall$E]

(22) Dc, w_2t_2 [7, $\blacksquare$E]

(23) $\neg\exists A(BcA\wedge Bc\neg A)$, w_2t_2 [21, 22, PL]

(24) $\neg(BcX\wedge Bc\neg X)$, w_2t_2 [23, $\neg\exists$E]

(25) $\forall x\forall A(KxA\rightarrow BxA)$, w_2t_2 [(KB), $\blacksquare$E]

(26) $\forall A(KcA\rightarrow BcA)$, w_2t_2 [25, $\forall$E]

(27) $Kc\neg X\rightarrow Bc\neg X$, w_2t_2 [26, $\forall$E]

(28) $Kc\neg X$, w_2t_2 [14, 19, MP]

(29) $Bc\neg X$, w_2t_2 [27, 28, MP]

(30) $BcX\wedge Bc\neg X$, w_2t_2 [13, 29, PL]

(31) $(BcX \wedge Bc\neg X) \wedge \neg(BcX \wedge Bc\neg X)$, w_2t_2 [30, 24, PL]

Clearly (31) is a contradiction. Hence, the assumption that (ODI) is false in some possible world at some moment in time must be false. It follows that (ODI) is valid. We are now in a position to establish that $\blacklozenge \exists x(\blacksquare Ox \wedge Ix)$ follows from $\blacklozenge \exists x(\blacksquare Ox \wedge \blacksquare Dx)$. Here is the proof:

(1) $\blacklozenge \exists x(\blacksquare Ox \wedge \blacksquare Dx)$, w_0t_0 [Assumption]

(2) $\neg\blacklozenge \exists x(\blacksquare Ox \wedge Ix)$, w_0t_0 [Assumption]

(3) $\exists x(\blacksquare Ox \wedge \blacksquare Dx)$, w_1t_1 [1, $\blacklozenge$E]

(4) $\blacksquare Oc \wedge \blacksquare Dc$, w_1t_1 [3, $\exists$E]

(5) $\blacksquare Oc$, w_1t_1 [4, PL]

(6) $\blacksquare Dc$, w_1t_1 [4, PL]

(7) $\forall x((\blacksquare Ox \wedge \blacksquare Dx) \rightarrow Ix)$, w_1t_1 [(ODI), $\blacksquare$E]

(8) $(\blacksquare Oc \wedge \blacksquare Dc) \rightarrow Ic$, w_1t_1 [7, $\forall$E]

(9) Ic, w_1t_1 [4, 8, MP]

(10) $\neg\exists x(\blacksquare Ox \wedge Ix)$, w_1t_1 [2, $\neg\blacklozenge$E]

(11) $\neg(\blacksquare Oc \wedge Ic)$, w_1t_1 [10, $\neg\exists$E]

(12) $\blacksquare Oc \wedge Ic$, w_1t_1 [5, 9, PL]

(13) $(\blacksquare Oc \wedge Ic) \wedge \neg(\blacksquare Oc \wedge Ic)$, w_1t_1 [12, 11, PL]

Arguments for (2): It is necessary that all beliefs are historically settled

Is it true that it is necessary that all beliefs are historically settled? In this section, I will consider an 'intuitive' argument for proposition (2) and a semantic argument.

The intuitive argument

Suppose that (2) is not true. Then it seems that what someone believes might depend on what will turn out to be the case in the future. This is counterintuitive. Consider the following example. It is not historically settled that a democrat will win the next election. It is historically possible that a democrat will win and it is historically possible that it is not the case that a democrat will win. Moreover, suppose that The Omniscient One is necessarily omniscient and infallible. Then whether The Omniscient One believes that a democrat will win or not is not yet historically settled. If a democrat will win, then it will be true once the democrat has won that it was the case (now) that The Omniscient One believed that a democrat would win; and if a democrat will not win, then it will be true once the

democrat has lost that it was the case (now) that The Omniscient One did not believe that a democrat would win. However, as of this moment it is not a settled fact whether The Omniscient One believes that a democrat will win or not. Scenarios such as this clearly suggest that (2) is true.

Furthermore, this is not the only possible reason for (2). There are several possible semantic arguments for this proposition. Let us consider one such argument, which I will call 'the argument from semantics.'

The argument from semantics

It is possible to show that $\blacksquare\forall x\forall A(BxA\rightarrow\square BxA)$ is valid if we assume the following semantic condition:

> (SC) If a possible world w' is historically accessible from a possible world w at a time t and a possible world w'' is doxastically accessible from w' at t for an individual c, then w'' is doxastically accessible from w at t for c.

Suppose that (SC) holds. Then we can prove that (2) is valid. Assume that (2) = $\blacksquare\forall x\forall A(BxA\rightarrow\square BxA)$ is not true in the possible world w_0 at the moment in time t_0. Then there is some possible world w_1 and some moment of time t_1, such that $\forall x\forall A(BxA\rightarrow\square BxA)$ is false in w_1 at t_1. Accordingly, $\forall A(BcA\rightarrow\square BcA)$ is false in w_1 at t_1 (for some arbitrary individual c). Consequently, $BcX\rightarrow\square BcX$ is false in w_1 at t_1 (where 'X' represents some arbitrary proposition). Therefore, BcX is true in w_1 at t_1, while $\square BcX$ is false in w_1 at t_1. It follows that there is a possible world w_2 that is historically accessible from w_1 at t_1 in which BcX is false at t_1, for $\square BcX$ is false in w_1 at t_1. Accordingly, there is some possible world w_3 that is doxastically accessible from w_2 at t_1 for c in which X is false at t_1. Since w_2 is historically accessible from w_1 at t_1 and w_3 is doxastically accessible from w_2 at t_1 for c, w_3 is doxastically accessible from w_1 at t_1 for c (condition (SC)). In conclusion, X is true in w_3 at t_1, for BcX is true in w_1 at t_1 and w_3 is doxastically accessible from w_1 at t_1 for c. Yet, this is contradictory. So, our assumption is false. It follows that $\blacksquare\forall x\forall A(BxA\rightarrow\square BxA)$ is valid.

Arguments for 3: It is possible that the future is open

Is it true that it is possible that the future is open? We have seen that there are arguments for (1) and (2). I will now consider two brief arguments for (3), which I will call 'the argument from science' and 'the argument from conceivability.'

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The argument from science

According to the argument from science, it is a scientific fact that the future is open. It is a scientific fact because according to the dominating interpretations of modern physics at least some processes in nature are indeterminate. It is, for example, historically possible that the photon will pass through the right slit and it is historically possible that it will pass through the left slit. Hence, the future is open. And if the future *is* open, then it is obvious that it is *possible* that it is open.[6]

The argument from conceivability

Suppose (contrary-to-the-facts?) that the future is not open. It is still *conceivable* that the future is open and that not everything that will happen is historically settled. Accordingly, we have a prima facie reason to believe that it is *possible* that the future is open (see the discussion about conceivability above). Even if it should turn out that the future is not open, it *could* have been open. Surely, our best current scientific theories at least *could* have been true.

All arguments in this section can be criticized and since {(1), (2), (3)} is inconsistent, it seems that at least some of the arguments must be unsound. However, together they show that we should take the aporia of omniscience seriously. How should we solve this puzzle? Let us now consider some possible solutions.

3. Possible Solutions

In this section, I will consider nine possible solutions to the aporia of omniscience. Some solutions seem more attractive than others, but no solution is entirely unproblematic. This gives further support to the claim that the aporia of omniscience really is an aporia. According to the first attempt, we should accept dialetheism.

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[6] This argument is not conclusive since there are many different interpretations of modern science. However, it clearly suggests that there is genuine randomness in nature and that the future is open. For more on different interpretations of modern science and some relevant references, see, for example, Birgitte Falkenburg and Friedel Weinert, "Indeterminism and Determinism in Quantum Mechanics," in *Compendium of Quantum Physics*, eds. Daniel Greenberger, Klaus Hentschel and Friedel Weinert (Springer, 2009), 307–311, David Hodgson, "Quantum Physics, Consciousness, and Free Will," in *The Oxford Handbook of Free Will*, ed. Robert Kane (Oxford: Oxford University Press, 2011), 57–83, Wesley C. Salmon, *Causality and Explanation* (Oxford: Oxford University Press, 1998), Ch. 2.

Solution 1: Accept dialetheism

One solution to the problem of omniscience is to accept the idea that there are true contradictions. Some philosophers, so-called dialetheists, believe that there are sentences that are both true and false. If we accept this idea, we might also accept the idea that it is possible to derive a contradiction from {(1), (2), (3)} even though all sentences in this set are true. This might be perfectly acceptable if there are true contradictions.

Nevertheless, this solution is quite problematic. Dialetheism is a very controversial theory and most dialetheists probably agree that not *every* contradiction is genuine (true). Hence, even a dialetheist might think that it is problematic that {(1), (2), (3)} is inconsistent.[7] So, it seems doubtful that this solution should turn out to be the most plausible overall.

Solution 2: Reject (1) because the concept of omniscience is incoherent

According to the second solution, we should reject (1) because the concept of omniscience is incoherent. It only seems to be possible that there is someone who is necessarily omniscient and infallible, but when we analyse the concept closer we can see that it is incoherent. There are other arguments in the literature that suggest that it is problematic to assume that it is possible that there is someone who is omniscient. It has, for example, been suggested that the concept of omniscience is inconsistent with so-called *de re* and *de se* beliefs, with human freedom and with the fact that there is no set of all truths.[8] So, maybe it is reasonable to reject (1) for many different reasons.

However, it certainly seems to be possible that there is someone who is necessarily omniscient and infallible. So, if we can reject (1) but accept some similar proposition instead of claiming that the concept of omniscience is incoherent, this appears to be a prima facie more plausible solution. Accordingly, our next solution might be better.

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[7] For more on dialetheism, see, for example, Graham Priest, Francesco Berto, Zach Weber, "Dialetheism," in *The Stanford Encyclopedia of Philosophy*, ed. Edward N. Zalta (2019 Edition), URL = < https://plato.stanford.edu/entries/dialetheism/>.

[8] For more on this, see, for example, Edward R. Wierenga, "Omniscience," in *The Oxford Handbook of Philosophical Theology*, eds. Thomas P. Flint and Michael C. Rea (Oxford and New York: Oxford University Press, 2009), 129–144. It is beyond the scope of this paper to discuss these arguments in detail.

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Solution 3: Reject (1) but accept some similar proposition

According to the third solution, we reject (1) not because the concept of omniscience is incoherent, but because we should not assume that it is possible that there is someone who is *necessarily* omniscient and infallible. This is compatible with accepting something similar, namely (1″).

> (1″) ◆∃x(Ox∧Ix). It is possible that there is some (individual) x such that x is omniscient and x is infallible.

If we reject (1) and accept (1″), the main argument does not go through; it breaks down at step (14). If this solution is correct, we can solve the aporia of omniscience and still assume that it is possible that there is someone who is omniscient and infallible. We do not have to claim that the concept of omniscience is incoherent. This suggests that solution 3 is better than solution 2. The concept of omniscience does not seem to be incoherent.

Nevertheless, this solution is not unproblematic, since we still have to reject (1), which is an intuitively attractive proposition. So, let us see if there are any other possible solutions to the puzzle.

Solution 4: Reject (2) because not all beliefs are historically settled

According to the fourth solution, we should reject the idea that it is necessary that all our beliefs are historically settled, that is, we should reject proposition (2). If (2) is not true, then the following sentence is true '◆∃x∃A(BxA∧¬□BxA),' which says that it is possible that there is someone who believes some proposition even though it is not historically settled that she believes this proposition. (2) might be false because at least some of our beliefs might depend on what happens in the future in the sense that what we believe now depends on what will actually happen later on. This idea is strange, but perhaps not so strange that we should reject it. Some propositions about the future certainly seem to be historically open, for example, the judgement that a democrat will win the next election. It is not (now) historically settled whether this proposition is true or not. If a democrat will win the next election, we can say that the proposition that a democrat will win was true (now) once it is settled that the democrat did win. Moreover, if a democrat will not win the next election, we can say that the proposition that a democrat will win was not true (now) once it is settled that the democrat did not win. Right now, there is no fact of the matter whether this proposition is settled true or not; we have to wait to see who is going to win to 'decide' its truth-value. Propositions about our beliefs could behave in a similar way. Perhaps we can believe some propositions even though it is not historically settled that we believe them. If this

is true, we can solve the aporia of omniscience. Given that we reject (2), we must also reject (SC), since (SC) entails (2). However, this might be a small price to pay if we are willing to give up the idea that our beliefs are historically settled.[9]

A problem with this solution is that there appear to be important and relevant differences between the proposition that a democrat will win the next election and the proposition that an individual *believes* that a democrat will win the next election. It is reasonable to assume that the first proposition is not historically settled if the future is open, for this proposition tells us something about the future. Nevertheless, the second proposition does not seem to assert anything about what will happen later on. It only says something about what someone *believes* about the future. Our current beliefs (whatever their contents) do not seem to depend on the future. We might have to give up this view since {(1), (2), (3)} is inconsistent, but before we do that, let us see if there are any other solutions to the aporia.

Solution 5: Reject (3) and assume some similar principle

According to the fifth solution, we should reject (3) but assume some similar principle. It is not possible that the future is open in the sense that (3) is true, but it might be possible in some other sense. (3) is not a plausible explication of what it means to say that it is possible that the future is open.

The problem with this solution is that it is not easy to come up with some other reasonable explication. We could perhaps say that the future is open in a possible world w at a time t iff there is some proposition A such that it is historically possible that it will be the case that A and it is historically possible that it will not be the case that A in w at t. In other words, the future is open in w at t iff $\exists A(\diamond FA \wedge \diamond \neg FA)$ is true in w at t. Maybe (1) and (2) is consistent with (3′) = $\blacklozenge \exists A(\diamond FA \wedge \diamond \neg FA)$ even though {(1), (2), (3)} is inconsistent. $\blacklozenge \exists A(FA \wedge \neg \Box FA)$ entails $\blacklozenge \exists A(\diamond FA \wedge \diamond \neg FA)$, but not vice versa. However, this particular suggestion does not solve the problem, for {(1), (2), (3′)} is also inconsistent (the proof of this is left to the reader). So, unless someone can come up with an alternative analysis of

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[9] Solution 4 can perhaps be called the Ockhamist solution since it is similar to a solution that Ockham suggested to a similar problem. For more on this, see William of Ockham, *Predestination, God's Foreknowledge, and Future Contingents. Translated with an Introduction, Notes, and Appendices by Marilyn McCord Adams and Norman Kretzmann*, 2nd ed. (Indianapolis: Hackett Publishing, 1983), especially Question II, Article IV. However, it is not obvious how Ockham should be interpreted and the problem he discusses is not exactly the same as the aporia of omniscience.

what it means to say that it is possible that the future is open that is plausible, it seems that we cannot use this solution to the aporia of omniscience.

Solution 6: Reject (3) and assume that it is not possible that the future is open

According to the sixth solution, we should reject (3) and not assume any similar principle. It is not possible that the future is open in the sense that (3) is true, and it is not possible in any other interesting sense either. Since {(1), (2), (3)} is inconsistent, we must probably give up some of our intuitions. So, maybe we should accept the 'fact' that it is not possible that the future is open.

However, this solution is counterintuitive and if we accept it, we must reject the argument from science and the argument from conceivability for (3) (see above). And those arguments appear to be particularly strong. If we accept this solution, it seems that we have to conclude that it is not conceivable that the future is open or that conceivability in this case does not entail possibility. It also appears to be the case that we must deny that our best current science shows that the future is open or else deny that the fact that the future is open entails that (3) is true. Consequently, this solution comes with a very high price.

Solution 7: Reject the definition of infallibility

It is possible to solve the puzzle by rejecting the definition of infallibility (I). If we use the following alternative analysis of this concept, the main argument does not go through any more:

> (I′) ■∀x(Ix↔∀A(BxA→A)). It is necessary that for every (individual) x: x is infallible if and only if everything x believes is true.

According to this definition, it is possible that someone is infallible in a possible world at a moment of time without being infallible in every possible world at every moment in time. Suppose that we use (I′) and not (I) to define the concept of infallibility. Then step (25) in the main argument breaks down. Hence, {(1), (2), (3)} is no longer inconsistent.

Nevertheless, it is doubtful that this solution is plausible, since we can show that some similar sets are inconsistent. Suppose that we replace (1) by (1′) = ◆∃x(■Ox∧■Dx) or by (1′′′) = ◆∃x(■Ox∧■Ix) and (I) by (I′). Then the following sets are inconsistent: {(1′), (2), (3)} and {(1′′′), (2), (3)} (proofs are left to the reader). So, even though {(1), (2), (3)} is no longer inconsistent if we use definition (I′), at least two other, equally problematic, sets of sentences are inconsistent given definition (I′). A more plausible solution might be to reject the definition of omniscience instead of the definition of infallibility.

Solution 8: Reject the definition of omniscience

According to the eight solution, we should reject the definition of omniscience. Perhaps omniscience does not require knowledge of absolutely *all* truths, perhaps it only requires knowledge of *some* truths. Consider the following alternative analysis of the concept:

> (O″) ■∀x(Ox↔∀A(□A→KxA)). It is necessary that for every (individual) x: x is omniscient if and only if (iff) for every (proposition) A, if it is historically settled that A (is true) then x knows that A.

If we use this definition of omniscience instead of (O), we cannot use the main argument to prove that {(1), (2), (3)} is inconsistent any longer. At step (16) we would arrive at '□FX→BxFX…' rather than 'FX→BxFX… .' According to this solution, an omniscient being will only have knowledge about truths about the future that are historically settled. Suppose it is not historically settled that a democrat will win the next election. Then an omniscient individual will not know that a democrat will win the next election (nor will this individual know that a democrat will not will). Such a person can still know many propositions about the future that are historically settled, for example, that it will be the case that a democrat will win or that a democrat will not win, that it will be the case that 1 + 1 = 2, that it will be the case that $E = mc^2$, etc. Nevertheless, if it is not historically settled that it will be the case that A, then not even an omniscient individual will know that it will be the case that A. Consequently, if there are truths about the future that are not historically settled, then not even an omniscient individual will have knowledge of such truths.

This solution is attractive in many respects. If it is not settled yet whether or not a democrat will win the next election, how could anyone know that a democrat will (or will not) win. Still, if we accept this solution, we have to assume that an omniscient individual's knowledge of the future is very limited. She (or he) will not know anything about the future that is not historically settled (or, at least, it is not necessary that she (he) has such knowledge). This seems counterintuitive.

Solution 9: Reject the principle of bivalence for future contingents

According to the ninth solution, we should reject the principle of bivalence for future contingents. There are no historically contingent truths about the future. If it is historically possible that a democrat will win and it is historically possible that a democrat will not win, then the sentence 'A democrat will win the next election' is neither true nor false (nor is the sentence 'A democrat will not win the next election' true or false). How does this solve the aporia of omniscience? Well, if

there are no truths about the (historically) contingent future, the propositional (or sentential) quantifier in the definition of omniscience (O) does not range over absolutely every proposition (sentence). Suppose that the individual i is omniscient. Then i knows every truth, that is, then $\forall A(A \rightarrow KiA)$ holds. However, since not every sentence about the future is either true or false, we cannot instantiate $\forall A(A \rightarrow KiA)$ with any sentence whatsoever. In particular, we cannot instantiate this sentence with FA, that is, $FA \rightarrow KiFA$ does not follow from $\forall A(A \rightarrow KiA)$. FA may speak about a contingent truth about the future, for example, that a democrat will win the next election. If this is correct, step (16) does not follow from step (15) in the main argument. Hence, the derivation does not go through. Consequently, we can avoid the aporia of omniscience by denying that an omniscient individual has knowledge of the (historically) contingent future. This does not entail that an individual of this kind is less than omniscient, for there are no truths about the contingent future for this individual to know.[10]

A problem with this solution is that we have to assume that the principle of bivalence is false for some sentences. And this principle is intuitively very reasonable and a part of standard (propositional) logic.

4. Conclusion

In this paper, I have discussed a new aporia, the aporia of omniscience. This aporia includes three propositions: (1) It is possible that there is someone who is necessarily omniscient and infallible, (2) It is necessary that all beliefs are historically settled, and (3) It is possible that the future is open. Every sentence in this set is intuitively reasonable and there are prima facie plausible arguments for each of them. However, the whole set {(1), (2), (3)} is inconsistent. Therefore, it seems to be the case that at least one of the propositions in this set must be false. I

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[10] The idea that propositions about the contingent future are neither true nor false is old. It has been argued that already Aristotle defended this position (even though there are other interpretations of the Greek philosopher). For more on this, see Aristotle, *Aristotle's "Categories" and "De Interpretatione," Translated with Notes and Glossary by John Lloyd Ackrill* (Oxford: Clarendon/ Oxford Press, 1963). In the 20th century Jan Łukasiewicz suggested that we need a three-valued logic to deal with similar problems (see papers in Storrs McCall (ed.), *Polish Logic 1920–1939* (Oxford: Clarendon Press, 1967)), a view that seems to have been shared by Arthur N. Prior in the 1950's (Arthur N. Prior, "On Three–Valued Logic and Future Contingents," *The Philosophical Quarterly*, Vol. 3 (1953): 317–326). For more on the history of this idea and many relevant references, see, for example, William Lane Craig, *The Problem of Divine Foreknowledge and Future Contingents from Aristotle to Suarez* (Leiden: E. J. Brill, 1988) and Peter Øhrstrøm and Per Frederik Vilhelm Hasle, *Temporal Logic: From Ancient Ideas to Artificial Intelligence* (Dordrecht/Boston/London: Kluwer Academic Publishers, 1995).

have discussed nine different possible solutions to this aporia and I have considered some arguments for and against these solutions. Some solutions seem more promising than others. The solutions that reject (3) are, for example, quite problematic since they seem to contradict our current best science. And the first solution is reasonable only if dialetheism, which is a quite controversial theory, is plausible. However, it is not obvious which solution is the best all things considered and there are arguments against all of them. The fact that it is difficult to tell which solution we should choose reinforces the claim that the aporia of omniscience really is an aporia. No matter how we choose to solve the puzzle, it seems that we have to give up some of our intuitions.[11]

DISCUSSION NOTE/ DEBATE

PROCESS RELIABILISM, PRIME NUMBERS AND THE GENERALITY PROBLEM

Frederik J. ANDERSEN, Klemens KAPPEL

ABSTRACT: This paper aims to show that Selim Berker's widely discussed *prime number case* is merely an instance of the well-known *generality problem* for process reliabilism and thus arguably not as interesting a case as one might have thought. Initially, Berker's case is introduced and interpreted. Then the most recent response to the case from the literature is presented. Eventually, it is argued that Berker's case is nothing but a straightforward consequence of the generality problem, i.e., the problematic aspect of the case for process reliabilism (if any) is already captured by the generality problem.

KEYWORDS: Berker's prime number case, process reliabilism, the generality problem

1. Introduction

In recent debate on *process reliabilism*[1] a specific case crafted by Selim Berker has gained attention.[2] The case, which concerns cognitive processes and prime numbers, is meant as a challenge leveled against reliabilism:[3]

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[1] In rough outline *process reliabilism* as proposed by Alvin Goldman is the following view: A belief-token *b* is epistemically justified if and only if *b* is caused/sustained by a reliable process. Here, a *reliable process* is a process of belief formation that (would) produce(s) a sufficiently high ratio of true to false beliefs, given a specified set of circumstances and a domain of application. Process reliabilism was first proposed and defended by Alvin Goldman. See for example Alvin Goldman, "What Is Justified Belief?," in *Justification and knowledge*, ed. G. Pappas (Springer, 1979), 1-23; Alvin Goldman, *Epistemology and Cognition* (Harvard University Press, 1986).

[2] The case was originally stated in Selim Berker, "The Rejection of Epistemic Consequentialism," *Philosophical Issues* 23 (2013): 363-387. It was then criticized in Alvin Goldman, "Reliabilism, Veritism, and Epistemic Consequentialism," *Episteme* 12, 2 (2015): 131-143. Berker responded in Selim Berker, "Reply to Goldman: Cutting Up the One to Save the Five in Epistemology," *Episteme* 12 (2015): 145-153. Recently, Berker's case was further criticized in Jeffrey Dunn and Kristoffer Ahlstrom-Vij, "Is Reliabilism a Form of Consequentialism?" *American Philosophical Quarterly* 54 (2) (2017): 183-194.

> **Prime numbers**. Suppose the following is true of me: whenever I contemplate whether a given natural number is prime, I form a belief that it is not. 'Is 25 prime? No, it is not.' 'Is 604 prime? No, it is not.' 'Is 7 prime? No, it is not.' Let us also stipulate that this is the only cognitive process by which I form beliefs about the primeness of natural numbers [...]. Since the ratio of prime to composite numbers less than n approaches 0 as n approaches infinity, my belief-forming process tends to yield a ratio of true to false beliefs that approaches 1. Therefore process reliabilists are forced to say that, because my belief-forming process is almost perfectly reliable, any belief formed on its basis is justified. But that's crazy! When I form a belief that 7 is not prime, it is simply not correct to say that, although that belief is false, it is epistemically redeemed by the truth of the other beliefs which would be formed via the process that led to it.[4]

Berker claims that a cognitive process like *whenever I contemplate whether a given natural number is prime, I form a belief that it is not* must be deemed reliable by the process reliabilist and that this is absurd. Berker is basing his claim on the observation that "Since the ratio of prime to composite numbers less than n approaches 0 as n approaches infinity, my belief-forming process tends to yield a ratio of true to false beliefs that approaches 1."[5] This will allegedly have the consequence that whenever an agent considers whether a given natural number n is prime or not, the agent will be epistemically justified in believing that it is not, even in cases where this is obviously false (e.g., in the case of 7). According to Berker this leads to a specific type of problematic trade-off between propositions for the reliabilist. Since the epistemic status of a concrete belief-token under evaluation, say the belief that *7 is not prime*, will be determined by the ratio of true to false beliefs – some of which will have different propositional content from the belief under evaluation – which the prime number-process outputs, it is claimed that in such cases process reliabilism sacrifices one proposition for other propositions, or that one instance of epistemic wrongdoing is perpetrated for the sake of a greater epistemic good, or, echoing Rawls, that the "separateness of propositions" is not respected.[6]

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[3] Note that we use the terms 'process reliabilism' and simply 'reliabilism' interchangeably throughout the paper.

[4] Berker, "The Rejection of Epistemic Consequentialism," 374-375.

[5] Berker, "The Rejection of Epistemic Consequentialism," 374-375.

[6] Berker, "Reply to Goldman," 145-153.

2. Recent Response by Dunn and Ahlstrom-Vij

Let us now consider the most recent response[7] to **Prime Numbers** by Jeffery Dunn and Kristoffer Ahlstrom-Vij (henceforth 'D & A').[8] D & A defend process reliabilism by observing that Berker helps himself to certain crucial implicit assumptions to make his case work, i.e., to make the prime number-process seem plausible and reliable:

> [...] Berker wants us to assume that (*a*) there are processes dedicated to generating beliefs about primehood, (*b*) he is relying on such a dedicated process – let us refer to it as *P* – in the scenario imagined, (*c*) for any number queried, *P* generates the output that it is not prime, and (*d*) numbers are queried in some quasi-random way among the natural numbers. Under those assumptions, *P* is reliable. And if so, reliabilism should say that the belief *that 7 is not prime*, generated by way of *P*, is justified. That, we claim, is the correct verdict. Any feeling that this is a counter-intuitive verdict should be traced, not to reliabilism, but to the psychological implausibility of (*a*), especially when paired with (*d*). For note that it is implausible indeed that some agent is as likely to contemplate whether 73,046,482,192,753 is prime as whether 53 is prime.[9]

Thus, D & A admit that given assumptions (a)-(d) stated above, the prime number-process will turn out to be reliable from the perspective of the reliabilist. However, they also point to an explanation of the counter-intuitiveness in **Prime Numbers**. They submit that the counter-intuitiveness is not due to process reliabilism *per se*, but to the psychological implausibility of assumption (a), and especially in conjunction with (d). It just seems implausible for there to be a human cognitive process dedicated to contemplating whether (more or less) random natural numbers are prime. Hence, D & A do appear to have an appealing defence of process reliabilism against Berker's case.

3. The Generality Problem

Even though we side with D & A with respect to **Prime Numbers**, we want to add some important qualifications in this section. In fact, we believe that D & A give Berker too much credit in their response. For it is clear, with or without assumptions (a)-(d) explicated, that Berker's proposed prime number-process has

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[7] Due to the limitations of space we have decided solely to present the newest response to Berker's case. Goldman gives three objections to the case in Goldman, "Reliabilism, Veritism, and Epistemic Consequentialism," 131-143. Note that none of Goldman's responses foresees our point about the generality problem below (cf. section 3).

[8] Dunn and Ahlstrom-Vij, "Is Reliabilism a Form of Consequentialism," 183-194.

[9] Dunn and Ahlstrom-Vij, "Is Reliabilism a Form of Consequentialism," 187.

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nothing to do with the standard cognitive human processes that are of interest to process reliabilists.[10] Cardinal examples of such processes are *visual perception*, *(long-term) memory*, *competent deduction* etc. Thus, we find it unnecessary for D & A to admit that the prime number-process will be deemed reliable by the process reliabilist under any circumstances. Berker simply misconstrues the reliabilist.

To give an extreme example illustrating why one should not give Berker any wriggle room in making a strawman out of reliabilism, consider the following cognitive process: *Every time I see a living individual on the streets of London, I will form the belief that this individual is not a crocodile*. Given various assumptions, e.g., a restriction to normal worlds, we could make this process look extremely reliable because only very rarely (if ever) crocodiles are seen on the streets of London, but this is still no reason for the process reliabilist to admit that this is a reliable cognitive process. An argument for this is straightforward. If one were to use the crocodile-process on a sample with 50% humans and 50% crocodiles, then the process would only result in true beliefs half of the time. Likewise, if we were to use the process on a sample with only 10% humans and 90% crocodiles, the process would be unreliable. As the crocodile-process has exactly the same structure as the prime number-process, we can conclude that the reliability of such processes is determined by the specific sample it processes. If **Prime Numbers** had been concerned with a sample of the natural numbers with 50% primes and 50% composite numbers, then the reliability of the prime number-process would have been fifty-fifty. Of course, this is not the kind of processes that the reliabilist accepts. That would indeed be absurd! In contrast, processes such as competent deduction do produce reliable results across various samples (in normal worlds), e.g., it does not matter whether a subject assesses a sample of 90% valid arguments and 10% invalid ones or *vice versa*; competent deduction would yield a reliable output of belief-tokes concerning the validity of the arguments in any case.

Now, let us make the following crucial observation. In so far as Berker's case is a problem for reliabilism at all, this is merely because of what follows from the *generality problem*,[11] which is already widely accepted as a genuine problem for

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[10] This point is also underscored by Goldman in his response to Berker: "No process should be confined to a single, specific subject-matter. Thus, what Berker calls a 'process' in his prime number example cannot really be accepted by process reliabilism. Agent S may be said to use a certain rule for deciding whether a particular natural number is prime (always saying 'no' for every natural number specified). But this rule should not be equated with a process." Goldman, "Reliabilism, Veritism, and Epistemic Consequentialism," 141-142.

[11] The generality problem was originally formulated by Goldman in "What Is Justified Belief?" 1-

process reliabilists. The generality problem points out the difficulties of individuating the relevant process in play in a given case. Suppose that you are glancing out of your bedroom window, forming the belief that *it is raining*. Presumably, you formed your belief via a reliable process – the question is what process exactly? Are you using *visual perception*, or *glancing*, or *glancing through a bedroom window*, or perhaps some fourth and somewhat different process? Goldman and Beddor describe the problem more generally as follows:

> Any particular belief is the product of a token causal process in the subject's mind/ brain, which occurs at a particular time and place. Such a process token can be 'typed,' however, in many broader or narrower ways. Each type will have its own associated level of reliability, commonly distinct from the levels of reliability of other types it instantiates. Which repeatable type should be selected for purposes of assigning a reliability number to the process token? If no (unique) type can be selected, what establishes the justificational status of the resulting belief?[12]

Berker's prime number case is merely an instantiation of this problem. Whenever the agent in the case comes across a natural number and wonders whether it is prime, she forms a particular negative belief-token, e.g., *7 is not prime*. If this can be counted as an acceptable cognitive process at all, it can plausibly be "typed" in different more or less broad ways. For example, the prime number-process might (less artificially) be seen as a narrow kind of heuristic, which is merely applied when one considers small natural numbers (say the numbers from 1 to 50). This narrowness would indeed impact the reliability associated with the process.[13] Hence, all Berker has shown with **Prime Numbers** is that it is possible to cook up a case individuating a very artificial cognitive process that yields a problem for the process reliabilist in terms of the process's associated level of reliability, but this is old news in epistemology as it is merely a consequence of the generality problem.

Finally, a quick remark on the kind of problematic, epistemic trade-offs Berker takes the reliabilist to face, violating the "separateness of propositions" (cf. section 1). Setting aside the difficulties of the generality problem and whether

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23, but was later developed in a more systematic way by Richard Feldman and Earl Conee. See, for example, Richard Feldman, "Reliability and Justification," *The Monist* 68 (1985): 159-174; Earl Conee and Richard Feldman, "The Generality Problem for Reliabilism," *Philosophical Studies* 89 (1998): 1-29.

[12] Alvin Goldman and Bob Beddor, "Reliabilist Epistemology," *The Stanford Encyclopedia of Philosophy* (Winter 2016 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/win2016/entries/reliabilism/>.

[13] Something similar is suggested by Goldman in "Reliabilism, Veritism, and Epistemic Consequentialism," 141.

process reliabilism would ever be committed to various peculiar cognitive processes with high reliability, it is no secret that process reliabilism is going to allow that processes with a high truth ratio can once in a while produce false but justified beliefs. Thus, it should be of no surprise to the reliabilist that she can sometimes end up in situations where she holds a belief that is epistemically justified even if blatantly false.

KNOWLEDGE, CERTAINTY, AND FACTIVITY: A POSSIBLE RAPPROCHEMENT

Jeffrey HOOPS

ABSTRACT: In recent discussions in this journal, Moti Mizrahi defends the claim that knowledge equals epistemic certainty. Howard Sankey finds Mizrahi's argument to be problematic, since, as he reads it, this would entail that justification must guarantee truth. In this article, I suggest that an account of the normativity of justification is able to bridge the gap between Mizrahi's proposal and Sankey's objections.

KEYWORDS: justification (epistemic), normativity, logic (justification), factivity, knowledge, certainty

1. Summarizing the Discussion

Moti Mizrahi puts forth the following argument for *epistemic certainty*, an argument which relies on the standard notion of factivity:

1) If S knows that p on the grounds that e, then p cannot be false given e.

2) If p cannot be false given e, then e makes p epistemically certain.

3) Therefore, if S knows that p on the grounds that e, then e makes p epistemically certain.[1]

Since p cannot be false given e, epistemic certainty can be understood as a property of propositions rather than of rational agents.

Mizrahi accepts the normative role that evidence plays in determining whether a belief should count as knowledge. He writes:

> The fact that a truth is difficult for people to accept is not evidence against it. Some religious believers find it difficult to accept the theory of evolution by natural selection, since they think that the theory is inconsistent with their religious beliefs. But the mere fact that those religious believers find it difficult to accept the theory of evolution by natural selection is not evidence against the

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[1] Moti Mizrahi, "You Can't Handle the Truth: Knowledge = Epistemic Certainty," *Logos & Episteme* X, 2 (2019): 225.

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theory itself.[2]

Furthermore, he rejects at least a variety of fallibilism, arguing that if we accept (3), we are committed to *e* securing knowledge that p.[3]

Howard Sankey objects that Mizrahi's argument "does not in fact proceed from the factivity of knowledge to knowledge being epistemic certainty."[4] Specifically, Sankey critiques (1). It is not, as Mizrahi claims, a premise about factivity, but rather, "[i]t is a claim about the relation between grounds (or evidence) and knowing."[5] The nature of that relationship is, as he notes, unclear. As he points out, the only thing really necessary for knowledge is truth.

In his reply to Sankey, Mizrahi agrees with Sankey that it is not possible to know a proposition that is false. However, unjustified propositions are also unknowable, Mizrahi notes. "[I]f *S* knows that *p*, then *p* must not only be true but also justified."[6] In other words, justification is implicit in claims to knowledge. Mizrahi then restates his argument without the appeal to *e*.

In Sankey's reply to Mizrahi's reply, he interprets Mizrahi's defense as claiming that "[i]t is not just that knowledge is factive, but that it is factive and requires justification" and that "Mizrahi takes the fact that knowledge requires both truth and justification to entail that justification must guarantee truth."[7] Tracing Mizrahi's argument, Sankey concludes that the level of justification required to conclude knowledge can be no less than certainty, a conclusion he finds problematic, for this would require that justification guarantee truth. He argues further that Mizrahi's contracted argument also fails, and that in the absence of a valid argument, we should assume that the infallibilist view of justification should not be believed.

An additional, though distinct, critique is offered by James Simpson. He argues that (2) of Mizrahi's original argument is false. To show why he thinks it is false, he presents the following scenario.

> **Math.** Suppose my mathematician dad, an honest and reliable fellow, tells me that 2+2=4. On this basis, I come to believe that 2+2=4.[8]

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[2] *Ibid*, 226.

[3] *Ibid*, 227.

[4] Howard Sankey, "Factivity or Grounds? Comment on Mizrahi," *Logos & Episteme* X, 3 (2019): 333.

[5] *Ibid*.

[6] *Ibid*, 444.

[7] Howard Sankey, "Why Must Justification Guarantee Truth? Reply to Mizrahi," *Logos & Episteme* X, 4 (2019): 446.

[8] James Simpson, "Knowledge Doesn't Require Epistemic Certainty: A Reply to Mizrahi," *Logos*

"2+2=4" is a necessary truth but, as Simpson argues, believing that "2+2=4" on the basis of testimony, even if that testimony is from an honest and reliable person, does not guarantee the truth of "2+2=4." If this is the case, then (2) of Mizrahi's argument fails.

2. Resolving Simpson's Objection

Before proceeding let's clear out Simpson's objection. He argues that in the case of a necessary truth like "2+2=4," believing it is true on the basis of reliable testimony is not enough to guarantee its truth, and thus it fails to be epistemically certain. However, Simpson himself notes that "what guarantees the truth of 2+2=4 isn't my dad telling me, in Math, that it's true that 2+2=4. It's that, in fact, 2+2=4."[9] He is right about this. But let's take this a step further: it is not possible that I could justifiably believe that "2+2=4" on the basis of my dad's testimony. Why is this the case? "2+2=4" is analytically true. When I understand the content of the proposition "2+2=4," I should immediately understand that it is true. In Math, when my dad transmits to me the proposition "2+2=4," if I understand the content of the proposition, then I immediately understand that it is true, without reference to the testimonial chain that led to my introduction to the proposition. In fact, the testimonial chain plays no normative role in regulating my belief and knowledge that "2+2=4." Simpson's objection to (2), then, is problematic.

3. What about *Normativity* Instead of *Certainty*?: A Rapprochement

As mentioned above, Mizrahi characterizes epistemic certainty as a property of propositions rather than of rational agents. A proposition is epistemically certain if a justification e is such that it guarantees that a rational agent S knows that p. One of Sankey's main quibbles with Mizrahi's argument is "Mizrahi takes the fact that knowledge requires both truth and justification to entail that justification must guarantee truth."[10] Sankey reacts against the kind of infallibilism promoted by Mizrahi's argument because he is skeptical about the nature of the relation between knowledge, justification, and truth. Specifically, he wonders whether justification should be included in our understanding of factivity. In what follows, I want to suggest a rapprochement between the Mizrahi's and Sankey's positions. Contra Mizrahi, I want to make the case that what we should really care about is the normativity of justification, rather than epistemic certainty. Contra Sankey, I

& Episteme X, 4 (2019): 449.

[9] *Ibid*, 450.

[10] Sankey, "Why Must Justification Guarantee Truth," 446.

argue that justification is important to factivity and I give a brief account of how justification can play this role.

Standard factivity, which is what I will call the notion of factivity that has been assumed in this debate, is treated axiomatically in various formal logics. For instance, knowledge is represented as

$$\mathrm{Ka}\varphi \to \varphi^{11}$$

which is the built out from the standard modal factivity axiom $\Box\varphi \to \varphi$. The knowledge formula is read informally as "if an agent a KNOWS that φ, then it is the case that φ." Knowledge implies truth, and truth is required for knowledge.

Hintikka[12] was among the first to use modal logics to express epistemic notions like the one in the previous paragraph. From his and others' formalizations into epistemic logic, Artemov and Fitting[13] developed a formal logic, Justification Logic, to help track the role that justifications play in knowledge ascriptions. Their Justification Logic builds on standard epistemic logic by "unfolding" the modal operator $\Box$ as the *justification variable*, t. For a logical formula P, the statement "t justifies P" is represented as $t\colon P$. The axioms familiar to modal logic have counterparts in Justification Logic. The axiom of modal logic $\Box F \to F$, read as "if it is necessarily the case that F, then F" Artemov and Fitting call the "Factivity Axiom" in Justification Logic. Consistent with the project of unfolding the necessity operator, $\Box$, of modal logic, the Factivity Axiom is stated as

Factivity Axiom: $\quad\quad\quad\quad t\colon F \to F$

How do Artemov and Fitting understand justification in relation to knowledge and, hence, truth? Let's call their position *justification factivity*, and it is characterized (at least partially) as follows: "Factivity states that justifications are sufficient to conclude truth."[14] They add elsewhere:

> Factivity is a strong assumption: justifications cannot be wrong. Nonetheless, if the justification is a mathematical proof, factivity is something mathematicians

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[11] Rasmus Rendsvig and John Symons, "Epistemic Logic," in *The Stanford Encyclopedia of Philosophy*, ed. Edward N. Zalta, Summer 2019 (Metaphysics Research Lab, Stanford University, 2019), https://plato.stanford.edu/archives/sum2019/entries/logic-epistemic/: §2.6.

[12] Jaakko Hintikka, *Knowledge and Belief* (Ithaca: Cornell University Press, 1962).

[13] Sergei Artemov and Melvin Fitting, "Justification Logic," in *The Stanford Encyclopedia of Philosophy*, ed. Edward N. Zalta, Winter 2016 (Metaphysics Research Lab, Stanford University, 2016), https://plato.stanford.edu/archives/win2016/entries/logic-justification/; Sergei Artemov and Melvin Fitting, *Justification Logic: Reasoning with Reasons*, Cambridge Tracts in Mathematics 216 (Cambridge, New York, NY: Cambridge University Press, 2019).

[14] Artemov and Fitting, "Justification Logic," §2.5.

are generally convinced of. If we think of knowledge as justified, true belief, factivity is built in. Philosophers generally understand justified, true belief to be inherent in knowledge, but not sufficient.[15]

Now, as Artemov and Fitting are quick to note, Justification Logic does not capture the whole of the discussion on justification in traditional epistemology. Standard discussions in epistemology about justification are often "from the ground up," i.e., concerned with how we can move from justification to knowledge. Justification Logic, on the other hand, is a "from the top down" approach: how can we characterize *justification* in actual cases of knowledge? Furthermore, Justification Logic, while providing a formalization of *justification*, "does not directly analyze what it means for *t* to justify [a formula] *F* beyond the format *t:F*, but rather attempts to characterize this relation axiomatically."[16] But Justification Logic is useful insofar as it attempts to formalize and structure the reasoning implicit in deliberations about justified belief.

How, then, can we analyze *t:P* in terms of standard epistemological discourse? Let's explore two options here. First, since *t* represents the unfolding of the *K* operator in epistemic logic (which is itself and unfolding of the necessity operator $\Box$ of modal logic), *t:P* is simply another way of representing **K**P (i.e., "*t* justifies *P*" is the analysis of "*P* is known" in Justification Logic). *t:P*, then, is just one way of representing knowledge.

Option two is this. Call *t* the *justificans* and *P* the *justificandum*. *t:P* formalizes a basic assumption of the *justifies* relation between the justificans and justificandum, namely that it is a necessary relation. One may find this claim to be jolting. But by axiomtically characterizing the relation between the *justificans* and *justificandum* as necessary, Justification Logic is simply stipulating that that relation is necessary. As an axiom, we can't really prove that it is the case that relation is necessary. However, it does seem both possible and desirable to show that this relation coheres with our normal understanding of justification. The necessary relation between the *justificans* and the *justificandum* is related to our believing that *P* on the basis of *t*. We can characterize this variously depending on the type of justification we have in mind. The situation is fairly straightforward on doxastic nonvoluntarism, e.g., reliablism, which is one way to characterize A's justification for her belief that *P*.

Consider the following scenario, borrowed from Jennifer Lackey.

PERCEPTION: Estelle, Edwin, and I, who have been roommates for the past eight

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[15] Artemov and Fitting, *Justification Logic: Reasoning with Reasons*, 24.

[16] Artemov and Fitting, "Justification Logic," §2.1; cf. Artemov and Fitting, *Justification Logic: Reasoning with Reasons*, 1-2.

> years, were eating lunch together at the dining room table in our apartment. When I asked Edwin to pass the wine to Estelle, he replies, "Estelle isn't here today." Prior to this disagreement, neither Edwin nor I had any reason to think that the other is evidentially or cognitively deficient in any way, and we both sincerely avowed our respective conflicting beliefs.[17]

"I" have a belief about Estelle's being present at the table. This belief was formed on the basis of normal perceptual practice, i.e., sight and hearing, and my visual and auditory percepts of Estelle give rise to my belief that Estelle is present with us. The crucial point at this juncture is this: it is *necessarily* the case that given my visual and auditory percepts of Estelle that I *believe* that Estelle is present at the table. It simply could not be the case that I could have percepts consistent with believing that Estelle is present without believing that Estelle is present. If it were to happen that I have percepts of Estelle but fail to believe she is present, we would surely think that something is wrong with my cognitive processing and, hence, I would fail to be justified in my belief that Estelle is not present. This serves to show that we typically think of the relation between the *justificans* and *justificandum* as *necessary*.

Voluntaristic views of belief-formation do not alter the picture radically. Suppose I am not sitting at the table with Edwin. I am in another room taking a nap while we wait for Estelle to arrive for dinner. She, Edwin and I plan to eat dinner and then go to a movie. I am awoken by the sound of the front door closing. I arise and enter the dining room, where I find two plates on the table and on the plates scrapes of food. On the coat rack I find a scarf that resembles Estelle's scarf. Edwin is nowhere to be found. I consider the evidence: the sound of the door, the dinner plates, Estelle's scarf. They ate dinner and left for the movie without me (and Estelle forgot her scarf!) On this evidential perspective, if my evidence e justifies my belief that P.

One may object that in this scenario the evidence is polyvalent: since it is subject to alternate interpretations, the same evidence base may support another conclusion. It may be the case that after beginning dinner, they realized that there was no wine, so they quickly left to get more. Since it appears that belief that "Edwin and Estelle have left for the movies" (P_{movies}) and "Edwin and Estelle have left to get more wine" (P_{wine}) both are supported by e, e does not *necessarily* support P_{movies} over P_{wine}. In other words, e does not conclusively support my believing one over the other. But for present purposes, that e is not conclusive for P_{movies} over

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[17] Jennifer Lackey, "A Justificationist View of Disagreement's Epistemic Significance," in *Social Epistemology*, eds. Adrian Haddock, Alan Millar, and Duncan Pritchard (Oxford: Oxford University Press, 2010), 306.

P_{wine} (or vice versa) does not change the picture: in both cases, e as a generic evidence base necessarily (though only partially) justifies both. That is really all that we're after at this juncture. Of course, when deliberating about whether to believe P_{wine} or P_{movies}, we acknowledge that evidence base e is incomplete. e will be necessary for whatever belief I end up forming, but I will need to gather additional evidence or make additional considerations over and above e to support belief that P_{movies} over P_{wine}, and vice versa. e will not do the work by itself. Both P_{movies} and P_{wine} have conclusive evidence bases, e_{movies} in the case of P_{movies} and e_{wine} in the case of P_{wine}. Both include e but include other crucial pieces of evidence such that, taken together, they become necessary and sufficient to conclude that P_{movies} or P_{wine}.

Thus, when an evidence base is complete, that evidence base necessarily justifies belief that P. If we want to be voluntarists about belief, my possession of evidence base e does not guarantee that I in fact believe that P; but it would seem to be the case that my possessing evidence base e obligates me to believe that P, such that failure for me to believe P is a failure for me to meet my epistemic duties.

It seems to me that part of the issue is Mizrahi, in characterizing certainty as a property of propositions and not of agents, does not attend to the role that this characterization ought to play in the epistemic deliberations of agents.

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