THE EFFECTS OF TERRAIN CORECTIONS ON GRAVITY DATA

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Abstract. Terrain corrections are part of the method of geoid determination when gravity data are used. Especially in mountainous regions, this corrections may have a significant impact on the gravity anomalies. Assuming that the observed gravity anomalies have been accurately estimated, there are still other error sources related to the digital elevation model accuracy, the error in the estimation of topographic effects and the effect that terrain correction errors have on geoidal height determinations. Also, there are differences between methods for determinations of terrain corrections. We computed a grid and map of classical terrain corrections from elevations on the territory of Romania. We used planar approximations by two dimensional Fast Fourier Transform. We evaluated the gravitational effect of topography using prisms and mass line models. Also, we compared this two methods and contribution of each term up to third-order. The output files contain the results up to the first, second and third order of the terrain correction series. We used the software of YECAI LI (1993) with the following steps: get input data filename, collect information about the data (header, number of rows, number of colons, grid spacing in both directions), display of statistics of the input data, choose one of the available formulas, decide integration limits size, choose topographic model (mass-prism representation, mass-line representation), compute the convolutions and output data.

Keywords: terrain corrections, gravitational effect, geoid, quasigeoid, co-geoid.

Rezumat. Efectele corecţiilor de teren asupra datelor de gravitaţie. Corecțiile de teren, fac parte din metoda de determinare a geoidului atunci când se utilizează date de gravitație. Mai ales în regiunile muntoase, aceste corecții pot avea un impact semnificativ asupra anomaliilor gravitaționale. Presupunând că anomaliile gravitaționale observate au fost estimate cu exactitate, există încă alte surse de eroare legate de precizia modelului de cote digitală, eroarea în estimarea efectelor topografice și efectul pe care îl au erorile de corecție a terenului asupra determinărilor înălțimii geoidale. De asemenea, există diferențe între metodele de determinare a corecțiilor de teren. Am calculat o grilă și o hartă a corecțiilor clasice de teren de la altitudini pe teritoriul României. Am folosit aproximări planare prin transformarea rapidă Fourier în două dimensiuni. Am evaluat efectul gravitațional al topografiei folosind prisme și modele de linii de masă. De asemenea, am comparat aceste două metode și contribuția fiecărui termen până la a ordinul trei. Fișierele de ieșire conțin rezultatele până la ordinul 1, 2 și 3 din seria de corecție a terenului. Am folosit software-ul lui YECAI LI (1993) cu următorii pași: introducerea numelui fișierului datelor de intrare, colectarea informațiilor despre date (antet, număr de rânduri, număr de coloane, spațierea gridului pe ambele direcții), afișarea statisticilor datelor de intrare, formulele disponibile, alegerea dimensiunilor limitelor de integrare, alegerea modelului topografic (reprezentarea cu prisme de masă, reprezentarea cu linii de masă), calcularea convoluțiilor și datelor de ieșire.

Cuvinte cheie: Corecții de teren, efect gravitațional, geoid, cvasigeoid, cogeoid.

INTRODUCTION

The analysis of the detailed surface gravity at a resolution sufficiently higher than the resolution of mean gravity anomalies data, depends critically on the availability of accurate topographic data.

For these computations to be made consistently, it is necessary to compile first a high-resolution global Digital Topographic Model (DTM), whose data will support the computation of all gravity parameters related to terrain heights.

If a geological structure is in isostatic balance, Free Air anomalies are approximately equal to zero, while Bouguer anomalies are correlated with the topography in the image (in the mirror). Thus, the Free Air and Bouguer gravimetric maps give us information about the isostatic equilibration of the geological structures.

In the calculation of the Earth's theoretical figure the mass distribution under the ellipsoid is assumed to be homogeneous. A local mass excess within the ellipsoid will deviate the local gravity.

As a result of the uneven distribution and heterogeneity of the Earth's internal mass, the geoid is a corrugated equipotential surface. Corrugations may be positive (the geoid being above the ellipsoid), when there is a mass excess, or they may be negative (the geoid being under the ellipsoid), when there is a mass deficiency in the basement.

Geoid sinking (negative height anomaly) and negative gravity Free-Air occur over the mass deficient regions. The high geoid and positive Free-Air gravity occur over the excess mass regions.

Where the degree of spherical harmonics n =6,...,16 , the positive geoidal anomalies mark the island ditch and arch, while the negative gravity zones are typical of both the oceanic and continental basins, especially the recent glacial ones, for which $2\pi R / \lambda \leq 40$, resulting λ greater than 1000 km, ie around 10-15% of the Earth's R radius (HEISKANEN & MORITZ, 1967; BARTHELMES & KOHLER, 2016; BALMINO et al., 2011).

Spectral approach has found that sources are predominantly shallow sources in the lithosphere, but to explain deeper sources, a number of spherical harmonics with a $n < 7$ degree are required.

Geoidal peaks (positive anomalies) are observed in subduction areas and island springs involving deep overflow.

Several studies use the "equivalent mass" technique to study geoidal anomalies in subduction areas. Here, the density distribution, which can explain the observed anomalies for $2 \leq n \leq 20$, is found for a non-rigid earth with induced loading and viscous deformation.

METHODS

The development of the methodology and the significance of the parameters is widely dealt with in many papers that we can enumerate (HEISKANEN & MORITZ, 1967; VAJDA et al., 2004; FORSBERG & TSCHERNING, 2008; SIDERIS, 2008). Very synthetically, we present the following methodological explanations (Fig.1).

Bruns' formula $N=\frac{T}{\gamma}$ (eq.1), with Stokes' integral $T=\frac{R}{4\pi}\iint_{\sigma}\Delta gS(\psi)d\sigma$ (eq.2) gives the undulation of the geoid *N* provided that there are no masses outside the geoidal surface.

R is the mean radius of the Earth, σ denotes the Earth's surface and *S(ψ)* is Stokes' function:

$$S(\psi)=\frac{1}{\sin\left(\frac{\psi}{2}\right)}-6sin\frac{\psi}{2}+1-5cos\psi-3cos\psi\ln\left(\sin\frac{\psi}{2}+sin^2\frac{\psi}{2}\right); sin^2\frac{\psi}{2}=sin^2\frac{\varphi P-\varphi}{2}+sin^2\frac{\lambda P-\lambda}{2}cos\varphi_P cos\varphi$$

where ψ is the spherical distance between the data point *(ϕ,λ)* and the computation point (ϕ_P,λ_P) .

One way to take care of the topographic masses of density ρ (usually assumed constant) is Helmert's condensation reduction, which is used here as a representative from a number of possible terrain reductions, applied as follows:

(a) remove all masses above the geoid;

(b) lower station from *P* to *Po* using the free-air reduction *F*;

(c) restore masses condensed on a layer on the geoid with density σ = ρ*H*.

This procedure gives *Δg* on the geoid computed from the expression

$\Delta g=\Delta g_P-A_P+F+A_{P0}^c=\Delta g_P+F+\delta A$ (eq.3), where (Δg_P+F) is the free-air gravity anomaly at *P*, *AP* is the attraction of the topography above the geoid at *P*, and $A_{P0}^c$is the attraction of the condensed topography at *Po*.

Due to the shifting of masses, the potential changes as well by an amount called the indirect effect on the potential, given by the following equation: $\delta T=T_{P0}-T_{P0}^c$ (eq.4), where T_{P0} is the potential of the topographic masses at *Po* and T_{P0}^c is the potential of the condensed masses at *Po*.

Due to this potential change, the use, (from eq.1 and eq.2) of equation $N=\frac{T}{\gamma}=\frac{R}{4\pi\gamma}\iint_{\sigma}\Delta gS(\psi)d\sigma$ (eq.5), with *Δg* from (eq.3) does not produce the geoid but a surface called the co-geoid. Thus, before applying Stokes' equation, the gravity anomalies must be transformed from the geoid to the co-geoid by applying a small correction δ*Δg* called the indirect effect on gravity $\delta\Delta g=-\frac{1}{\gamma}\frac{\partial\gamma}{\partial h}\delta T$ (eq.6)

The final expression giving *N* can now be written as: $N=N^c+\delta N$, where: *Nc* is the co-geoidal height and δ*N* is the indirect effect on the geoid (Fig. 1).

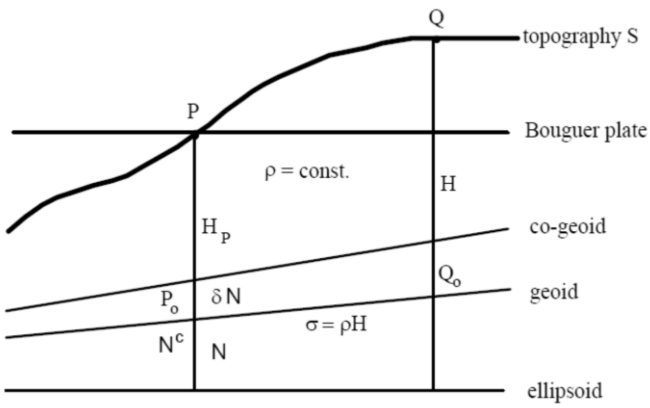

Figure 1. Actual and condensed topography, in planar approximation (after SIDERIS, 2008).

We use the vertical derivative operator **L** (SIDERIS, 2008) defined in planar approximation, as: $\boldsymbol{L}f=\frac{1}{2\pi}\iint\frac{f-f_P}{l^3}dxdy$, $L^n=\frac{\partial^n}{\partial z^n}$, where P is the computational point. The potential change is:

$\delta T=-\pi k\rho H_P^2-2\pi k\rho\sum_{r=1}^{\infty}\frac{1}{(2r+1)!}\boldsymbol{L}^{2r-1}H^{2r+1}$ (eq.7) and the attraction change is equal to the classical terrain correction *c:*

$\delta A=c=2\pi k\rho\sum_{r=1}^{\infty}\frac{1}{(2r)!}\boldsymbol{L}^{2r-1}(H-H_P)^{2r}$ (eq.8), where *k* denotes Newton's gravitational constant attraction of the condensed.

Topography in (eq. 3) must be computed on the geoidal surface in order for the reduced gravity to refer to the geoid (actually, the co-geoid) and be used as input to Stokes' formula.

Keeping only the terms for *r* in (eq.7) and (eq.8), the terrain effect on Δ*g* and the indirect effect on *N* take the following form (formulas for the Direct and Inverse Terrain Contribution):

$\delta A_P = c_P = -\Delta g_P^H = \pi k\rho \boldsymbol{L}(H - H_P)^2 = \pi k\rho[\boldsymbol{L}H^2 - 2H_P\boldsymbol{L}H] = \frac{1}{2}k\rho \iint_E \frac{(H-H_P)^2}{l^3}dxdy = \frac{1}{2}k\rho \iint_E \frac{H^2-H_P^2}{l^3}dxdy - H_P k\rho \iint_E \frac{H-H_P}{l^3}dxdy$ (eq.9)

$\delta N_P = -\frac{\pi k\rho}{\gamma}H_P^2 - \frac{\pi k\rho}{3\gamma}LH^3 = -\frac{\pi k\rho}{\gamma}H_P^2 - \frac{k\rho}{6\gamma}\iint_E \frac{H^3-H_P^3}{l^3}dxdy$ (eq.10)

Terrain correction given in (eq.9) can be written in any of the following two convolution forms for 3D and 2D:

$c(X_P, Y_P, Z_P) = G\rho \int_E \iint_{h_P}^{h} \frac{(h-h_P)^2}{[(X-X_P)^2+(Y-Y_P)^2+(Z-Z_P)^2]^{3/2}}dxdy\,dz$ (eq.11)

Expanding the integral at the level z=h(x_P, y_P) and omitting the 2nd and higher order terms:

$c(X_P, Y_P) = \frac{1}{2}G\rho \iint_\sigma \frac{(h-h_P)^2}{[(X-X_P)^2+(Y-Y_P)^2]^{3/2}}dxdy$ (eq.12)

In practical applications, topography is digitized on a regular grid. The height within each cell is represented by a prism with mean height and mean density of the topography, which is called the mass prism topographic model. If the mass of the prism is mathematically concentrated along its vertical symmetric axis, then the topography within the prism is represented by a line, which gives the mass line topographic model.

For rectangular prisms, the calculation algorithm is the following (YAMAMOTO, 2002 apud FORSBERG, 2008):

The vertical component of the gravitational attraction of the chosen prism is:

$g_P = G\rho\{F(x_2, y_2, h) - F(x_1, y_2, h) - F(x_2, y_1, h) + F(x_1, y_1, h)\}$, (eq.13) where:

$$F(x, y, h) = x ln\left(\frac{y + \sqrt{x^2 + y^2}}{y + \sqrt{x^2 + y^2 + h^2}}\right)$$

delimited by the planes : x=x1, x=x2, y=y1, y=y2, z=0, z=h, x2>x1, y2>y1,h>0

To approximate the straight rectangular prism through a mass line, we used the following relationships (YAMAMOTO, 2002 apud FORSBERG, 2008):

$g_l = G\rho s_1 s_2 \left(\frac{1}{u} - \frac{1}{\sqrt{u^2+h^2}}\right)$, (eq.14) where: $s_1 = x_2 - x_1$, $s_1 = y_2 - y_1$, $u = \sqrt{\left(\frac{x_1+x_2}{2}\right)^2 + \left(\frac{y_1+y_2}{2}\right)^2}$

RESULTS AND DISCUSSIONS

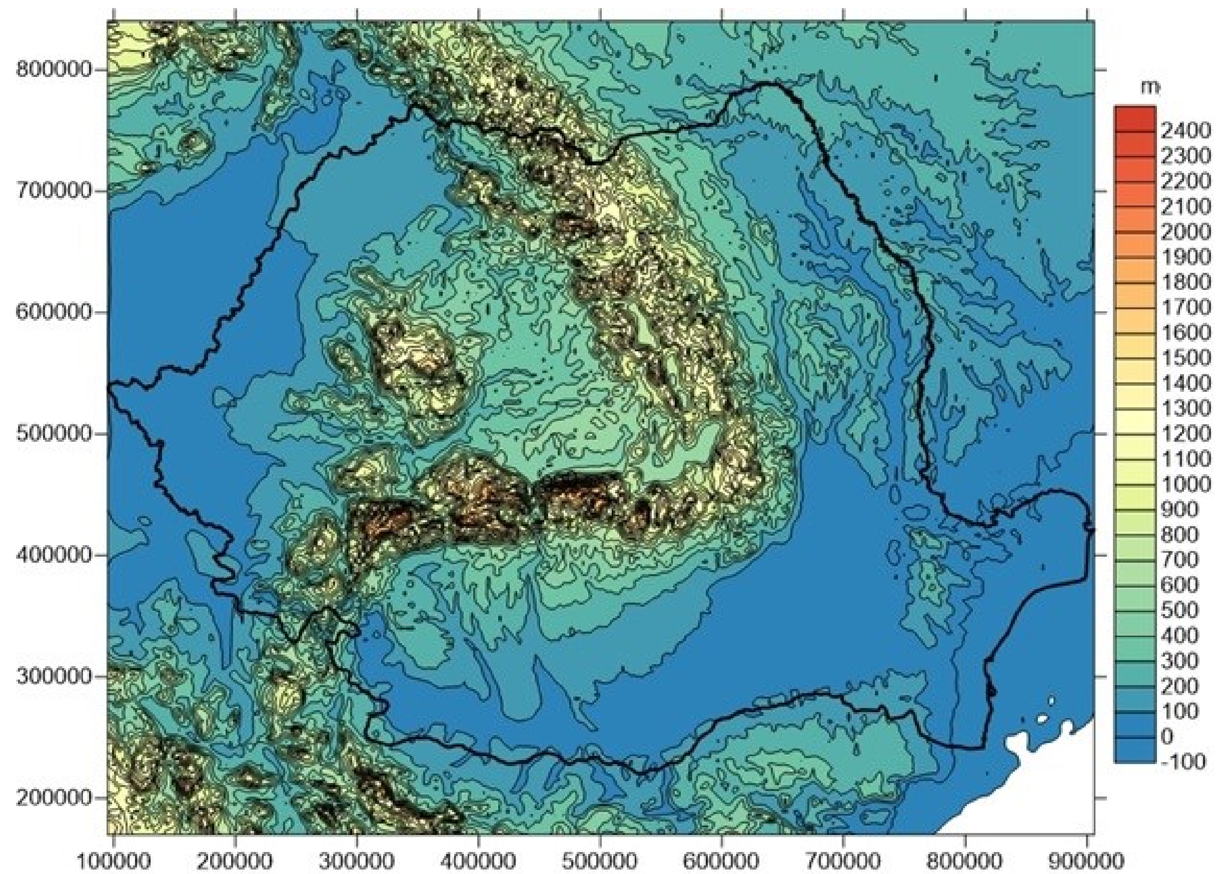

Figure 2. Topography map of Romania (from ETOPO 1 data).

We used the ETOPO 1 file for the topographical input data (Fig. 2), taken over by the Gravimetric International Bureau site, for the territory of Romania. We converted these data to STEREO 70 metric coordinates.

ETOPO1 is a 1 arc-minute global relief model of Earth's surface that integrates land topography and ocean bathymetry. It was built from numerous global and regional data sets.

The utilization of digital terrain models is essential for obtaining good gravity field modelling results in mountainous areas.

We used the general software to make data files and primary processing (Surfer, Global Mapper, Google Earth, Excel and Arc Gis).

The software programs specialized in calculating geodetic and gravimetric parameters, accredited by the International Gravimetric Bureau, were accessed using the Pyton interface.

A brief presentation of these programs (YECAI & SIDERIS, 1993 apud FORSBERG, 2008):

• With the tc2dftpl program, a classical ground correction grid is calculated from a grid of the digital terrain model using the 2D Fast Fourier Transform. The input file contains a grid (height.dat) and at output we obtain three field correction files (Tc2DFT1.MP, Tc2DFT2.MP, Tc2DFT3.MP) from developments in Fourier series up to third order for each of the adopted methods for topography models: using rectangular prism models or using mass-line models.

The user can select the size of the integration domain.

• With the com_data program, we can calculate the differences between the output files and the corrections of the terrain with different degrees of expansion. The calculated statistics can give us an idea of the differences that occur between different degrees of expansion.

• With the fftgeoid program, we can calculate the geoid corrugations in a user-selected grid using a grid with gravity anomalies as input data using the Stokes integral. We can select the planar approximation or spherical approximation using 2D or 1D Fast Fourier Transform. This program can run both with averaged gravity anomalies or punctual values. It is also possible to determine an error grid calculated according to the standard errors of the used geopotential model coefficients and the standard errors of the gravity anomalies.

Also, the GRAVSOFT programs TC, TCFOUR and SPFOUR will produce various kinds of terrain effects by Newtonian integration of density effects (FORSBERG & TSCHERNING, 2008). Using these programs described above, we obtained the data files that led us to get the following images (Figs. 3, 4, 5).

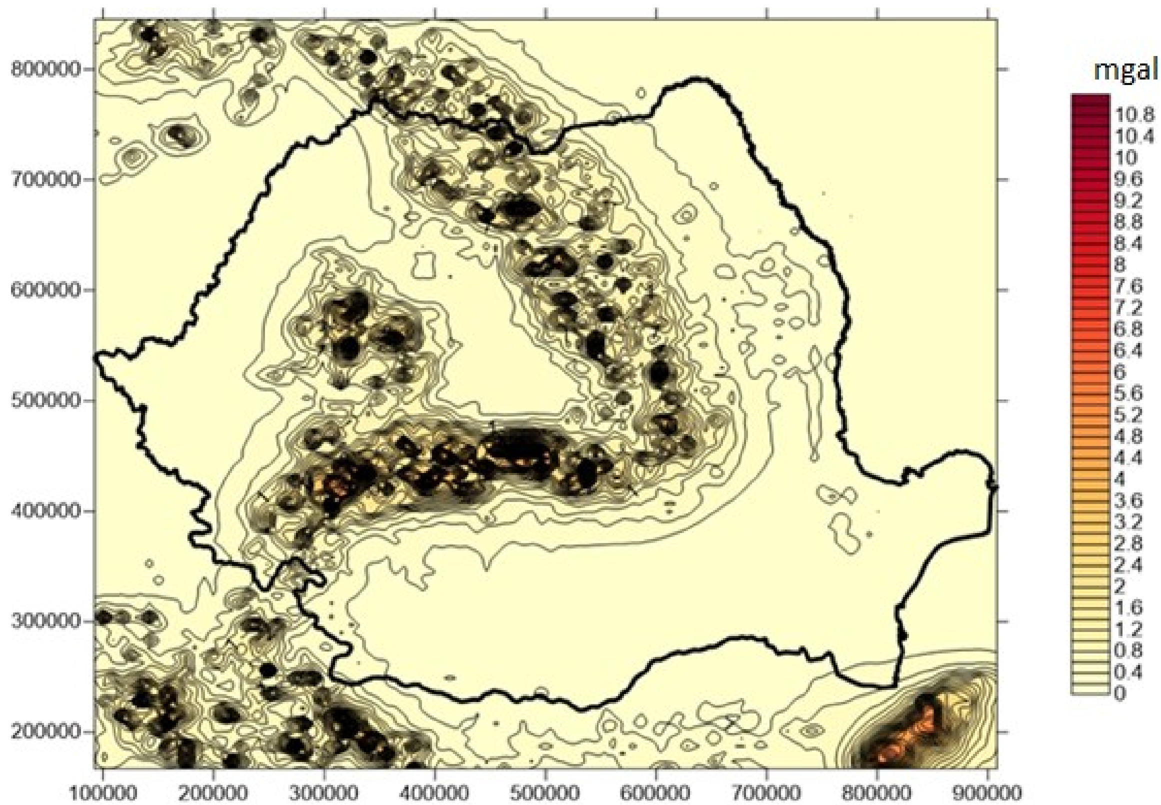

Figure 3. The representation of the output data, containing the sum of terms with first order, second order and third order from the development of Fourier Series, representing the direct gravity correction given by topography (mgal) by the rectangular prism integration method.

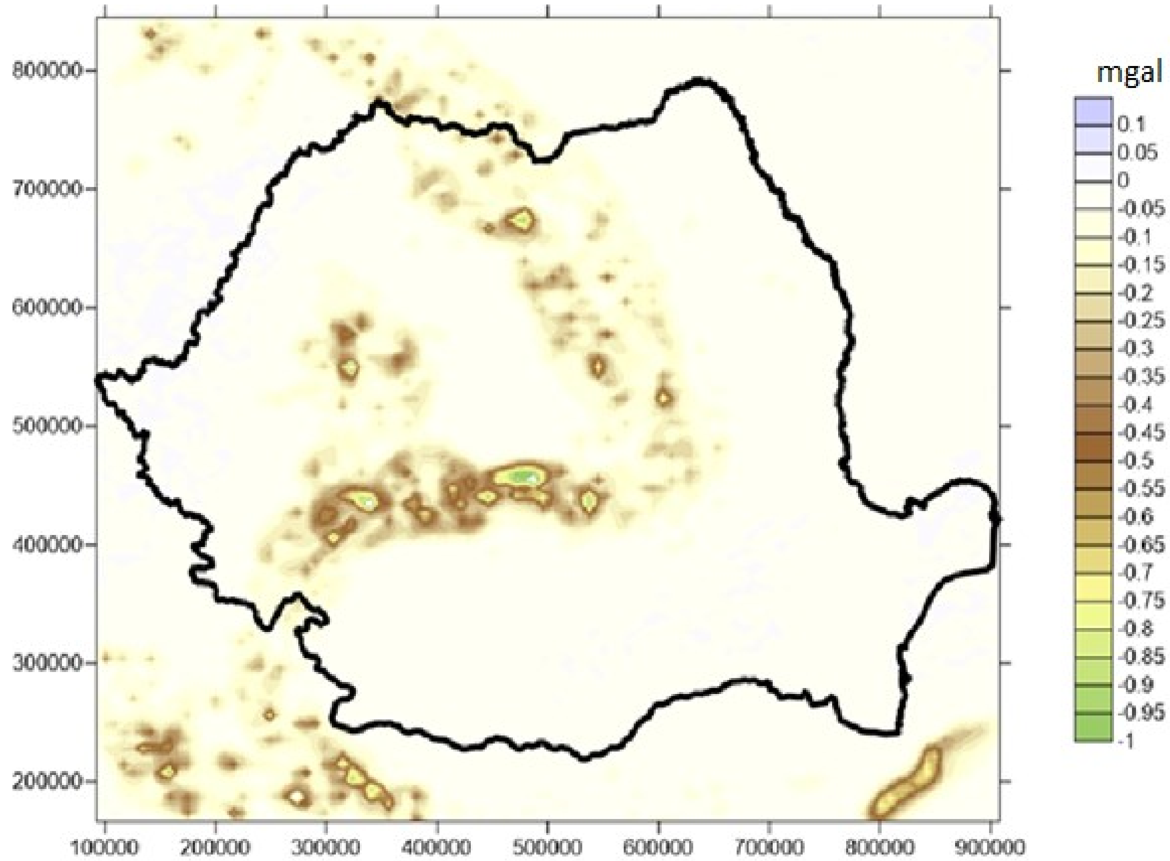

Figure 4. The difference between the correction given by the topography (mgal) by the rectangular prism integration method and the correction given by the topography (mgal) by the line-mass integration method. The data, contain the sum of terms with first order, second order and third order from the development of Fourier Series, for both methods.

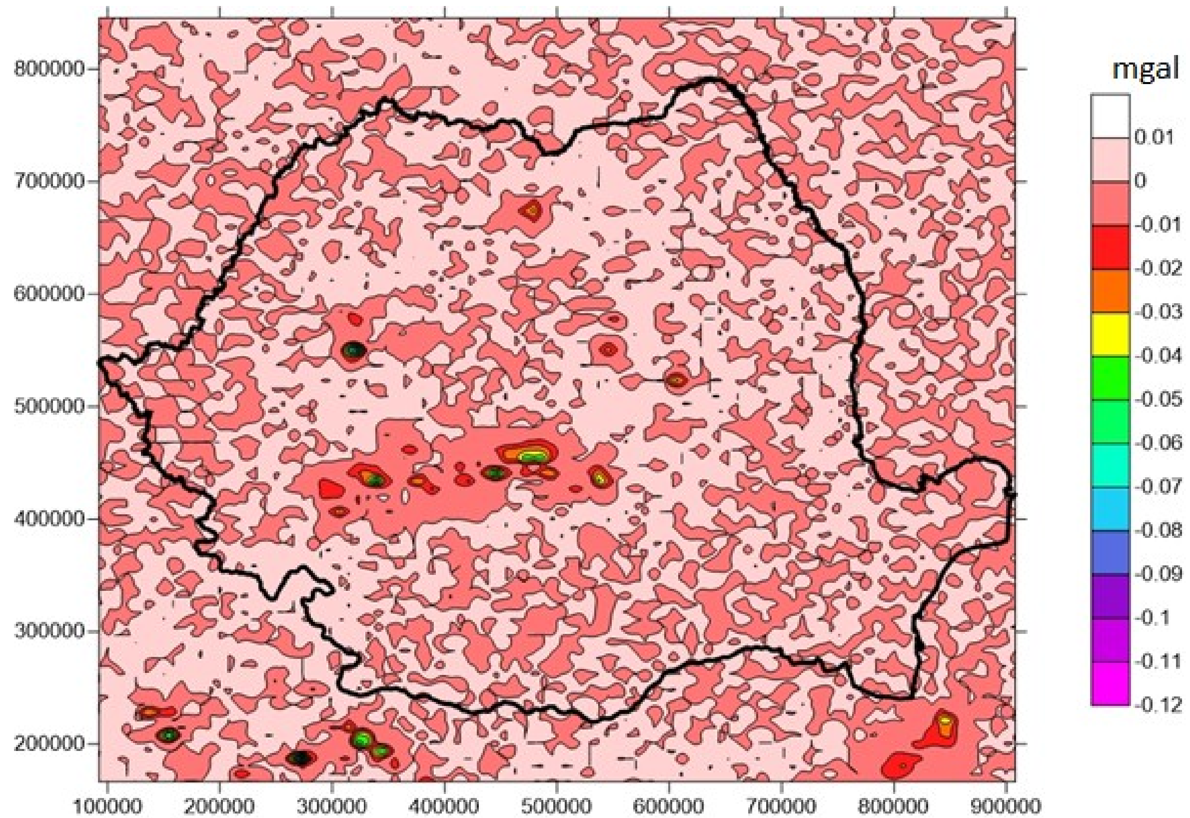

Figure 5. Contribution of terms only with second order and third order from the development of Fourier Series in gravity correction (mgal), given by the topography through the line-mass integration method.

The correction of gravity by the rectangular prism method (Fig. 3), taking into account the first three terms of the Fourier development, has a variation of maximum 11 mgal for the territory of Romania. In low areas, this correction is up to 1 mgal, increasing in the mountain areas (with highest altitude) to 11 mgal.

An almost equal correction is obtained with the line-mass method.

In Fig. 4, we note that the difference between the two methods, taking into account the first three terms of the Fourier development, has a variation in the range of -1 to 0.15 mgal for the territory of Romania. The greatest differences of -1 mgal also occur in the highest mountain areas.

In Fig. 5, we note that the contribution of the upper order terms (2nd order and 3rd order) of the Fourier development, in the case of the line-mass method, has a variation in the range of -0.12 to 0.02 mgal for the territory of Romania. The highest differences of -0.12 mgal also occur in the highest mountain areas.

Also, the results with line-mass method are very close with results of rectangular prism method.

CONCLUSIONS

For stricter accuracy requirements for gravity anomalies, terrain corrections must be considered, especially in high mountain regions.

From the physical point of view, the mass line model is less realistic than the mass prism model. However, for the Romanian territory, both methods gave similar results.

Therefore, it is useful to investigate how big the effect on the terrain corrections will be when the mass line model is used instead of the mass prism model. The values of the terrain correction vary from fractions of a miligal (in lower zone) to tens of miligals (in mountain ranges) and differences between this two methods are less of 1 mgal.

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