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CATACLYSMIC VARIABLE TT ARIETIS OBSERVED AT BUCHAREST OBSERVATORY DURING 1994

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Abstract. New photoelectric observations of the cataclysmic variable TT Arietis obtained in 1994's observational season are presented. The observations were performed in filter B only, using a 50 cm Cassegrain telescope of the Bucharest Observatory.

Key words: stellar photometry, cataclysmic variables.

1. INTRODUCTION

TT Ari (BD + 14°341) is one of the brightest cataclysmic variables. Smak and Stepień (1969) found the period of light variation to be around 3.2 hours. Spectroscopic observations carried out by Cowley et al. (1975) indicate radial velocity variations: the obtained spectroscopic period was around 4% longer than the photometric one. The difference between the orbital and photometric periods led to the new classification of TT Ari as an intermediate polar (Jameson et al. 1982).

During the bright state, the spectrum of TT Ari exhibits moderately strong H emission in the optical range, which are placed in wide shallow absorption. The UV is dominated by the usual strong resonance lines in absorption with CIV exhibiting a P Cygni profile. In spectra taken during the normal brightness (~ 12.0 mag) no pronounced absorption around the H emission is visible. During the intermediate state (~ 14.5 mag) He UV lines went into emission and the continuum is bluer than in the bright state: the optical lines became narrower and much stronger in emission.

A quasi-periodic variability on time scale of minutes was observed by many observers. Semeniuk et al. (1987) found out a 20 min period which appears in the majority of nights of all observed seasons. The period seems to decrease from 27 min in 1962 to 17 min in 1985.

When TT Ari fades into the intermediate state, the three hour modulation disappears. Due to small orbital inclination, only non-periodic flickering is seen then. In Jameson et al's opinion (1982) the photometric period reflects rotation of

the white dwarf and the spectroscopic one the orbital motion of the system. However, Semeniuk et al. (1987) pointed out that this period decreases too fast.

A more detailed analysis of TT Ari can be found in the works given by Udalski (1988) and Wenzel et al. (1992).

2. OBSERVATIONS

The observations were carried out during the period 27 Oct.–7 Nov. 1994 on 6 nights with a single-channel photometer fed by a 50 cm Cassegrain telescope. Our work is a part of an international campaign “TT Ari-1994” initiated by I. L. Andronov. An EMI 9502B unrefrigerated photomultiplier and filter B (close to Johnson’s system) were used. Our comparison star was BD + 14°336. In four observational nights (Oct. 31, Nov. 1, 5, 7) the whole 3.2 h orbital period was covered in order to spotlight the light variation due to the orbital and rotational motions. The absolute errors of the individual points were not larger than 0.01 mag. The measurements were made differentially with respect to the comparison star. All the observations were corrected for the first order differential extinction. A total of 602 observed points were obtained. In Table 1 are given the differences *comp-var* in filter B obtained in the observers’ system. Figs 1–4 present the observed light curves. Phase calculations were performed using the ephemeris given by Udalski (1988).

At first sight one can observe the following main features:

- 1) 3.2 h modulation (Oct. 31).
- 2) 20 min modulation (Oct. 31, Nov. 5, 7).
- 3) deep dips (Nov. 1, 5, 7)

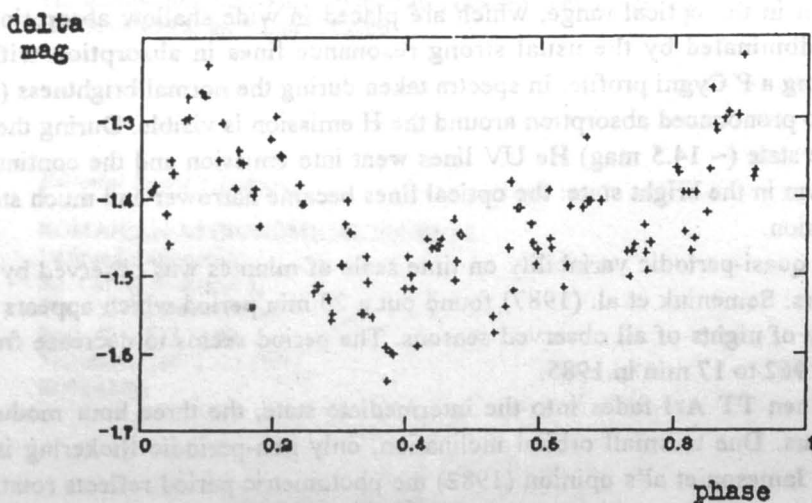


Fig. 1 — Light curve of TT Ari observed on Oct. 31, 1994.

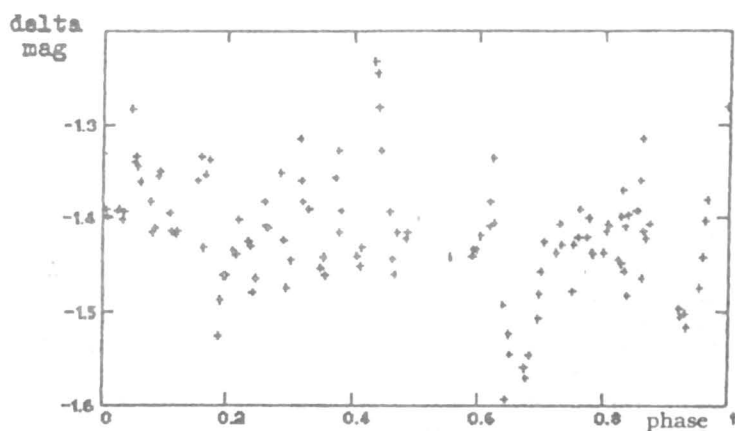


Fig. 2 — Light curve of TT Ari observed on Nov. 1, 1994.

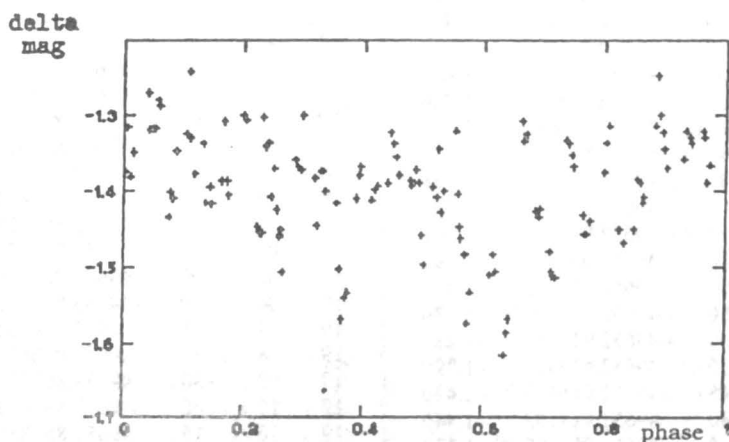


Fig. 3 — Light curve of TT Ari observed on Nov. 5, 1994.

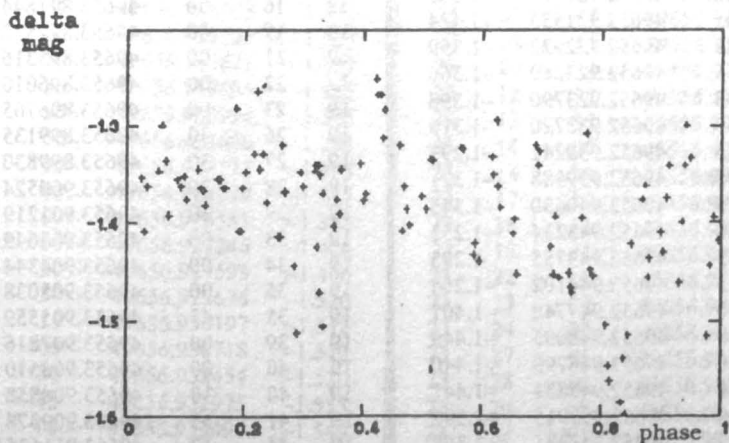


Fig. 4 — Light curve of TT Ari observed on Nov. 7, 1994.

Table 1

Individual magnitude differences (*comp-var*) of TT Ari observed during 1994 (filter B)

U.T.			Hel. MJD	delta	U.T.			Hel. MJD	delta
27 Oct. 1994					28 Oct. 1994				
h	m	s							
21	10	00	49652.887679	-1.368	22	45	15	49652.953824	-1.387
21	12	30	49652.889415	-1.408	22	46	00	49652.954345	-1.335
21	13	30	49652.890109	-1.408	22	53	00	49652.959206	-1.466
21	14	00	49652.890456	-1.458	22	54	00	49652.959901	-1.494
21	14	45	49652.890977	-1.493	22	55	00	49652.960595	-1.442
21	16	00	49652.891845	-1.467	22	56	00	49652.961290	-1.448
21	16	45	49652.892366	-1.547	22	56	30	49652.961637	-1.452
21	17	30	49652.892887	-1.547	23	01	30	49652.965109	-1.522
21	18	15	49652.893408	-1.511					
21	22	45	49652.896533	-1.463	18	37	00	49653.864761	-1.409
21	23	15	49652.896880	-1.496	18	38	00	49653.865455	-1.411
21	24	00	49652.897401	-1.439	18	45	30	49653.870663	-1.524
21	24	30	49652.897748	-1.411	18	47	30	49653.872052	-1.469
21	25	00	49652.898095	-1.454	18	51	30	49653.874830	-1.405
21	25	30	49652.898442	-1.446	18	52	00	49653.875177	-1.393
21	26	15	49652.898963	-1.443	18	53	00	49653.875872	-1.395
21	26	45	49652.899310	-1.483	18	56	30	49653.878302	-1.405
21	31	30	49652.902609	-1.358	18	57	30	49653.878997	-1.436
21	32	00	49652.902956	-1.366	18	58	30	49653.879691	-1.601
21	33	15	49652.903824	-1.461	18	59	00	49653.880038	-1.611
21	38	00	49652.907123	-1.495	19	00	00	49653.880735	-1.645
21	47	00	49652.913373	-1.373	19	03	00	49653.882816	-1.567
21	47	30	49652.913720	-1.336	19	04	00	49653.883510	-1.521
21	48	00	49652.914067	-1.283	19	04	45	49653.884031	-1.564
21	48	30	49652.914415	-1.299	19	08	30	49653.886635	-1.499
21	51	45	49652.916672	-1.420	19	09	30	49653.887330	-1.468
21	52	30	49652.917192	-1.449	19	10	00	49653.887677	-1.483
21	54	15	49652.918408	-1.380	19	10	15	49653.887851	-1.490
21	58	15	49652.921185	-1.365	19	11	00	49653.888372	-1.516
21	58	45	49652.921533	-1.424	19	16	00	49653.891844	-1.642
22	00	45	49652.922922	-1.359	19	17	00	49653.892538	-1.711
22	01	15	49652.923269	-1.366	19	21	00	49653.895316	-1.578
22	02	00	49652.923790	-1.398	19	22	00	49653.896010	-1.566
22	23	30	49652.938720	-1.319	19	23	00	49653.896705	-1.554
22	24	15	49652.939241	-1.297	19	26	30	49653.899135	-1.573
22	24	45	49652.939588	-1.321	19	27	30	49653.899830	-1.594
22	26	15	49652.940630	-1.346	19	28	30	49653.900524	-1.543
22	30	00	49652.943234	-1.253	19	29	30	49653.901219	-1.582
22	30	45	49652.943755	-1.293	19	33	00	49653.903649	-1.532
22	31	15	49652.944102	-1.264	19	34	00	49653.904344	-1.571
22	36	30	49652.947748	-1.401	19	35	00	49653.905038	-1.602
22	37	00	49652.948095	-1.449	19	35	45	49653.905559	-1.579
22	38	00	49652.948790	-1.440	19	39	00	49653.907816	-1.507
22	39	30	49652.949831	-1.441	19	40	00	49653.908510	-1.499
22	42	30	49652.951915	-1.392	19	40	30	49653.908858	-1.504
22	43	15	49652.952435	-1.398	19	41	15	49653.909378	-1.531
22	44	30	49652.953303	-1.373	19	44	30	49653.911635	-1.438
					19	45	30	49653.912330	-1.467

Table 1 (continued)

U.T.			Hel. MJD	delta	U.T.			Hel. MJD	delta
19	46	00	49653.912677	-1.441	22	19	30	49656.935926	-1.434
19	47	00	49653.913372	-1.465	22	20	00	49656.936273	-1.403
19	52	00	49653.916844	-1.589	22	21	00	49656.936968	-1.443
19	53	00	49653.917538	-1.576	22	24	00	49656.939051	-1.548
19	54	00	49653.918233	-1.566	22	24	45	49656.939572	-1.548
19	58	15	49653.921184	-1.582	22	25	30	49656.940093	-1.476
19	59	00	49653.921705	-1.605	22	26	00	49656.940440	-1.509
19	59	45	49653.922226	-1.607	22	27	00	49656.941134	-1.553
20	02	30	49653.924135	-1.595	22	31	00	49656.943912	-1.591
20	16	30	49653.933858	-1.544	22	31	45	49656.944433	-1.635
20	17	30	49653.934552	-1.510	22	32	30	49656.944954	-1.598
20	18	30	49653.935246	-1.533	22	33	15	49656.945475	-1.600
31 Oct. 1994					22	36	45	49656.947905	-1.517
21	23	30	49656.897037	-1.390	22	37	30	49656.948426	-1.474
21	24	00	49656.897385	-1.390	22	38	30	49656.949120	-1.500
21	24	45	49656.897905	-1.381	22	39	00	49656.949468	-1.517
21	28	00	49656.900162	-1.420	22	40	15	49656.950336	-1.589
21	28	45	49656.900683	-1.459	22	43	00	49656.952245	-1.451
21	29	30	49656.901204	-1.350	22	43	30	49656.952593	-1.485
21	30	00	49656.901551	-1.367	22	44	30	49656.953287	-1.465
21	33	00	49656.903635	-1.297	22	45	30	49656.953981	-1.462
21	33	45	49656.904155	-1.270	22	46	30	49656.954676	-1.462
21	34	30	49656.904676	-1.296	22	47	00	49656.955023	-1.453
21	38	00	49656.907107	-1.264	22	50	30	49656.957454	-1.426
21	38	45	49656.907628	-1.268	22	51	00	49656.957801	-1.506
21	39	45	49656.908322	-1.232	22	51	30	49656.958148	-1.482
21	43	00	49656.910579	-1.350	22	52	15	49656.958669	-1.445
21	44	00	49656.911273	-1.302	22	53	00	49656.959190	-1.448
21	47	30	49656.913704	-1.386	23	01	45	49656.965266	-1.552
21	48	15	49656.914225	-1.337	23	02	30	49656.965787	-1.573
21	48	45	49656.914572	-1.352	23	05	30	49656.967870	-1.540
21	49	30	49656.915093	-1.355	23	06	30	49656.968565	-1.464
21	53	00	49656.917523	-1.396	23	07	00	49656.968912	-1.379
21	53	30	49656.917871	-1.389	23	07	45	49656.969433	-1.518
21	54	30	49656.918565	-1.435	23	08	45	49656.970127	-1.412
21	58	00	49656.920996	-1.323	23	09	30	49656.970648	-1.412
21	59	00	49656.921690	-1.293	23	12	30	49656.972731	-1.433
22	00	30	49656.922732	-1.343	23	13	15	49656.973252	-1.458
22	01	00	49656.923079	-1.347	23	14	00	49656.973773	-1.485
22	06	00	49656.926551	-1.382	23	14	30	49656.974120	-1.467
22	07	00	49656.927246	-1.406	23	17	30	49656.976204	-1.389
22	07	30	49656.927593	-1.406	23	18	15	49656.976724	-1.452
22	10	30	49656.929676	-1.520	23	18	45	49656.977072	-1.465
22	11	15	49656.930197	-1.512	23	22	15	49656.979502	-1.515
22	12	00	49656.930718	-1.512	23	23	00	49656.980023	-1.488
22	14	30	49656.932454	-1.530	23	24	00	49656.980717	-1.410
22	15	15	49656.932975	-1.555	23	27	45	49656.983322	-1.413
22	16	00	49656.933496	-1.549	23	28	30	49656.983842	-1.323
22	18	00	49656.934884	-1.484	23	29	00	49656.984190	-1.404
22	18	30	49656.935232	-1.485	23	29	45	49656.984710	-1.404
					23	32	45	49656.986794	-1.405
					23	40	15	49656.992002	-1.468

Table 1 (continued)

U.T.			Hel. MJD	delta	U.T.			Hel. MJD	delta
23	41	00	49656.992523	-1.502	19	45	30	49657.828974	-1.394
23	41	30	49656.992870	-1.468	19	47	45	49657.830537	-1.283
23	42	15	49656.993391	-1.460	19	48	45	49657.831231	-1.340
23	45	00	49656.995301	-1.435	19	49	15	49657.831578	-1.333
23	45	45	49656.995821	-1.470	19	49	45	49657.831926	-1.346
23	46	15	49656.996169	-1.494	19	50	30	49657.832446	-1.361
23	47	00	49656.996689	-1.458	19	53	30	49657.834530	-1.382
23	54	15	49657.001724	-1.363	19	54	00	49657.834877	-1.417
23	55	00	49657.002245	-1.369	19	55	00	49657.835571	-1.410
23	56	15	49657.003113	-1.393	19	55	45	49657.836092	-1.355
23	57	00	49657.003634	-1.443	19	56	30	49657.836613	-1.350
23	58	00	49657.004328	-1.393	19	59	30	49657.838696	-1.394
23	58	45	49657.004849	-1.470	20	00	00	49657.839044	-1.414
23	59	45	49657.005544	-1.452	20	01	15	49657.839912	-1.418
24	00	15	49657.005891	-1.394	20	01	45	49657.840259	-1.414
24	03	15	49657.007974	-1.419	20	08	15	49657.844773	-1.359
24	04	00	49657.008495	-1.364	20	09	00	49657.845293	-1.333
24	04	30	49657.008842	-1.263	20	10	00	49657.845988	-1.433
24	05	15	49657.009363	-1.308	20	10	30	49657.846335	-1.352
24	06	00	49657.009884	-1.315	20	11	30	49657.847030	-1.337
24	08	15	49657.011446	-1.301	20	14	30	49657.849113	-1.524
24	09	30	49657.012314	-1.292	20	15	00	49657.849460	-1.487
24	12	15	49657.014224	-1.295	20	15	45	49657.849981	-1.461
24	13	30	49657.015092	-1.223	20	16	30	49657.850502	-1.461
24	16	30	49657.017175	-1.375	20	19	00	49657.852238	-1.435
24	17	00	49657.017523	-1.367	20	20	00	49657.852932	-1.438
1 Nov. 1994					20	20	45	49657.853453	-1.401
19	06	15	49657.801717	-1.448	20	23	30	49657.855363	-1.426
19	07	15	49657.802412	-1.456	20	24	15	49657.855884	-1.431
19	08	15	49657.803106	-1.483	20	24	45	49657.856231	-1.479
19	08	45	49657.803454	-1.397	20	25	30	49657.856752	-1.464
19	12	30	49657.806058	-1.465	20	28	15	49657.858661	-1.382
19	13	15	49657.806579	-1.414	20	28	45	49657.859009	-1.410
19	13	45	49657.806926	-1.422	20	29	45	49657.859703	-1.412
19	14	45	49657.807620	-1.406	20	33	30	49657.862307	-1.351
19	24	00	49657.814044	-1.496	20	34	15	49657.862828	-1.424
19	24	30	49657.814391	-1.504	20	35	15	49657.863522	-1.475
19	25	30	49657.815085	-1.501	20	36	15	49657.864217	-1.446
19	26	15	49657.815606	-1.516	20	39	15	49657.866300	-1.314
19	30	15	49657.818384	-1.474	20	39	30	49657.866474	-1.359
19	31	15	49657.819078	-1.441	20	40	15	49657.866995	-1.382
19	32	00	49657.819599	-1.402	20	42	00	49657.868210	-1.390
19	32	30	49657.819946	-1.380	20	45	30	49657.870640	-1.453
19	38	15	49657.823939	-1.281	20	46	30	49657.871335	-1.442
19	39	15	49657.824634	-1.330	20	47	00	49657.871682	-1.462
19	40	00	49657.825155	-1.390	20	50	00	49657.873765	-1.356
19	40	45	49657.825676	-1.398	20	50	45	49657.874286	-1.327
19	43	15	49657.827412	-1.393	20	51	30	49657.874807	-1.416
19	44	00	49657.827933	-1.390	20	52	00	49657.875154	-1.392
19	45	00	49657.828627	-1.401	20	56	45	49657.878453	-1.440
					20	57	30	49657.878974	-1.452
					20	58	15	49657.879495	-1.432

Table 1 (continued)

U.T.			Hel. MJD	delta	U.T.			Hel. MJD	delta
21	01	45	49657.881925	-1.233	22	23	15	49657.938522	-1.360
21	02	30	49657.882446	-1.245	22	23	45	49657.938869	-1.314
21	03	00	49657.882793	-1.281	5 Nov. 1994				
21	03	45	49657.883314	-1.327	19	59	00	49661.838299	-1.303
21	06	45	49657.885397	-1.394	20	00	00	49661.838993	-1.342
21	07	30	49657.885918	-1.443	20	00	45	49661.839514	-1.336
21	08	15	49657.886439	-1.460	20	04	30	49661.842118	-1.461
21	08	45	49657.886786	-1.416	20	05	15	49661.842639	-1.452
21	11	45	49657.888869	-1.422	20	09	45	49661.845764	-1.359
21	12	15	49657.889217	-1.416	20	10	30	49661.846285	-1.368
21	31	45	49657.902758	-1.440	20	11	15	49661.846806	-1.372
21	32	30	49657.903279	-1.434	20	12	00	49661.847326	-1.300
21	33	15	49657.903800	-1.435	20	16	00	49661.850104	-1.384
21	34	30	49657.904668	-1.419	20	16	30	49661.850451	-1.446
21	37	15	49657.906578	-1.382	20	17	30	49661.851146	-1.375
21	37	30	49657.906751	-1.409	20	18	15	49661.851667	-1.375
21	38	15	49657.907272	-1.336	20	19	00	49661.852187	-1.401
21	38	45	49657.907619	-1.406	20	22	45	49661.854792	-1.416
21	41	30	49657.909529	-1.493	20	23	30	49661.855312	-1.503
21	42	15	49657.910050	-1.591	20	24	30	49661.856007	-1.568
21	43	00	49657.910571	-1.522	20	25	15	49661.856528	-1.542
21	43	45	49657.911091	-1.544	20	26	00	49661.857049	-1.534
21	47	30	49657.913696	-1.557	20	29	00	49661.859132	-1.411
21	48	00	49657.914043	-1.557	20	30	00	49661.859826	-1.379
21	48	45	49657.914564	-1.569	20	30	30	49661.860173	-1.368
21	49	30	49657.915084	-1.545	20	34	00	49661.862604	-1.413
21	52	15	49657.916994	-1.507	20	34	45	49661.863125	-1.398
21	52	45	49657.917341	-1.481	20	35	30	49661.863646	-1.392
21	53	00	49657.917515	-1.456	20	39	00	49661.866076	-1.389
21	54	15	49657.918383	-1.426	20	39	45	49661.866597	-1.322
21	57	45	49657.920814	-1.437	20	40	30	49661.867118	-1.338
21	58	40	49657.921450	-1.406	20	41	30	49661.867812	-1.354
21	59	00	49657.921682	-1.406	20	42	30	49661.868507	-1.379
21	59	30	49657.922029	-1.429	20	46	00	49661.870937	-1.388
22	02	30	49657.924112	-1.478	20	46	30	49661.871284	-1.394
22	03	15	49657.924633	-1.429	20	47	15	49661.871805	-1.390
22	04	30	49657.925501	-1.421	20	48	00	49661.872326	-1.373
22	04	45	49657.925675	-1.390	20	48	45	49661.872847	-1.390
22	07	00	49657.927237	-1.421	20	49	45	49661.873541	-1.460
22	07	30	49657.927584	-1.400	20	50	30	49661.874062	-1.497
22	08	15	49657.928105	-1.436	20	53	30	49661.876145	-1.394
22	09	00	49657.928626	-1.439	20	54	30	49661.876840	-1.409
22	12	00	49657.930709	-1.437	20	55	00	49661.877187	-1.347
22	12	45	49657.931230	-1.415	20	56	00	49661.877882	-1.429
22	13	15	49657.931577	-1.407	20	57	00	49661.878576	-1.400
22	13	45	49657.931925	-1.409	21	00	30	49661.881006	-1.320
22	16	45	49657.934008	-1.445	21	01	15	49661.881527	-1.404
22	17	15	49657.934355	-1.398	21	02	00	49661.882048	-1.449
22	18	00	49657.934876	-1.369	21	02	30	49661.882395	-1.464
22	18	45	49657.935397	-1.410	21	03	30	49661.883090	-1.485
22	22	00	49657.937654	-1.392	21	04	30	49661.883784	-1.575
22	22	30	49657.938001	-1.392					

Table 1 (continued)

U.T.			Hel. MJD	delta	U.T.			Hel. MJD	delta
21	05	30	49661.884479	-1.533	22	28	30	49661.942117	-1.382
21	11	45	49661.888819	-1.511	22	29	15	49661.942637	-1.349
21	12	45	49661.889513	-1.485	22	33	45	49661.945762	-1.270
21	13	30	49661.890034	-1.507	22	34	30	49661.946283	-1.320
21	16	15	49661.891944	-1.616	22	35	15	49661.946804	-1.317
21	17	00	49661.892465	-1.588	22	36	00	49661.947325	-1.317
21	17	45	49661.892985	-1.569	22	37	00	49661.948019	-1.279
21	21	30	49661.895590	-1.308	22	37	30	49661.948367	-1.287
21	22	15	49661.896110	-1.334	22	40	30	49661.950450	-1.435
21	23	15	49661.896805	-1.324	22	41	15	49661.950971	-1.403
21	26	15	49661.898888	-1.427	22	42	00	49661.951491	-1.411
21	27	00	49661.899409	-1.434	22	43	00	49661.952186	-1.348
21	27	45	49661.899930	-1.425	22	46	00	49661.954269	-1.325
21	30	30	49661.901839	-1.480	22	46	45	49661.954790	-1.241
21	31	15	49661.902360	-1.506	22	47	30	49661.955311	-1.331
21	31	45	49661.902708	-1.512	22	48	30	49661.956005	-1.378
21	32	30	49661.903228	-1.515	22	51	30	49661.958089	-1.337
21	35	45	49661.905485	-1.332	22	52	30	49661.958783	-1.416
21	36	30	49661.906006	-1.339	22	53	30	49661.959477	-1.395
21	37	15	49661.906527	-1.354	22	54	00	49661.959825	-1.418
21	38	00	49661.907048	-1.369	22	57	15	49661.962082	-1.387
21	41	00	49661.909131	-1.433	22	58	00	49661.962602	-1.309
21	41	45	49661.909652	-1.458	22	59	00	49661.963297	-1.388
21	42	30	49661.910173	-1.458	22	59	45	49661.963818	-1.407
21	43	30	49661.910867	-1.441	23	04	00	49661.966769	-1.301
21	47	45	49661.913818	-1.377	23	04	45	49661.967290	-1.307
21	48	30	49661.914339	-1.337	23	08	45	49661.970068	-1.447
21	49	15	49661.914860	-1.313	23	09	30	49661.970588	-1.456
21	52	15	49661.916943	-1.451	23	10	15	49661.971109	-1.456
21	53	00	49661.917464	-1.451	23	13	15	49661.973192	-1.409
21	54	00	49661.918159	-1.468	23	14	00	49661.973713	-1.371
21	57	15	49661.920416	-1.451	23	15	00	49661.974408	-1.425
21	58	00	49661.920936	-1.386	23	15	45	49661.974929	-1.460
21	59	00	49661.921631	-1.388	23	16	30	49661.975449	-1.507
22	00	00	49661.922325	-1.416	7 Nov. 1994				
22	00	30	49661.922672	-1.408					
22	04	00	49661.925103	-1.314					
22	04	45	49661.925624	-1.247	20	03	45	49663.841562	-1.342
22	05	30	49661.926145	-1.301	20	04	30	49663.842083	-1.342
22	06	15	49661.926665	-1.324	20	05	15	49663.842604	-1.351
22	07	00	49661.927186	-1.346	20	06	00	49663.843124	-1.370
22	07	45	49661.927707	-1.370	20	09	45	49663.845729	-1.405
22	13	00	49661.931353	-1.360	20	10	30	49663.846249	-1.377
22	13	45	49661.931874	-1.322	20	11	15	49663.846770	-1.390
22	14	45	49661.932568	-1.330	20	11	45	49663.847117	-1.346
22	15	30	49661.933089	-1.338	20	18	15	49663.851631	-1.340
22	19	00	49661.935519	-1.320	20	19	15	49663.852326	-1.378
22	19	30	49661.935867	-1.328	20	20	00	49663.852846	-1.371
22	20	15	49661.936388	-1.390	20	23	15	49663.855103	-1.252
22	21	00	49661.936908	-1.368	20	23	45	49663.855451	-1.268
22	26	30	49661.940728	-1.375	20	24	30	49663.855971	-1.268
22	27	30	49661.941422	-1.315	20	25	15	49663.856492	-1.313

Table 1 (continued)

U.T.			Hel. MJD	delta	U.T.			Hel. MJD	delta
20	26	00	49663.857013	-1.284	21	57	30	49663.920553	-1.341
20	30	45	49663.860312	-1.320	22	00	30	49663.922637	-1.462
20	31	30	49663.860832	-1.409	22	01	15	49663.923158	-1.400
20	32	00	49663.861180	-1.363	22	02	30	49663.924026	-1.342
20	33	00	49663.861874	-1.387	22	03	00	49663.924373	-1.300
20	33	45	49663.862395	-1.402	22	04	00	49663.925067	-1.334
20	38	15	49663.865520	-1.374	22	09	45	49663.929060	-1.516
20	39	30	49663.866388	-1.387	22	10	30	49663.929581	-1.444
20	40	45	49663.867256	-1.336	22	12	00	49663.930623	-1.395
20	41	30	49663.867777	-1.337	22	13	00	49663.931317	-1.419
20	50	30	49663.874027	-1.323	22	19	00	49663.935484	-1.354
20	51	15	49663.874547	-1.324	22	20	00	49663.936178	-1.408
20	54	30	49663.876804	-1.433	22	21	00	49663.936873	-1.363
20	55	15	49663.877325	-1.422	22	24	30	49663.939303	-1.348
20	56	30	49663.878193	-1.440	22	25	30	49663.939998	-1.300
21	00	30	49663.880971	-1.332	22	26	30	49663.940692	-1.337
21	01	15	49663.881492	-1.352	22	27	30	49663.941386	-1.393
21	01	45	49663.881839	-1.295	22	30	30	49663.943470	-1.356
21	02	30	49663.882360	-1.363	22	31	30	49663.944164	-1.363
21	03	15	49663.882881	-1.386	22	32	30	49663.944859	-1.352
21	07	15	49663.885658	-1.412	22	33	15	49663.945379	-1.376
21	08	00	49663.886179	-1.397	22	37	15	49663.948157	-1.346
21	08	45	49663.886700	-1.453	22	38	00	49663.948678	-1.365
21	14	30	49663.890693	-1.320	22	39	00	49663.949372	-1.368
21	15	15	49663.891214	-1.341	22	39	30	49663.949720	-1.383
21	16	00	49663.891735	-1.326	22	43	30	49663.952497	-1.357
21	19	30	49663.894165	-1.424	22	44	30	49663.953192	-1.356
21	20	30	49663.894859	-1.453	22	45	15	49663.953713	-1.365
21	21	00	49663.895207	-1.394	22	46	00	49663.954233	-1.365
21	22	00	49663.895901	-1.456	22	49	15	49663.956490	-1.283
21	25	15	49663.898158	-1.451	22	50	00	49663.957011	-1.408
21	26	00	49663.898679	-1.466	22	51	00	49663.957705	-1.408
21	26	30	49663.899026	-1.471	22	51	30	49663.958053	-1.365
21	29	30	49663.901109	-1.396	22	52	15	49663.958574	-1.349
21	31	00	49663.902151	-1.309	22	55	15	49663.960657	-1.329
21	32	00	49663.902845	-1.448	22	56	00	49663.961178	-1.265
21	33	00	49663.903540	-1.455	22	57	00	49663.961872	-1.345
21	37	15	49663.906491	-1.505	22	58	00	49663.962567	-1.329
21	38	00	49663.907012	-1.547	22	59	00	49663.963261	-1.380
21	38	30	49663.907359	-1.564	23	03	00	49663.966039	-1.414
21	42	00	49663.909790	-1.585	23	03	45	49663.966559	-1.414
21	43	00	49663.910484	-1.540	23	04	15	49663.966907	-1.321
21	43	30	49663.910831	-1.597	23	05	00	49663.967428	-1.369
21	44	30	49663.911526	-1.515	23	08	15	49663.969684	-1.444
21	45	00	49663.911873	-1.422	23	09	00	49663.970205	-1.514
21	49	00	49663.914651	-1.376	23	10	00	49663.970900	-1.348
21	49	45	49663.915172	-1.361	23	11	00	49663.971594	-1.364
21	50	30	49663.915692	-1.456	23	14	30	49663.974025	-1.348
21	51	00	49663.916040	-1.437	23	15	15	49663.974545	-1.424
21	54	30	49663.918470	-1.410	23	16	15	49663.975240	-1.471
21	55	30	49663.919165	-1.430	23	17	30	49663.976108	-1.505
21	56	30	49663.919859	-1.375					

A power spectra analysis of our data will be carried out in the next future, in order to determine the short time scale oscillations.

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DIFFERENTIAL PHOTOMETRY OF NSV5798

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Abstract. Blue and yellow photometric observations of the eclipsing binary system NSV5798 were made on 21 nights during 1984 to 1992. A listing of all the observations and the corresponding light curve and ephemeris are given.

Key words: Variable star — W UMa system — photometry.

1. INTRODUCTION

NSV5798 is classified in the New Catalogue of Suspected Variable Stars (1982) as a W Ursae Majoris system.

Figure 1 gives an identification chart where 1 and 2 are the comparison and check stars, respectively. Table 1 contains the B and V magnitudes of the stars.

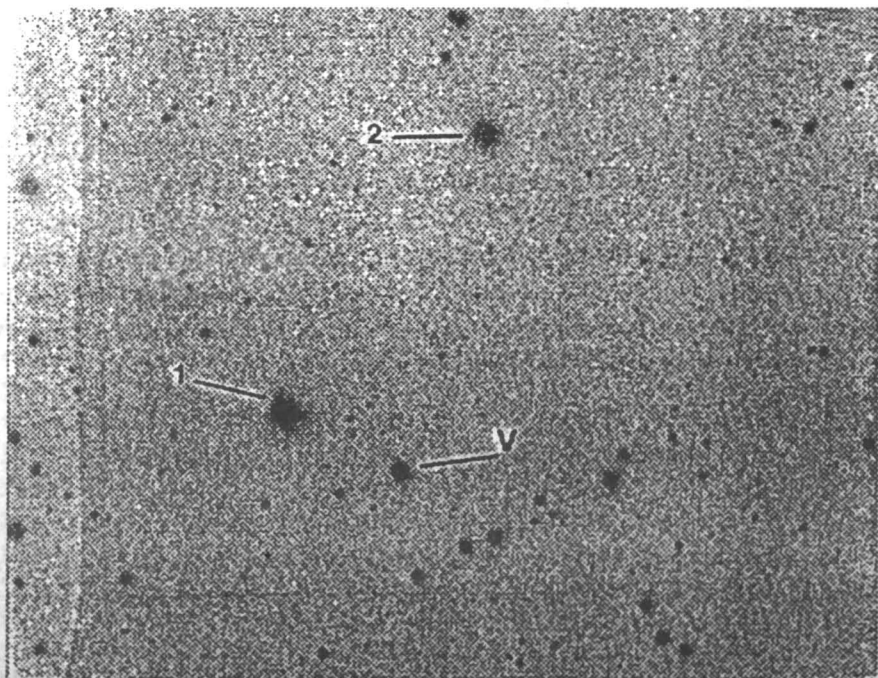


Fig. 1. — Identification chart reproduced from the print of the Palomar Sky Survey. North is at the top and east is to the left.

Table 1

B and *V* magnitudes of the variable, comparison and check stars

Star	RA (1950)	Dec	<i>B</i>	<i>V</i>
NSV5798	12h 35 m 33 s	-3° 42' 6	12.29-12.96	11.62-12.24
1	43s	41' 7	10.75	9.75
2	22s	35' 7	10.78	10.20

2. OBSERVATIONS

The present observations were made on 21 nights in the years 1984-92 with the 150 cm reflecting telescope of the National Astronomical Observatory at Calar Alto (Almería). The Cassegrain focus of this telescope was equipped with a photometer housing a RCA313034A photomultiplier refrigerated with dry ice. The filters used were the standard ones for the Johnson system

B: BG 12/1 + BG 18/1 + GG 400/1

V: GG 495/2 + BG 18/1

and the output of the photometer is fed to a digital counting system. A complete description of this photometer is elsewhere (Lahulla and Pensado, 1982).

An observation normally consisted in measuring for 20 seconds in each of the filters the variable and the comparison star. After each star the sky was measured. During each night, observations of the check star established the constancy of the comparison star. Standard stars in the Neckel list (Neckel and Chini, 1980) were observed for transformation to Johnson's system.

3. EPOCHS OF MINIMUM AND LIGHT ELEMENTS

Plots of the individual nightly of ΔV versus Julian date allowed to derive epochs of minimum and maximum light. These values are listed in Table 2. From the plots of observations versus phase a provisory epoch and period were determined.

The epochs of minimum light of Table 2 were introduced into a least-squares solution to obtain improved light elements for the system, namely

$$\text{JD Hel Min} = 5798.2189 + 0.369379 \text{ E.}$$

Table 2

Magnitudes: NSV 5798 — Comparison

JD (Hel.) 2440000+	ΔV	ΔB	JD (Hel.) 2440000+	ΔV	ΔB
5798.4384	2.147	1.849	6155.5269	1.905	1.604
5798.4515	2.039	1.702	6155.5327	1.952	1.625
5798.5295	1.880	1.624	6155.5380	1.974	1.647
5798.5415	1.989	1.676	6185.4661	1.992	1.608
5798.5847	2.341	2.035	6185.4743	2.034	1.707
5798.5978	2.490	2.192	6185.4833	2.133	1.788
5798.6015	2.459	2.145	6185.4908	2.201	1.870
5798.6149	2.258	1.926	6185.4978	2.316	1.980
5802.3728	1.849	1.474	6185.5064	2.430	2.024
5802.3785	1.828	1.475	6185.5122	2.495	2.116
5802.3835	1.867	1.593	6185.5174	2.544	2.252
5802.3884	1.851	1.485	6185.5230	2.456	2.142
5802.3934	1.881	1.537	6185.5353	2.330	2.014
5802.3979	1.951	1.624	6185.5414	2.194	1.878
5802.4023	1.910	1.558	6492.5017	2.103	1.808
5802.4534	2.430	2.104	6492.5134	2.090	1.735
5802.4583	2.382	2.115	6492.5420	1.900	1.584
5802.4632	2.372	2.108	6492.5503	1.881	1.565
5802.4678	2.496	2.201	6492.6028	1.992	1.690
5802.4726	2.522	2.302	6492.6074	2.014	1.703
5802.4805	2.497	2.145	6492.6206	2.132	1.831
6149.4800	2.249	1.956	6492.6256	2.197	1.884
6149.4909	2.422	2.085	6492.6546	2.482	2.222
6149.4961	2.469	2.152	6492.6617	2.431	2.143
6149.5011	2.512	2.218	6492.6663	2.374	2.092
6149.5085	2.467	2.208	7189.6835	2.514	2.201
6154.5453	1.904	1.577	7189.6871	2.492	2.133
6154.5506	1.909	1.558	7189.6903	2.447	2.127
6154.5551	1.912	1.585	7189.6934	2.364	2.060
6155.4291	2.314	2.041	7189.6961	2.335	2.023
6155.4405	2.195	1.903	7189.6994	2.303	2.010
6155.4448	2.155	1.805	7189.7032	2.293	1.943
6155.4512	2.125	1.823	7189.7067	2.264	1.908
6155.4557	2.038	1.721	7189.7347	2.059	1.721
6155.4766	1.909	1.581	7189.7380	2.068	1.733
6155.5047	1.961	1.636	7182.6654	2.548	2.217
6155.5103	1.944	1.593	7182.6724	2.418	2.111
6155.5157	1.912	1.583	7182.6787	2.394	2.045
6155.5212	1.925	1.604	7182.6855	2.275	1.945

Table 2 (continued)

JD (Hel.) 2440000+	ΔV	ΔB	JD (Hel.) 2440000+	ΔV	ΔB
7201.6430	2.177	1.848	7573.7011	1.982	1.651
7201.6501	2.285	1.854	7571.6000	2.425	2.152
7201.6580	2.278	1.997	7571.6355	2.143	1.865
7201.6670	2.612	2.298	7571.6755	1.925	1.611
7201.6762	2.476	2.193	7571.7012	1.870	1.554
7201.6837	2.388	2.082	7572.5653	2.133	1.800
7201.6913	2.256	1.948	7572.5977	1.953	1.610
7201.6986	2.165	1.847	7572.6404	1.928	1.598
7201.7059	2.050	1.744	7572.6746	2.028	1.724
7201.7129	2.021	1.705	7572.6911	2.228	1.909
7202.5988	2.489	2.204	7647.3750	2.012	1.689
7202.6057	2.425	2.131	7647.3787	2.000	1.666
7202.6102	2.370	2.083	7647.3816	1.978	1.654
7202.6147	2.313	2.027	7647.3847	1.960	1.643
7202.6312	2.132	1.830	7647.3893	1.948	1.610
7202.6356	2.102	1.795	7647.3922	1.935	1.612
7204.5672	1.977	1.651	7647.3976	1.936	1.622
7204.5720	1.992	1.655	7647.4010	1.926	1.622
7204.5768	2.004	1.677	7647.4051	1.930	1.620
7204.5814	2.033	1.699	7647.4081	1.910	1.597
7204.5860	2.064	1.737	7647.4112	1.916	1.600
7204.5907	2.106	1.774	7647.4140	1.911	1.597
7204.5961	2.151	1.823	7647.4186	1.890	1.573
7204.6000	2.207	1.887	7647.4225	1.889	1.569
7204.6048	2.276	1.958	7647.4252	1.880	1.565
7204.6096	2.339	2.020	7647.4280	1.889	1.563
7204.6145	2.419	2.102	7647.4307	1.903	1.567
7204.6192	2.480	2.176	7647.4335	1.898	1.575
7204.6241	2.499	2.200	7647.4368	1.901	1.571
7204.6286	2.484	2.190	7647.4415	1.926	1.564
7204.6331	2.387	2.087	7647.4445	1.917	1.578
7204.6377	2.391	2.114	7647.4476	1.928	1.594
7204.6424	2.302	1.985	7647.4504	1.914	1.576
7204.6473	2.230	1.912	7647.4535	1.942	1.599
7204.6517	2.210	1.868	7647.4563	1.960	1.629
7204.6564	2.160	1.788	7647.4616	2.023	1.703
7204.6612	2.101	1.825	7647.4660	1.997	1.668
7204.6663	2.006	1.608	7647.4688	1.989	1.651
7573.5401	1.893	1.569	7647.4713	1.982	1.653
7573.5946	2.032	1.703	7647.4754	2.060	1.751
7573.6553	2.352	2.058	7647.4784	2.089	1.758

Table 2 (continued)

JD (Hel.) 2440000+	ΔV	ΔB	JD (Hel.) 2440000+	ΔV	ΔB
7647.4811	2.097	1.776	7648.4399	2.467	2.178
7647.4844	2.163	1.828	7648.4447	2.435	2.133
7647.4888	2.220	1.897	7648.4471	2.387	2.091
7647.4921	2.263	1.938	7648.4495	2.387	2.055
7647.4961	2.323	1.996	7648.4522	2.328	2.037
7647.4994	2.377	2.076	7648.4546	2.299	2.000
7647.5022	2.424	2.116	7648.4580	2.265	1.951
7647.5054	2.458	2.152	7648.4613	2.215	1.911
7647.5091	2.505	2.217	7648.4659	2.155	1.835
7647.5119	2.529	2.236	7648.4694	2.121	1.801
7647.5154	2.527	2.225	7648.4730	2.089	1.761
7647.5182	2.511	2.218	7648.4772	2.060	1.727
7647.5211	2.470	2.169	7648.4801	2.049	1.727
7647.5239	2.408	2.105	7648.4913	1.979	1.666
7647.5270	2.381	2.099	7648.4989	1.948	1.625
7647.5296	2.352	2.051	7648.5012	1.944	1.620
7647.5350	2.267	1.966	7648.5037	1.945	1.605
7647.5375	2.241	1.934	7648.5066	1.920	1.585
7647.5410	2.195	1.878	7648.5094	1.907	1.591
7647.5437	2.163	1.845	7648.5138	1.907	1.582
7647.5471	2.134	1.804	7648.5162	1.909	1.561
7648.3735	1.927	1.609	7648.5199	1.888	1.573
7648.3760	1.944	1.627	7648.5223	1.888	1.557
7648.3784	1.946	1.637	7648.5248	1.890	1.555
7648.3812	1.965	1.652	7648.5278	1.877	1.548
7648.3837	1.979	1.654	7648.5302	1.891	1.542
7648.3884	1.989	1.683	7648.5308	1.888	1.553
7648.3908	2.016	1.694	7648.5314	1.885	1.541
7648.3947	2.033	1.710	7648.5319	1.881	1.544
7648.3972	2.046	1.733	7648.5350	1.879	1.552
7648.4008	2.073	1.757	7648.5356	1.887	1.550
7648.4108	2.195	1.867	7648.5380	1.890	1.552
7648.4133	2.215	1.889	7648.5410	1.887	1.555
7648.4177	2.263	1.947	7648.5434	1.895	1.567
7648.4200	2.287	1.984	7648.5469	1.907	1.607
7648.4228	2.343	2.027	7648.5492	1.906	1.575
7648.4252	2.377	2.045	7648.5516	1.912	1.580
7648.4279	2.401	2.084	7649.3480	2.388	2.031
7648.4308	2.427	2.162	7649.3509	2.428	2.111
7648.4337	2.451	2.146	7649.3540	2.473	2.176
7648.4379	2.484	2.171	7649.4601	1.894	1.567

Table 2 (continued)

JD (Hel.) 2440000+	ΔV	ΔB	JD (Hel.) 2440000+	ΔV	ΔB
7649.4643	1.896	1.570	7650.4217	2.004	1.679
7649.4672	1.893	1.584	7650.4239	2.036	1.702
7649.4698	1.901	1.586	7650.4263	2.035	1.706
7649.4721	1.890	1.577	7650.4286	2.054	1.723
7649.4752	1.913	1.589	7650.4309	2.073	1.758
7649.4777	1.925	1.602	7650.4331	2.088	1.761
7649.4801	1.913	1.578	7650.4354	2.111	1.776
7649.4827	1.896	1.579	7650.4377	2.143	1.829
7649.4857	1.954	1.637	7650.4400	2.157	1.841
7649.4881	1.962	1.638	7650.4424	2.202	1.873
7649.4932	1.989	1.670	7650.4450	2.220	1.908
7649.4958	1.988	1.673	7650.4475	2.280	1.977
7649.4983	2.039	1.716	7650.4501	2.304	1.999
7649.5013	2.022	1.711	7650.4526	2.349	2.036
7649.5039	2.048	1.729	7650.4550	2.395	2.071
7650.3567	1.912	1.577	7650.4575	2.447	2.125
7650.3603	1.904	1.583	7650.4599	2.462	2.157
7650.3633	1.900	1.557	7650.4624	2.482	2.191
7650.3660	1.904	1.571	7650.4648	2.522	2.212
7650.3685	1.895	1.572	7650.4673	2.493	2.225
7650.3710	1.886	1.553	7650.4697	2.517	2.279
7650.3733	1.886	1.552	7650.4721	2.521	2.227
7650.3758	1.889	1.553	7650.4745	2.490	2.196
7650.3785	1.879	1.556	7650.4768	2.449	2.181
7650.3810	1.881	1.545	7650.4794	2.461	2.165
7650.3834	1.875	1.551	7650.4820	2.385	2.094
7650.3858	1.883	1.549	7650.4844	2.353	2.053
7650.3883	1.894	1.541	7650.4868	2.313	2.015
7650.3907	1.886	1.563	7651.3617	2.141	1.822
7650.3932	1.900	1.571	7651.3640	2.160	1.853
7650.3958	1.901	1.567	7651.3664	2.195	1.885
7650.3984	1.915	1.593	7651.4145	2.243	1.939
7650.4009	1.929	1.588	7651.4173	2.209	1.901
7650.4034	1.926	1.606	7651.4197	2.164	1.855
7650.4057	1.934	1.607	7651.4220	2.119	1.822
7650.4082	1.946	1.603	7651.4243	2.223	1.905
7650.4105	1.954	1.635	7651.4267	2.104	1.798
7650.4128	1.965	1.630	7651.4290	2.084	1.771
7650.4151	1.979	1.646	7651.4318	2.061	1.748
7650.4173	1.991	1.641	7651.4351	1.999	1.702
7650.4194	1.991	1.672	7651.4381	2.098	1.783

Table 2 (continued)

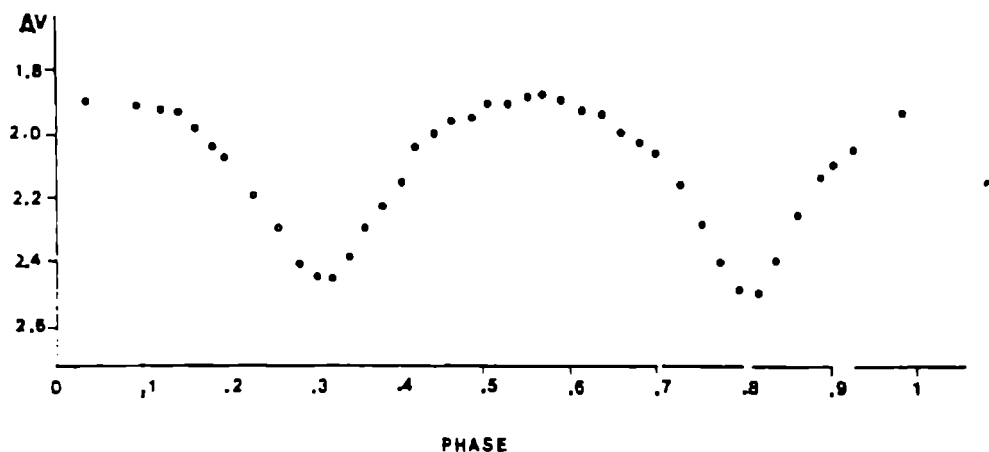
JD (Hel.) 2440000+	ΔV	ΔB	JD (Hel.) 2440000+	ΔV	ΔB
7651.4409	2.013	1.693	8751.3519	2.004	1.519
7651.4434	1.996	1.681	8751.3555	2.094	1.684
7651.4447	1.995	1.676	8751.3594	2.097	1.740
7651.4461	1.982	1.669	8751.3627	2.014	1.726
7651.4474	1.987	1.669	8751.3661	2.148	1.813
7651.4487	2.005	1.679	8751.4142	2.250	1.905
7651.4501	1.965	1.718	8751.4177	2.198	1.879
7651.4525	1.959	1.642	8751.4210	2.140	1.829
7651.4552	1.944	1.634	8751.4242	2.108	1.758
7651.4575	1.907	1.593	8751.4360	2.033	1.702
7651.4598	1.983	1.651	8751.4400	2.005	1.678
7651.4624	1.921	1.606	8751.4428	1.983	1.670
7651.4647	1.913	1.601	8751.4464	1.980	1.648
7651.4668	1.900	1.585	8751.4508	1.972	1.608
7651.4689	1.872	1.549	8751.4542	1.972	1.621
7651.4714	1.972	1.641	8751.4576	1.944	1.617
7651.4729	1.896	1.580	8751.4612	1.935	1.571
7651.4761	1.892	1.573	8751.4644	1.921	1.582
7651.4775	1.888	1.573	8751.4673	1.895	1.567
7651.4781	1.892	1.573	8751.4707	1.904	1.545
7651.4786	1.895	1.567	8751.4736	1.893	1.543
7651.4799	1.812	1.497	8751.4767	1.888	1.539
7651.4812	1.839	1.523	8751.4797	1.887	1.553
7651.4897	1.792	1.480	8751.4831	1.878	1.551
7651.4902	1.796	1.479	8751.4870	1.869	1.522
7651.4908	1.818	1.484	8751.4906	1.864	1.533
7651.4921	1.849	1.511	8751.4941	1.876	1.543
7651.4948	1.884	1.561	8751.4973	1.892	1.547
7651.4973	1.889	1.561	8751.5007	1.899	1.525
7651.4998	1.914	1.579	8751.5038	1.898	1.561
7651.5027	1.885	1.567	8751.5072	1.895	1.554
7651.5051	1.885	1.552	8751.5106	1.926	1.565
7651.5075	2.010	1.679	8751.5139	1.930	1.603
7651.5099	1.918	1.601	8751.5175	1.939	1.602
7651.5122	1.936	1.612	8751.5207	1.963	1.629
7651.5145	1.928	1.600	8751.5245	1.976	1.623
7651.5170	1.929	1.602	8751.5277	2.000	1.682
7651.5192	1.919	1.599	8751.5311	2.009	1.643
7651.5214	2.111	1.779	8751.5342	2.039	1.680
7651.5237	1.983	1.651	8751.5373	2.054	1.688
7651.5259	1.987	1.686	8751.5410	2.084	1.744
			8751.5441	2.104	1.774

With the above ephemeris 41 normal points were derived. They are contained in Table 3 and plotted versus phase in Figure 2. Table 4 shows the individual primary and secondary minima.

Table 3

Normal Points

Phase	ΔV	ΔB	Phase	ΔV	ΔB
0.038	1.892	1.572	0.526	1.900	1.574
0.096	1.902	1.583	0.547	1.877	1.545
0.122	1.920	1.595	0.568	1.873	1.540
0.142	1.930	1.624	0.589	1.897	1.566
0.162	1.986	1.642	0.612	1.923	1.591
0.180	2.033	1.703	0.633	1.940	1.609
0.196	2.070	1.748	0.655	1.993	1.660
0.231	2.183	1.868	0.677	2.029	1.690
0.257	2.298	1.986	0.697	2.059	1.733
0.281	2.410	2.106	0.722	2.160	1.833
0.302	2.450	2.148	0.747	2.284	1.956
0.323	2.455	2.165	0.770	2.408	2.092
0.340	2.383	2.080	0.791	2.482	2.183
0.359	2.296	1.953	0.814	2.493	2.204
0.380	2.232	1.921	0.838	2.394	2.091
0.400	2.144	1.820	0.861	2.252	1.938
0.418	2.041	1.724	0.885	2.142	1.823
0.442	1.988	1.661	0.901	2.097	1.766
0.461	1.959	1.638	0.926	2.050	1.714
0.483	1.951	1.623	0.982	1.932	1.596
0.503	1.905	1.581			

Fig. 2. — Plot of ΔV normal points versus phase.

The run of the light curve confirms the classification of this binary as a W UMa system.

Table 4

Individual minima JD (Hel.) 2440000+

I	II
6149.5011	5802.4726
6492.6546	6185.5174
7202.5988	7201.6670
7648.4379	7204.6241
8751.4142	7647.5119
	7649.3540
	7650.4648

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TOTAL SOLAR IRRADIANCE IN THE MAXIMUM PHASE OF THE ELEVEN YEARS SOLAR CYCLE

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Abstract. The authors set out to study the link between *IR* (registering performed on ERB/NIMBUS 7) and certain solar activity indices, in the maximum phase for solar cycles 21 (1979–1980) and 22 (1989–1990). From among the solar activity indices we have used: the daily sunspot relative number (R), the radio flux on 10.7 cm (F), the coronal index in green line (I_{c}), the daily flare index (I_f), the daily scaled flare number (SFN), the daily weighted flare index ($I_f P$) and the daily flare total area index ($I_f A$).

The results presented proof of the existence of real correlations between the total irradiance variation and the manifestations of the solar activity in the active regions (sunspots, flares, radio emission).

Key words: total solar irradiance, activity of the Sun.

1. INTRODUCTION

The observations of the total solar irradiance (*IR*) in the extraterrestrial space for 16 years permitted the analysis of the data series by the most variation methods. The detection of some periods of *IR* variation, their correlation with other periodicities in the solar activity and the utilization of the results in the explanation of the mysterious determinism solar interior-solar activity have been followed.

By using spectral analysis techniques periods of the order of days: 13; 22–30; 49–51 but also 154; 227 or 315 days (Frohlich et al., 1991) have been made evident. The variations are of the order of several tenths of percent, being characterized by decreases of the *IR* value throughout several days which appear in registries under the form of “deeps”. These deeps correlate best with the projected surface index (*PSI*), an index which depends on the area of the sunspots on the visible disk (Frohlich et al., 1991). The *PSI* is defined as:

$$PSI = \alpha \cdot \sum \{[(3 \cdot \mu + 2)/2] \cdot \mu \cdot a\} \quad (1)$$

where μ is the center angle of the sunspot group and a its area. The factor α takes into account the umbra/penumbra area ratio and the effective temperature of the

sunspot photosphere, in the simplest possible way. This influence of the sunspot does not explain entirely the *IR* variation, part of which is due to the variation of the faculae area in the active regions. Thus, the spectral analysis of the calcium plage index (Ca-K) presents perplexing similitudes with that of the data of the experiment from NIMBUS 7 in different spectral domains (Pap et al., 1990). In the power spectra of the Ca plage index the periods of 28 and 13.7 days are identical with those in the registries from NIMBUS in SBUV Mg II.

Less evident in the calcium plage data, the periods of 151 and 51 days are nevertheless present and the most evident periods in the two series of indices (327 days for Ca-K and 315 days for SBUV Mg II) are of the same magnitude order.

Besides the variations of the order of days or decades, in the *IR* registries a distinct modulation of 11 years (Foukal, 1992) can be observed. This long-term variation has been the first signal that the *IR* variation can be significant for the terrestrial climate. So far this variation could not be explained; there is only the hypothesis that it might be due more to the variation of the faculae during a cycle of 11 years than to the sunspot area variation.

More and more researchers consider that the elucidation of the causes of the *IR* variation might be the key to the explanation of several problems concerning the solar interior, as well as to the way in which the internal solar processes determine the surface activity and the activity in the solar atmosphere.

2. DATA AND METHOD OF ANALYSIS

By following the evolution of the solar activity phenomena throughout an eleven years solar cycle (SC) we can distinguish the following phases: minimum, ascending phase, maximum, descending phase. They differ by duration and by different levels of the solar activity due to the evolution of the solar magnetic field in a way specific to each phase of the cycle. For this reason studies of correlations between various manifestations of the solar activity should be made on the solar cycle phases and afterwards the results should be analysed taking into consideration the solar cycle as a whole.

We set out to study the link between *IR* (registering performed on the ERB/NIMBUS 7) and certain solar activity indices, in the maximum phase period for SCs 21 (1979–1980) and 22 (1989–1990). In the maximum phase of a cycle no manifestation of the magnetic fields specific to the preceding cycle is practically any longer registered and neither have the magnetic fluxes of the following cycle appeared yet on the surface, which means that this is a period proper to the solar cycle to which it belongs.

From among the solar activity indices we have used:

$\bar{R}$ — the daily sunspot relative number (Sunspot Bulletin, SIDC-Bruxelles), defined as:

$$\bar{R} = k(10g + f) \quad (2)$$

where g is number of sunspot group, f is total number of spot on the visible hemisphere and k reduces the observed values to the standard sunspot number.

F — the radio flux on 10.7 cm (2800 MHz), daily values expressed in units of $10^{-22} \text{ Wm}^{-2}\text{Hz}^{-1}$ (Solar-Geophysical Data, NGDC — Boulder, Colorado) (SGD);

I_c — the coronal index in green line, 530.3 nm (Tatranska Lomnica Obs.);

I_f — The daily flare index (Solar-Geophysical Data), defined as:

$$I_f = \frac{0.76}{T} \sum (A_d)^2 \quad (3)$$

where individual flare areas A_d are measured in millionths of solar disk, T is the effective observing time in minutes (I_f corresponds closely to the flare index developed at the High Altitude Observatory to measure the integrated intensity of flare radiation; the flare areas are not corrected for geometric foreshortening, so the definition of I_f place great weight on large flares located near the center of the Sun's disk);

SFN — the daily scaled flare number (Pap and Vrsnak, 1989) as:

$$SFN = \sum k_i N_i \quad (4)$$

where k_i is the scaling coefficient with values ranging between 0.5 and 50, by the importance class of the flares, and N_i is the daily number of flares in a particular class of importance, calculated by us for the period studied;

$I_f P$ — the daily weighted flare index defined as:

$$I_f P = \frac{\sum k_i N_i}{\sum k_i} \quad (5)$$

$I_f A$ — the daily flare total areas index defined as:

$$I_f A = \sum A_i \quad (6)$$

where $\sum A_i$ is the sum of the flare areas in one day, expressed in millionths of solar disk.

In our study we use the relative sunspot number instead of the sunspot areas because so far the data about SC 22 have not been available. Besides, between $\bar{R}$ and the sunspot areas S there is a good correlation (the correlation coefficient being of +0.85) expressed by:

$$S = 16.7 \bar{R} \quad (7)$$

so that the use of $\bar{R}$ gives a good approximation of the sunspot areas at a given moment.

The radio flux density of 10.7 cm includes the emission integrated from all sources existing on the solar disk at a given moment: active regions, large-scale magnetic field structures localized in the low corona and overheated regions. This index is well correlated with $\bar{R}$, which suggests the connection with the active regions, but also with indices which characterize the solar emission in lines of Ca-K, MgII, EUV, X-rays, which suggests the connection with the large-scale phenomena in the low corona.

It is well known that the intensity of the green coronal line depends on the solar activity level. The coronal index I_c used in this study represents the energy emitted by the entire solar corona in the green line and is expressed in Wsr^{-1} . The coronal index is deduced from the daily intensities measured around the solar disk (Rybansky, 1992).

The index SFN has been used by various authors for the correlation of the flares with other phenomena being considered as index which makes a good evaluation of the energy released by the flares due to the k_i scalars, which have been introduced in order to give priority to evaluations of the brightness class of the flares. For studies of correlation with IR we appreciate that a more realistic evaluation of the flare flux can be obtained through index $I_f P$ defined by (5), because it gives priority to the importance class of the flares to the detriment of the brightness class which is a subjective parameter that depends on the resolution power of the instrument, as well as on the quality and on the interpretation way of the observations.

We have considered that the eruptive area could be a good appreciation of the energy flux emitted in a solar flare, being consequently an adequate index for studies of correlations with the daily values of IR .

The correlation between IR and the above mentioned solar indices has been analysed as follows:

- a) in the maximum phases of the SCs 21 (1979–1980) and 22 (1989–1990);
- b) on annual periods: 1979, 1980; 1989; 1990;
- c) on Carrington rotations, from the rotation no. 1809 to the rotation no. 1836;
- d) in the deeps of Irradiance (D1: Feb. 27–May 2, 1980; D2: quarterly IV, 1980; D3: quarterly II, 1989; D4: quarterly III, 1990).

The analysis has been made on the basis of the indices mentioned above, the index series being organized for the proposed intervals of study: a, b, c and d. The correlation coefficient between the IR series and each series of considered indices has been calculated according to the following formula:

$$r(x, y) = \frac{\sum (x_i - \bar{x}) \cdot (y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \cdot \sum (y_i - \bar{y})^2}} \quad (8)$$

where x_i and y_i are the daily values of the correlated indices; $\bar{x}$ and $\bar{y}$ are the average values of the indices over the interval analysed.

3. RESULTS AND DISCUSSION

The graphic representation of the series of indices used for various intervals shows the existence of more or less periodical variations. This type of representations has been useful for making evident some "deeps" of irradiance. From among all, for the correlation study we have chosen only the "deeps" marked at point d).

The correlation coefficients calculated between the series of IR indices and each of the series of indices considered are presented in Tables 1–4, for all the analysed intervals.

For the maximum phases of the cycles, the correlation is stronger between IR and $\bar{R}$, on the one hand, and, between IR and F on the other hand, the values of the correlation coefficients being very close (Table 1).

Table 1

The Correlation Coefficients between the Total Solar Irradiance and the Solar Activity Indices in the Maximum Phase of SCs 21 and 22

Index	Max. SC 21	Max. SC 22
$\bar{R}$	-0.399	-0.376
F	-0.504	-0.364
I_c	+0.057	-0.424
I_f	-0.251	—
SFN	-0.274	+0.028
I_fP	-0.049	-0.280
I_fA	-0.409	-0.105

In the periods of solar activity maxima the contribution of the active regions in the radio flux component of 10.7 cm is greater than the large-scale coronal structures which are strongly perturbed by the magnetic structures of the active regions extended high in the corona. For this reason the correlation coefficients $r(IR, \bar{R})$ and $r(IR, F)$ have close values. As for the correlation between IR and flares, the best result can be obtained by taking into consideration the total flare area for the maximum of SC 21. The correlations between IR and the flare indices: I_f , SFN , I_fP do not give significant values. It can be observed that indices SFN and I_f give similar results in the correlations of the maximum of SC 21, which proves that index SFN is a good approximation for the flares index (I_f) calculated by the World Data Center, Boulder, Co. For correlations with IR these indices do not give enough weight to the eruptive area which gives the best evaluation of the energy flux released by flares. The discrepancy between the values 0.409 and 0.105 of the correlation coefficient $r(IR, I_fA)$ for SCs 21 and 22 respectively, is unexpected at first sight.

The analysis of the solar cycles from the point of view of the total energy power of the flares indicates a strong difference between the two maximum phase:

the total number of flares registered in the maximum of SC 21 being of 20144, while for the maximum of solar cycle 22 it is only of 14213. This difference is reflected also in the summed flare area, $I_f A$, consequently the correlation between IR and $I_f A$ modifies significantly from one maximum to another. The difference between the two maxima is maintained also in the annual correlation analyses (Table 2) and in the "deeps" (Table 3).

Table 2

The Correlations Coefficients between the Total Solar Irradiance and the Solar Activity Indices for Annual Periods

Index	1979	1980	1989	1990
$\bar{R}$	-0.528	-0.412	-0.419	-0.280
F	-0.497	-0.566	-0.375	-0.182
I_c	+0.176	-0.006	-0.467	-0.249
I_f	-0.205	-0.326	—	—
SFN	-0.264	-0.385	+0.094	+0.236
$I_f P$	-0.091	-0.009	-0.254	-0.166
$I_f A$	-0.302	-0.465	-0.117	-0.139

Table 3

The Correlation Coefficients between the Total Solar Irradiance and the Solar Activity Indices in Irradiance "Deeps"

Index	D1	D2	D3	D4
$\bar{R}$	-0.450	-0.506	-0.655	-0.761
F	-0.666	-0.831	-0.399	-0.339
I_c	-0.696	-0.022	-0.606	-0.718
I_f	-0.518	-0.353	—	—
SFN	-0.558	-0.519	+0.117	-0.191
$I_f P$	-0.197	+0.195	-0.393	-0.379
$I_f A$	-0.584	-0.588	-0.190	-0.252

The energy contribution of the flares emission in the radio field, on 10.7 cm, is found under a different form also in the correlation between IR and F for the two analysed maxima.

The sunspot relative numbers, $\bar{R}$, for the two cycle maxima have close annual values (155 and respectively 157), which is reflected also in the almost equal values of the correlation coefficients $r(IR, \bar{R})$ for the two cycles.

A great discrepancy is observed between the values of the correlation index $r(IR, I_c)$: +0.0557 in the maximum of SC 21 and -0.424 in the maximum of SC 22. The variations of I_c during the two cycles are different: the maximum of index I_c in

SC 21 was reached in the second half of the year 1981, two years after the maximum of sunspot relative number $\bar{R}$, while in SC 22 the maximum of I_c coincided with the maximum of $\bar{R}$ in 1989 (Rusin and Rybansky, 1991). The time difference between the two maxima of SC 21 is reflected also in the analysed correlations. This assertion is supported by the great value of $r(IR, I_c)$ in the "deep" D1 of the maximum of SC 21, which corresponds nevertheless to an interval with an "activity impulse" in the emission of the green coronal line (Rybansky et al., 1988).

For correlation analysis it is important to choose an appropriate interval in which the analysis is performed, because the fragmentations of some intervals or the choice of extending long intervals can lead to ambiguous results.

As concerns the solar activity, the most important intervals are: the eleven years cycle, the phases of eleven years solar cycle, the magnetic cycles of 22 years and the duration of the solar Carrington rotation (27 days).

Given the intrinsic connection between the solar differential rotation and the generation of toroidal magnetic fields which determine the small-scale magnetic activity of the solar atmosphere (active regions) we have detailed the correlation analysis on solar rotations. Table 4 presents the results obtained for the period of the SC 22 maximum between November 15, 1988–December 19, 1990. One can remark solar rotations in which the correlation index $r(IR, \bar{R})$ exceeds very much the value obtained in the entire maximum phase or in the annual periods (for example: -0.788 in rotation no. 1817 or -0.857 in rotation no. 1830), or others in which the correlation between IR and $\bar{R}$ disappears or changes its sign. By an attentive analysis, rotation by rotation, we have found in the rotations with great values of the negative correlation a sunspot activity that is constantly high throughout the entire rotation, sunspot groups of large area or several sunspot groups of moderate area, so that the index $\bar{R}$ has been high throughout the entire interval.

In the rotations with a positive correlation or with a correlation close to zero, the sunspot groups were not evenly distributed over the entire duration of the rotation; there were days with many groups and/or with a large area which disappeared at the western edge of the disk, after which there followed a period with small values of $\bar{R}$. In such rotations the solar irradiance did not follow rapidly the variation of the spotted area because the faculae areas remained behind the sunspot groups, influencing on their turn the irradiance variation. For this study, the period of 27 days, the duration of a solar rotation, does not bring a significant result.

From among the periods in which the total solar irradiance presented a strong decrease throughout several days, followed by a slow return to the previous values, ("deeps"), we have selected four intervals marked D1; D2; D3 and D4, two of which belong to the maximum of SC 21 and the other two, to the maximum of SC 22.

Table 4

The Correlation Coefficients between the Total Solar Irradiance and the Solar Activity Indices on Carrington Rotations

Rotation	$\bar{R}$	F	I_c	SFN	I_pP	I_pA
1809	-0.335	-0.116	-0.174	-0.348	-0.321	-0.300
1810	-0.632	-0.641	+0.302	-0.184	+0.174	-0.080
1811	-0.289	-0.545	+0.413	-0.332	-0.042	-0.528
1812	-0.170	-0.330	+0.211	-0.211	-0.169	-0.378
1813	-0.382	-0.411	-0.131	-0.166	+0.190	-0.496
1814	-0.332	-0.268	+0.616	-0.157	+0.048	-0.277
1815	-0.434	-0.003	+0.808	-0.380	-0.227	-0.373
1816	-0.657	-0.697	-0.251	-0.339	-0.090	-0.272
1817	-0.788	-0.737	+0.383	-0.473	-0.516	-0.493
1818	-0.160	+0.140	-0.013	+0.051	-0.174	+0.087
1819	-0.639	-0.736	+0.206	-0.310	-0.255	-0.259
1820	-0.001	+0.077	+0.764	+0.107	-0.016	-0.067
1821	-0.265	-0.532	+0.290	-0.523	+0.209	-0.398
1822	-0.600	-0.246	+0.207	+0.011	+0.087	-0.143
1823	-0.646	-0.715	+0.096	-0.412	-0.108	-0.462
1824	-0.160	-0.307	-0.137	+0.043	+0.217	+0.109
1825	-0.530	-0.448	-0.043	+0.263	-0.171	+0.167
1826	-0.006	+0.040	+0.318	-0.126	+0.042	-0.034
1827	-0.267	+0.094	+0.151	-0.015	-0.396	-0.082
1828	+0.447	+0.653	+0.633	+0.498	-0.062	+0.429
1829	-0.742	-0.646	-0.305	-0.380	-0.077	-0.437
1830	-0.857	-0.593	+0.248	-0.292	-0.420	-0.321
1831	-0.589	-0.320	+0.168	-0.264	-0.013	-0.140
1832	-0.744	-0.924	-0.555	-0.315	-0.291	-0.272
1833	-0.326	+0.423	+0.325	+0.092	-0.404	+0.088
1834	-0.382	-0.332	+0.475	-0.446	-0.075	-0.286
1835	+0.409	-0.192	+0.637	+0.102	-0.110	-0.078
1836	+0.647	+0.326	+0.466	+0.139	+0.058	-0.028

In Table 3 it can be observed that the general tendencies of the correlation coefficients for the maximum phases or the annual periods hold and become more prominent. For these "deeps" a statistical analysis through the method of the superposed epochs, which can also evaluate the degree of IR dependence on the indices analysed, is necessary.

4. CONCLUDING REMARKS

The results presented prove the existence of a real correlation between the total irradiance variation and the solar activity phenomena in the active regions (sunspots, flares, radio emission).

Although a study of this kind is necessary for each phase of the solar cycle, we can nevertheless state that:

- the increase of the flare activity in an active region, through the depositing and the consumption of magnetic energy in the coronal microprocesses which make up a solar flare, leads to the decrease of energy in other spectral fields producing irradiance “deeps”;

- although the energy released during the flares is smaller with at least a magnitude order than the energy absent in the irradiance “deeps” (Pap and Vrsnak, 1989), the research of such correlations can lead to the elucidation of the causes of the total solar irradiance variability;

- the periodic and quasiperiodic variations of *IR* should be directly compared with the manifestation of the magnetic structures on the surface and in the solar atmosphere;

- the causes of the *IR* variations are of crucial importance for the understanding of the way in which the solar interior determines the surface activity, and eventually for the understanding of the Sun’s evolution.

- apart from these causes the more precise understanding of the average and long term variations of *IR* is important for the understanding of the terrestrial climate variation.

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BLACK HOLES AS ELECTRIC CONDENSERS

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Abstract. A nonrotating (Schwarzschild) black hole of mass greater than 10^{17-18} g is being considered. It is shown that thermal and quantum effects lead to the electric charging of the black hole in a plasma containing particles with different masses. The notion of electric capacitance for a black hole is introduced.

Key words: astrophysics — black holes.

1. ELECTRIC CHARGING OF A BLACK HOLE IN A PLASMA

I shall consider a spherical shell of a plasma in the field of a Schwarzschild black hole. A plasma is scattered due to the thermal motion. The existence of bound orbits in the Schwarzschild gravitational field with $r > 3R_g \equiv 6GM/c^2$ (the notations, as throughout this paper, are the usual ones) leads to the availability of metastable states. Such orbits do exist (see e.g. Zeldovich and Novikov 1971) for values of the angular momentum $L > L_{ms} = R_g mc \sqrt{3}$. The corresponding lifetime is very large and is determined in general case by both quantum tunneling (Găină 1988, 1990; Găină and Zaslavskii 1992) and thermal fluctuations (Găină and Zaslavskii 1990).

In case $kT < (1 - 2\sqrt{2}/3)mc^2 \approx 0.057mc^2$, the flux probability of the particles on the orbits in the neighbourhood of the marginally stable orbit ($\Delta L \equiv \hbar \Delta l \equiv L - L_{ms} \ll L_{ms}$) into the black hole is

$$\gamma_{ih} \approx \frac{c^3}{12GM} \left(\frac{kT}{mc^2} \right)^{1/4} \frac{\exp[-6^{-3/2} \hbar c^3 \Delta l / (GMkT)]}{3^{1/4} 2^{1/8} \Gamma(3/4)}. \quad (1)$$

In the immediate neighbourhood of the marginally stable orbit ($\Delta L/L_{ms} < kT/(mc^2)$) the exponential in (1) comes close to unity, so the fall time of particles ($\tau = 1/\gamma$) becomes maximal (Găină and Zaslavskii 1990):

$$\tau_{ih} \approx 21.1 (GM/c^3) [mc^2/(kT)]^{1/4}. \quad (2)$$

This last formula was derived for an ideal gas, that is, it may be applied for plasma when the mean potential energy of interaction of the electrons of plasma ($\langle V_{pot} \rangle = Ze^2/\langle N \rangle^{1/3}$) is much smaller than kT , namely

$$\langle N \rangle \ll [kT/(Ze^2)]^3 \quad (3)$$

(condition of ideality of plasma). One may show that the tidal action of the black hole on a plasma at Debye scale length will be small enough when $\langle N \rangle \gg \gg kT/(4\pi e^2 R_g^2)$, that is, both conditions are fulfilled when $kT \gg e^2/(4\pi R_g)$.

The ionization time dependence on the rest mass of the particles of plasma has a very interesting and simple consequence. The absorption rate of light particles is larger than the corresponding one for heavy particles. It gives rise to electric charging of the black hole. The charging will continue until it is compensated by the electric attraction between the charges of the black hole (negative) and the plasma shell (positive) for $r \cong 3R_g$. The flux is dominated by the particles with height of the potential barrier $\Delta V_L \cong kT$, so one can obtain $Qe/(3R_g) \cong kT$, that is, the equilibrium charge of the black hole will be (in units of elementary charge e):

$$Q = 3(T/T_h)/(4\pi\alpha) = 5.4 \times 10^8 MT, \quad (4)$$

with M expressed in solar masses, T in Kelvin degrees, while $T_h = \hbar c^3/(8\pi GkM)$ is the temperature of the black hole.

The other process which may constitute a contribution to the electric charging of a black hole (in the nonrotating case, too) is the quantum tunneling of light particles (electrons) of plasma onto the black hole near the top of the potential barrier when $\Delta L \ll L_{ms}$. The corresponding probabilities were calculated by Găină (1988, 1990); see also Găină and Zaslavskii (1992):

$$\gamma_L = \frac{4}{\sqrt{\pi}} \left(\frac{4}{3}\right)^{3/16} \frac{c^3 (\Delta L)^{7/8}}{GM} \left(\frac{\hbar c}{GMm}\right)^{3/8} \exp \left[\frac{-72(\Delta L)^{5/4}}{5 \times 3^{1/8}} \left(\frac{\hbar c}{GmM}\right)^{1/4} \right] \quad (5)$$

The minimal lifetime of a particle closest to the centre of the Schwarzschild field orbit is

$$\tau_q = 2^{17/40} 3^{3/2} \sqrt{\pi} \left(\frac{e}{7}\right)^{7/10} \frac{GM}{c^3} \left(\frac{GmM}{\hbar c}\right)^{1/5} \cong 6.38 \frac{Gm}{c^3} \left(\frac{GmM}{\hbar c}\right)^{1/5} \quad (6)$$

Comparing (6) with (1), we get

$$\tau_q / \tau_{ih} \cong 0.302 \left(\frac{GmM}{\hbar c}\right)^{1/5} \left(\frac{kT}{mc^2}\right)^{1/4} \quad (7)$$

The quantum tunneling will be dominant when the condition $R_g \leq 793\lambda_c \times [mc^2/(kT)]^{5/4}$ is fulfilled, that is, for fully ionized hydrogen plasma at $kT \cong \alpha^2 mc^2/2$ one requires $R_g \leq 1.6 \times 10^8 \lambda_c \cong 0.006$ cm. It is easy to calculate the equilibrium charge of a black hole caused by quantum tunneling of electrons of plasma localized at $r \geq 3R_g$ onto the black hole (expressed in e units)

$$Q \cong (0.469/\alpha)[GmM/(hc)]^{2/5} = 64.2M^{2/5}, \quad (8)$$

with the last M expressed in units of 5.19×10^{17} g.

For a collapsed body of Earth's mass $M = 5.98 \times 10^{27}$ g the charge of plasma condenser will be $Q \cong 6.8 \times 10^5 e$ by quantum tunneling. For a black hole of the solar mass $M = 1.99 \times 10^{33}$ g the electric charge will be $8.5 \times 10^{15} e \cong 1.37 \times 10^{-3}$ C by thermal ionization of hydrogen plasma at $T \cong 1.6 \times 10^7$ K.

Notice that at limit $R_g \rightarrow \lambda_c$ the black hole charge is lower than the critical value $Q \cong 137e$, where the production of pairs may occur in the electric field of a black hole. But in case $R_g \lesssim 2 \times 10^3 \lambda_c$ our approximation fails because the electric field of the black hole becomes sufficiently strong to equal the gravitational force on a particle, that is, $eQ \cong GmM$. On the other hand, in this case, the strong gravitational field of a black hole makes decrease considerably the temperature of ionization of atoms by tidal disruption of the Coulombian force.

2. ELECTRIC CAPACITANCE OF A BLACK HOLE IN A PLASMA

Let us define the notion of electric capacitance for a system black hole + plasma. Since the total energy of the system is given by $E = Q^2/(3R_g)$, one gets

$$C = Q^2/(2E) = 3R_g/2 = 0.490M, \quad (9)$$

M being expressed in solar masses, and C in mcF.

The voltage between the spherical plates of a condenser is

$$U = Q/C = (2E/C)^{1/2} = 2Q/(3R_g). \quad (10)$$

In particular, $U = 1.725 \times 10^{-4} T$ for thermal ionization of plasma and $U = 8 \times 10^5 M^{-3/5}$ for quantum tunneling of electrons onto the black hole (U expressed in Volts, T in Kelvin degrees, M in units of 5.19×10^{17} g). The voltage can be of appreciable value in this case.

It is easy to estimate the nonlinear corrections to the electric capacitance (9) of a black hole in a plasma for $R_g \lesssim 10^3 \lambda_c$:

$$C = (3R_g/2)[1 + (eQ/(24GmM))^2]. \quad (11)$$

3. DISCUSSION AND CONCLUSION

The question which may arise is: may really existing black holes carry electric charges? The answer is yes for black holes of stellar origin. The condition (2) for the ideality of plasma is observed whenever black holes appear as a result of stellar matter collapse (e.g. Zeldovich and Novikov 1971). The conditions in a plasma surrounding the collapsar approach these ones, so the thermal condenser is possible in this case.

For smaller (primordial) black holes, which were formed from ultrarelativistic, ultrahot matter in the early Universe (e.g. Hawking 1971; Zeldovich and Novikov 1971; Carr and Hawking 1974) and survived at actual epoch, quantum tunneling of particles may lead to the effect of electric condenser, even if they are not surrounded by hot plasma. In both cases the collapsar rotation might be taken into account for an accurate estimation of the electric capacitance and charge of the black hole.

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AN INVERSE PROBLEM FOR ISOLATED ORBITS

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Abstract. We examine the question of finding all potentials $V(x, y)$ which can produce a specific *isolated* planar orbit $f(x, y) = 0$, given in advance and traced by a material point for adequate initial conditions. We study in some detail an example to clarify the difference of this version of the problem from the usual version dealing with a *monoparametric family* of orbits $f(x, y) = \text{constant}$.

Key words: celestial mechanics — inverse problem of dynamics.

1. INTRODUCTION

We shall refer to the version of the inverse problem of dynamics which seeks the force fields or, most frequently, the potentials $V(x, y)$ which can give rise, for adequate initial conditions, to certain orbits given in advance and traced in the Cartesian xy -plane by a material point of units mass. Regarding the multitude of the given orbits we shall refer either to a *monoparametric family of curves* described by the equation

$$f(x, y) = c \quad (1)$$

or to a *single curve*

$$F(x, y) = 0. \quad (2)$$

Whittaker (1944), for instance, presents Dainelli's work for the family (1) and offers the force components generating all orbits (1). He also states clearly that "on varying the constant c , this equation will represent a family of curves". But Dainelli himself in his 1880 report refers to the single orbit (2).

Working in an inertial frame, Szebehely (1974) derived his partial differential equations in $V(x, y)$ for a given family of curves (1) but his derivation holds for *isoenergetic* families, i.e. having $E = \text{constant}$ for all members of (1). Few years later, Morrison (1977) generalized this equation to account for energy dependence

$$E = E(c) \quad (3)$$

given in advance along the family.

Besides, from the old days up to now, one can locate in the literature references to the inverse problem for *one single orbit* (2), given in advance. We quote, for instance, Suslov (1890), Rajaraman (1979), P. du T. der Merwe (1991), Antonov and Timoshkova (1993), who formulate the inverse problem with equation (2). In the framework of the restricted three-body problem Drămbă also used equation (2) in a rotating frame. We mention also Kasner's (1909) work on the corresponding *direct problem* and the third order ordinary differential equation in $y(x)$ for "the definite trajectory" $y = y(x)$ with coefficients depending on the components.

So, in some cases, the presence of the varying parameter c in the right hand side of equation (1) was simply ignored or taken as equivalent to its absence. But, at certain instances (e.g. Szebehely et al. (1980)) the role of c was misinterpreted and, in the pertinent formulae, instead of equation (1), the equation $F(x, y, c) = 0$ was used. As a result, one ended up with potentials depending on c , which, of course, cannot be.

From the viewpoint of an astronomer, having *one* specific observed orbit (perhaps, *some* orbits) is the usual situation, whereas it is practically impossible to possess a *continuous* set of such observed orbits.

The aim of this paper is to clarify the role of the parameter c and comment on the essential difference between the two inverse problems dealing with *one isolated orbit* (2) and a *family of orbits* (1). It will become clear that the solution of the *one-orbit problem* is conveyed to the known framework where the pertinent partial differential equations hold true.

2. THE TWO BASIC EQUATIONS

All potentials $V(x, y)$ which can generate the monparametric family (1) traced with a preassigned energy dependence (3) are given by Szebehely's first order partial differential equation

$$V_x + \gamma V_y + \frac{2\Gamma}{1 + \gamma^2}(E - V) = 0 \quad (4)$$

where

$$\gamma = \frac{f_y}{f_x}, \quad \Gamma = \gamma \gamma_x - \gamma_y. \quad (5)$$

It is easily seen from the above equations that

(i) to each function $f(x, y)$ there corresponds *one* function $\gamma(x, y)$ and, vice versa, to each $\gamma(x, y)$ there corresponds *one* family (1). So, assigning the function $\gamma(x, y)$ is equivalent to specifying the family (1) and, since, E being given, the function $f(x, y)$ no longer appears in equation (4), we can refer to the "family of orbits $\gamma(x, y)$ ".

(ii) in general, as many potentials as an arbitrary function of a certain argument would allow can produce, for adequate initial conditions, the given family $\gamma(x, y)$. Each of these potentials, of course, produces, for random initial conditions, other orbits not included in (1).

On the other hand, not all the given geometrical information (1) becomes mechanical reality too. For actual motion to take place, of course, one needs to make sure that $E \geq V(x, y)$ as the massive point is moving and this requirement leads to the inequality (Bozis and Ichtiaroglou, 1994)

$$\frac{V_x + \gamma V_y}{\Gamma} \leq 0. \quad (6)$$

The so-called *family boundary curves* define *regions of the xy-plane* where motion along the various members of (1) is allowed. Only these curves (1) or those parts of the curves (1) which lie inside the allowed region (6) are then admitted as *actual orbits*. Since the (constant along *each* orbit) energy E varies from orbit to orbit, the above curves do not, generally, coincide with the well-known *zero velocity curves* which refer to orbits of the same energy.

The second equation, relating only potentials and families of orbits (1), is (Bozis, 1984)

$$-V_{xx} + kV_{xy} + V_{yy} = \lambda V_x + \mu V_y \quad (7)$$

with

$$k = \frac{1 - \gamma^2}{\gamma}, \quad \lambda = \frac{\Gamma_y - \gamma \Gamma_x}{\gamma \Gamma}, \quad \mu = \lambda \gamma + \frac{3\Gamma}{\gamma}. \quad (8)$$

Equation (7) is linear of the second order and its general solution introduces *two* arbitrary functions. This is in agreement with the multitude of solutions of equation (4) which is solved, each time, for an arbitrary selection of the function $E = E(c)$.

Usually we make equations (7) and (4) cooperate as follows: if a pair of orbits $\gamma(x, y)$ and potentials $V(x, y)$ is compatible (i.e. if it satisfies equation (7)), the corresponding total energy E can be found from equation (4). The energy must be of the form (3) and this constitutes a criterion of the correctness of the calculations involved.

3. POTENTIALS GENERATING ISOLATED TRAJECTORIES

Let us now face an inverse problem on the basis of *one isolated trajectory* (2) given in advance. We assume that the equation $F(x, y) = 0$ stands for *one* smooth (closed or not) geometrical curve and we put the question: Which potentials are compatible with this specific trajectory?

At this stage we do not care as to whether actual motion takes place on the entire curve (2) or is a libration or an asymptotic motion on a certain arc of (2).

First of all, let us make clear that our equations (4) and (7) hold true for families of orbits which are *at least* monparametric, expressed by equation (1).

Comment: For autonomous systems, the given family (1), may be *at most* three-parametric. Two and three-parametric families $F(x, y, b) = c$ and $F(x, y, a, b) = c$ were studied by Bozis (1983) and by Xanthopoulos and Bozis (1983) respectively. It was shown that, in general, no potential exists which can give rise to such families given in advance arbitrarily. If, however, the given functions $F(x, y, b)$ or $F(x, y, a, b)$ satisfy certain conditions, then there exists a potential which can be determined "almost" uniquely.

So or otherwise the solutions of equations (4) and (7) are defined and the equations themselves are meaningful in open domains of the xy -plane and by no means on certain curves (2) of the plane. Clearly, all members of (1), for values of c taken from an open interval of R , lie inside such a domain.

So then, to answer the above question we are forced to classify the given trajectory (2) to a certain family (1), compute from (5) the corresponding function $\gamma(x, y)$ and solve for $V(x, y)$ equation (7). Every solution of (7) will also generate the given trajectory (2) which belongs to the family.

In classifying (2) one first feels tempted to consider that the given curve $F(x, y) = 0$ is simply the member of the family $F(x, y) = c$ for $c = 0$. The problem then is transferred to the problem of solving equation (7) with $\gamma = F_y/F_x$. This is of course correct and, in fact, people mentioned in the Introduction treat $F(x, y) = 0$ as if it were the same with $F(x, y) = c$. Yet it is only one way, out of many possible ways, of answering our question because the single curve $F(x, y) = 0$ can be classified as a member of many other monparametric families (1), having different functions $\gamma(x, y)$ and, consequently, leading to different differential equations (7). These equations are not expected to have the same solutions, although some solutions may be common. Therefore, new admissible potentials (in the sense that they create the isolated orbit with equation $F(x, y) = 0$) will be added, due to the new selection of the monparametric family to which $F(x, y) = 0$ belongs.

To clarify and support our argument we propose to work out in some detail a simple, yet typical example.

4. EXAMPLE

The isolated curve

$$x^2 + y^2 = 1 \quad (9)$$

belongs to the monoparametric family of concentric circles

$$x^2 + y^2 = c_1 \quad (9a)$$

for $c_1 = 1$. However, the same curve (9) may be taken, *e.g.* as a member of the monoparametric family of conic sections

$$\frac{1 - x^2}{y^2} = c_2 \quad (9b)$$

for $c_2 = 1$, not to mention other possible classifications as, for instance, the family $\frac{-y^2 + \sqrt{y^4 + 4x^2}}{x^2} = c_3$ for $c_3 = 2$, etc.

In what follows we shall deal with the two families (9a, b). The functions $\gamma(x, y)$ are, respectively: $\gamma_1 = y/x$, and $\gamma_2 = (1 - x^2)/xy$. Calculating the coefficients k, λ, μ , from equations (8) and inserting into equation (7) we obtain, respectively,

$$-V_{xx} + \frac{x^2 - y^2}{xy} V_{xy} + V_{yy} - \frac{3}{x} V_x + \frac{3}{y} V_y = 0 \quad (10a)$$

and

$$-V_{xx} + \frac{(x^2 - 1)^2 - x^2 y^2}{xy(x^2 - 1)} V_{xy} + V_{yy} - \frac{3}{x} V_x + \frac{3}{y} V_y = 0. \quad (10b)$$

The general solution of equation (10a) is known from Broucke and Lass (1977) and Molnár (1981). In polar coordinates, r, θ it reads

$$V(r, \theta) = g(r) + \frac{1}{r^2} h(\theta) \quad (11)$$

where g and h are arbitrary functions of their respective arguments.

The second equation (10b) is, obviously, different from (10a). Actually, it so happens that $k_2 \neq k_1$, whereas $\lambda_2 = \lambda_1, \mu_2 = \mu_1$. One then understands that all *sepa-*

table (in the coordinates x, y) solutions $V(x, y)$ of equation (10a), i.e. all solutions with $V_{xy} = 0$, are also solutions of (10b). Being easily detected, these common solutions are, apart from an additive constant,

$$V_{sep} = \frac{a_1}{x^2} + \frac{a_2}{y^2} + a_3(x^2 + y^2) \quad (12)$$

where a_1, a_2, a_3 are constants.

The set of potentials (12) is a subset of (11) corresponding to $g(r) = a_3 r^2$ and $h(\theta) = \frac{a_1}{\cos^2 \theta} + \frac{a_2}{\sin^2 \theta}$. From equation (4) it can be found that the circles (9a) are traced by the potential (12) with energy

$$E_1 = 2a_3 c_1 \quad (13a)$$

whereas the conics (9b) are traced by the same potential (12) with

$$E_2 = a_3 + a_1 + \frac{a_3 - a_1}{c_2} + a_2 c_2 (1 - c_2). \quad (13b)$$

Notice that the unit circle (9) is traced with $E = 2a_3$.

We did not manage to find the general solution of the second order equation (10b). However, we did find a set of solutions of (10b) whose multiplicity is introduced through one arbitrary function (not two). What we did was to solve Szebehely's first order equation (4) for the family (9b) under the additional assumption that all members of this family are traced with energy $E = 0$. For this case equation (4) reads

$$V_x + \frac{1 - x^2}{xy} V_y = \frac{2(x^2 - 1)}{x[x^2 y^2 + (1 - x^2)^2]} V. \quad (14)$$

Forming and solving the corresponding system of subsidiary equations we found the general solution of (14)

$$V_{isoen} = \left[2 - \left(x^2 + y^2 + \frac{1}{x^2} \right) \right] H[2 \ln|x| - (x^2 + y^2)] \quad (15)$$

where H is an arbitrary function of its argument.

As expected, all potentials (15) are also solutions of equation (7), as applied for the family (9b). In general, these solutions are not separable, i.e. $V_{xy} \neq 0$. Each

of these potentials can create (9b) as a family of *isoenergetic orbits*, with $E = 0$. On the other hand, we have shown that each potential (12) creates the family (9b) with energy E , given by (13b). Then, due to the linearity in $V(x, y)$ of the partial differential equation (7), the two solutions can be added to obtain a "richer" solution

$$V = V_{sep} + V_{isoen} \quad (16)$$

producing orbits (9b) with total energy $E = 0 + E_2 = E_2$, given by (13b).

In general, a potential (16) does not create the entire family (9a) and, also, a potential (11) does not create the entire family (9b). However, all potentials (16) and (11) can produce the single orbit (9) which belongs to both families (9a) and (9b).

Needless to say that, in addition to the above, one could find other potentials giving rise to the specific orbit (9). To this end one should classify the orbit (9) to other monparametric families and attempt to solve the corresponding equation (7). New solutions would then appear.

Comment: The expression (16) does not even include all potentials producing (9) considered as a member of the family (9b). Other solutions would spring if, in solving equation (4) as we did, instead of $E = 0$, we had selected another energy dependence $E_2 = E_2(c_2)$, not given by (13b) for a certain triplet a_1, a_2, a_3 . But no one could guarantee that the corresponding equation (4) would actually be solvable, as it happened in the zero energy case.

5. ARCS OF ALLOWED MOTION

Our main concern in sections 3 and 4 has been to find all potentials which are compatible with the isolated trajectory (9). To this end, having classified (9) in various families (9a), (9b) etc., we tried to solve the corresponding differential equations (7).

But does, actually, the entire curve (9) stand for an *orbit* of a massive point? And is the motion a rotation on the entire circle $x^2 + y^2 = 1$ or a libration or an asymptotic motion on a certain arc of it?

The answer to these questions, coming from inequality (6), goes as follows: It all depends on the potential and the family to which (9) has been classified. Only this part of the curve $x^2 + y^2 = 1$ (if any) which happens to lie inside allowed region defined by (6) will be admitted as a *real orbit*.

Take, for instance, the potential

$$V = \left[2 - \left(x^2 + y^2 + \frac{1}{x^2} \right) \right] [2 - (x^2 + y^2) 2 \ln|x|] \quad (17)$$

of the form (15). For $c_2 > 0$, among the ellipses (9b), all traced with zero energy, the circle (9) is included for $c_2 = 1$. Since $2 - (x^2 + y^2 + 1/x^2) \leq 0$ everywhere in the plane, the requirement $0 \geq V(x, y)$ or, equivalently, because all motions are isopergetic, inequality (6) leads to

$$x^2 + y^2 - 2 \ln |x| \leq 2. \quad (18)$$

The two (symmetrical with respect to both axes Ox, Oy) shaded domains in Fig. 1 are defined by (18) and represent the allowed regions of motion for members of (9b) traced by (17). On the other hand, as can be checked easily, the points $A_0(1, 0)$ and $A'_0(-1, 0)$ of Fig. 1 (which belong to all conics $x^2 + c_2 y^2 = 1$) are *unstable equilibrium points of the potential* (17). Since, except for $V_x = V_y = 0$ at A_0 (and A'_0), it is also $V(A_0) = 0$ (and $V(A'_0) = 0$), one understands that, as regards e.g. the unit circle (9), real motion on it with zero energy will be *asymptotic*, tending to A_0 (or A'_0) from $y > 0$ or from $y < 0$, depending on the initial position.

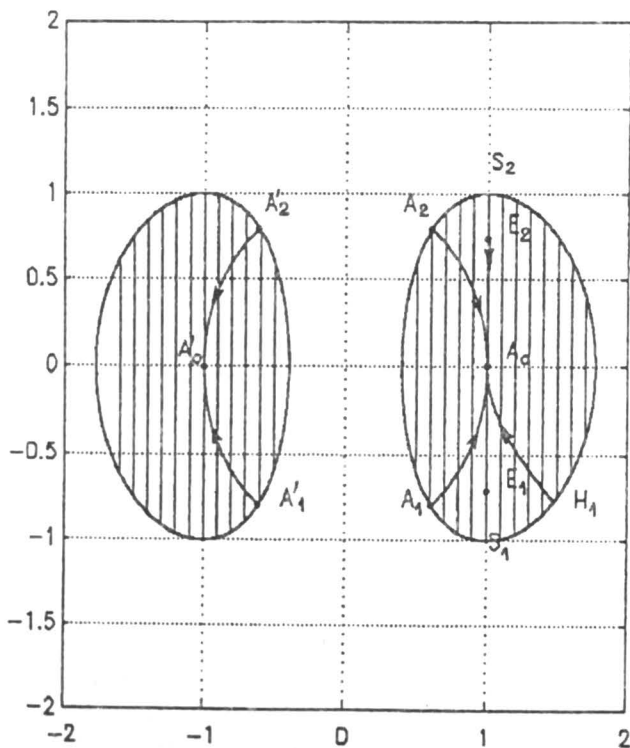


Fig. 1. — Conic sections (9b), generated by the potential (17) represent real orbits (arcs of asymptotic motion) only inside the shaded regions. In particular the asymptotic motion on the arc A_2A_0 of the unit circle is shown by a heavy line. The line H_1A_0 with $H_1(1.47951, -0.77102)$ corresponds to asymptotic motion on the hyperbola (9b) with $c_2 = -2$.

To verify this result we integrated numerically the system

$$\ddot{x} = -V_x, \quad \ddot{y} = -V_y \quad (19)$$

with V given by (17) and with initial conditions

$$x_0 = 0.6065306597, \quad y_0 = 0.7950600976, \quad \dot{x}_0 = 0, \quad \dot{y}_0 = 0.$$

Starting with zero velocity at the point A_2 of Fig. 1, the unit mass moved indeed very accurately on the circular arc A_2A_0 tending “asymptotically” to A_0 . In fact it reached the closed vicinity of A_0 in finite time with velocity very close to zero, it stayed there for a time longer than it took it to trace the entire arc A_2A_0 and then, due to the instability of the equilibrium point A_0 , it went away from A_0 . The energy remained constant but the unit mass no longer travelled along the unit circular arc. It remained, however, inside the right shaded region of Fig. 1 because the total energy is zero and inequality (18) gives not only the family boundary curve (6) but the zero velocity curve $0 \geq V(x, y)$, as well.

Since the point $A_0(1, 0)$ belongs to all conics (9b), the motion is asymptotic not only on the arc A_2A_0 but on all arcs (9b), terminating in A_0 , as e.g. on the arc H_1A_0 of the hyperbola $x^2 - 2y^2 = 1$, as well as on the straight line segment S_2A_0 which corresponds to $c_2 > 0$ and which separates the elliptical ($c_2 > 0$) from the hyperbolic ($c_2 < 0$) arcs of the family (9b).

The rectilinear motions on S_2A_0 can be found analytically: Indeed, for $x = 1$, the first equation (19), applied for the potential (17), becomes an identity whereas the second equation (19) gives

$$\ddot{y} = 2y(1 - 2y^2). \quad (20)$$

For the zero energy asymptotic motion on S_2A_0 , starting with zero velocity at the point $S_2(1, 1)$, the solution of (20) is

$$y = \frac{2e^{-\sqrt{2}t}}{1 + e^{-2\sqrt{2}t}}, \quad (21)$$

giving, for $t \rightarrow \infty$, $y \rightarrow 0$.

The motion on S_1A_0 (Fig. 1) is symmetrical to (21) and, of course, analogous results can be found for $x < 0$.

The question now arises: are there potentials creating *librational* motion on the arc A_1A_2 of Fig. 1? The answer is in the affirmative. Actually, there are several such potentials. As an example we give the potential (Bozis and Ichtiaroglou, 1994)

$$V(r, \theta) = \frac{1}{r} - \frac{a}{2r^2}(1 + \cos \theta) \quad (22)$$

of the form (11) creating arcs of circles (9a) *inside* the cardioid

$$r = a(1 + \cos \theta). \quad (23)$$

We only need adjust the value of $a = (1 + e^{-1/2})^{-1}$ so that (22) intersects the boundary of (18) at the same points $A_1(e^{-1/2}, -(1 - e^{-1})^{1/2})$ and $A_2(e^{-1/2}, (1 - e^{-1})^{1/2})$.

Comments: 1. The points $E_1(1, -\sqrt{2}/2)$, $E_2(1, \sqrt{2}/2)$ are also unstable equilibrium points of the *two-dimensional potential* (17) but they correspond to total energy $-1/4$. So, they play no role in zero energy motions created by (17). Actually E_2 is reached by the mass moving according to (21) in time

$$t = \frac{\ln(\sqrt{2} + 1)}{\sqrt{2}} \text{ with velocity } v = -\frac{\sqrt{2}}{2}.$$

2. A detailed consideration reveals that as regards *rectilinear motions* on the line $x = 1$, the point A_0 is unstable, whereas the points E_1 , E_2 are stable.

3. The energy of the asymptotic motions on A_1A_0 or A_2A_0 due to the potential (17) is zero, whereas the energy of the librational motion due to (22) is $1/2$.

4. The two potentials (17) and (22) have in common the arc A_1A_2 but not A_1A_2' on which real motion is not allowed by (22).

5. Potentials can be found allowing for rotational motion on the entire circle $x^2 + y^2 = 1$. Such is, e.g., the Newtonian potential $V = -1/r$.

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FREE-FALL COLLAPSE OF A HOMOGENEOUS SPHERE IN MANEFF'S GRAVITATIONAL FIELD

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Abstract. The free-fall collapse of a homogeneous spherical cloud in Maneff's gravitational field is analysed. The exact analytic solution of the collapse is obtained. The free-fall time is determined and compared with Newtonian free-fall time. For an interstellar cloud the effect of the supplementary term in Maneff's potential is negligible, but this is important for the collapse of a compact object. In Maneff's field the collapse is accelerated.

Key words: astrophysics, stellar evolution, gravitational collapse, Maneff's field.

1. INTRODUCTION

The importance of the gravitational collapse in the study of the stellar evolution is well known. This process was analysed in the Newtonian theory of gravitation, as well as in the relativistic (Einsteinian) gravitation (Kippenhahn and Weigert 1991).

In 1924 G. Maneff considered a post-Newtonian nonrelativistic law of gravitation (Maneff 1924), assuming that the gravitational interaction between the masses m_1 and m_2 is given by the "force function"

$$U = \frac{Gm_1m_2}{r} \left[1 + \frac{3G(m_1 + m_2)}{2c^2r} \right] \quad (1)$$

where r is the distance between m_1 and m_2 , G is the Newtonian gravitational constant, and c is the speed of light.

Recently (Mioc and Stoica 1995; Diacu et al. 1995) the two-body problem in the Maneff gravitational field featured by (1) was studied. The complete solution of the problem was given.

In this paper the problem of the free-fall collapse of a homogeneous sphere in Maneff's gravitational field is being analysed.

2. FREE-FALL COLLAPSE OF A HOMOGENEOUS SPHERE

We shall consider a homogeneous spherical cosmic cloud. Neglecting the effect of pressure, the cloud will collapse in free-fall.

Let M be the mass of the cloud, and let $m \ll M$ be the mass of a particle which falls in the gravitational field of M . To study the motion of m , the following potential will be attached to U from (1) ($m_1 = M$, $m_2 = m$)

$$\Phi(r) = -\frac{GM}{r} - \frac{3G^2 M^2}{2c^2 r^2} \quad (2)$$

The motion of the particle is described by the equation

$$\ddot{r} = -d\Phi(r)/dr, \quad (3)$$

where dots indicate time-derivatives. From (2) and (3) it results the following equation of motion for a particle situated at the surface of the cloud

$$\ddot{r} = -\frac{GM}{r^2} - \frac{3G^2 M^2}{c^2 r^3}. \quad (4)$$

Supposing that, during the collapse, the mass of the cloud is conserved, the density $\rho(t)$ of the cloud (which is constant in the cloud at a given instant) is connected to the initial density ρ_0 by

$$\rho(t) = \rho_0 r_0^3 / r^3, \quad (5)$$

where r_0 is the initial value of r (at the beginning of the collapse).

Taking into account (5) and $M = (4\pi/3)r^3\rho$, the equation of motion (4) becomes

$$\ddot{r} = -\frac{4\pi G \rho_0 r_0^3}{3r^2} - \frac{16\pi^2 G^2 \rho_0^2 r_0^6}{3c^2 r^3}. \quad (6)$$

The integral of energy for (6) gives

$$\dot{r}^2 = \frac{8\pi G \rho_0 r_0^3}{3r} + \frac{16\pi^2 G^2 \rho_0^2 r_0^6}{3c^2 r^2} + C \quad (7)$$

where the integration constant will be determined from the conditions at the beginning of the collapse. At the moment $t = 0$ we have

$$r(0) = r_0, \quad \dot{r}(0) = 0. \quad (8)$$

With the initial conditions (8) we have

$$C = -\frac{8\pi G \rho_0 r_0^2}{3} - \frac{16\pi^2 G^2 \rho_0^2 r_0^4}{3c^2} \quad (9)$$

which, replaced in (7), yields

$$\dot{r}^2 = \frac{8\pi G\rho_0 r_0^2}{3} \left(\frac{r_0}{r} - 1 \right) \left[\frac{2\pi G\rho_0 r_0^2}{c^2} \left(\frac{r_0}{r} + 1 \right) + 1 \right] \quad (10)$$

Here we shall introduce the Schwarzschild gravitational radius of the cloud $r_g = 2GM/c^2 = 8\pi G\rho_0 r_0^3/3c^2$ and the parameter, $a = 3r_g/4r_0$, which is a measure of the strength of the field. Then, from (10) it results

$$\frac{\dot{r}}{r_0} = -\sqrt{\frac{8G\pi\rho_0}{3}} \sqrt{\frac{r_0-r}{r}} \sqrt{\frac{a(r_0+r)+r}{r}} \quad (11)$$

where during the collapse $r < r_0$ and we took the sign $(-)$ in (11) because $r(t)$ is a decreasing function.

With the change of variable

$$r = r_0 \frac{u^2 - a}{u^2 + 1 + a}, \quad (12)$$

and taking into account the initial condition $u(0) = \infty$ (corresponding to $r(0) = r_0$), equation (11) can be integrated exactly, obtaining

$$\frac{1}{1+a} \left[\frac{(1+2a)u}{u^2+1+a} + \frac{1}{\sqrt{1+a}} \left(\frac{\pi}{2} - \arctan \frac{u}{\sqrt{1+a}} \right) \right] = \left(\sqrt{\frac{8\pi G\rho_0}{3}} \right) t. \quad (13)$$

Equations (12), (13) provide the solution of the collapse.

In Maneff's gravitational field the solution depends on r_0 . This means that the collapse is *not a (strictly) homologous contraction* as in the Newtonian field. But, generally, a is small in (13), that is, the dependence on r_0 is weak. Therefore, the collapse described by (13) can be considered as a *quasihomologous contraction*.

For $a = 0$, with the change of variable $\zeta = \pi/2 - \arctan u$, we obtain from (13) the Newtonian solution of collapse (Kippenhahn and Weigert 1991)

$$\zeta + \frac{1}{2} \sin(2\zeta) = \sqrt{\frac{8\pi G\rho_0}{3}} t. \quad (14)$$

3. FREE-FALL TIME

The collapse will continue until the particle will reach the center, that is $r = 0$ or $u = \sqrt{a}$. Then from (13) the free-fall time t_{ff} is

$$t_{ff} = \sqrt{\frac{3}{8\pi G\rho_0}} \frac{1}{1+a} \left[\sqrt{a} + \frac{1}{\sqrt{1+a}} \left(\frac{\pi}{2} - \arctan \sqrt{\frac{a}{1+a}} \right) \right] \quad (15)$$

For $a = 0$ we obtain the Newtonian expression of the free-fall time t_{ff}^{Newt} , that is (Kippenhahn and Weigert 1991)

$$t_{ff}^{Newt} = \sqrt{\frac{3\pi}{32G\rho_0}} \quad (16)$$

The last expression is comparable with the free-fall time for a star

$$t_{ff}^{star} = \sqrt{\frac{R}{g}} = \sqrt{\frac{3}{4\pi G\rho_0}}, \quad (17)$$

where R is the radius of the star, ρ_0 is its mean density, and g is the (Newtonian) gravitational acceleration at the surface of the star.

In the case of a small a , (15) allows the following asymptotic expression

$$t_{ff} = t_{ff}^{Newt} \left(1 - \frac{3a}{2} \right). \quad (18)$$

For an interstellar cloud of about solar mass, with $\rho_0 \cong 4 \times 10^{-20} \text{ kg/m}^3$, $r_0 \cong 2$ l. y. the difference $t_{ff}^{Newt} - t_{ff}$ is negligible. But for a neutron star in a close binary system, for which the accretion can be the cause of the collapse, the above difference might represent some tens of percents from t_{ff}^{Newt} .

In conclusion, the supplementary term in the potential of the Maneff gravitational field will accelerate the collapse.

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ON THE INVERSE PROBLEM OF DYNAMICS FOR GENERALIZED LAGRANGIAN SYSTEMS

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Abstract. A system of partial differential equations for the force function U (hence for the potential function $-U$) is given when one knows a family of orbits for the system

$$\frac{d}{dt} L_{\dot{q}} = L_q$$

where $L : D' \times D \subseteq \mathbb{R}^{2n} \rightarrow \mathbb{R}$,

$$L(\dot{q}, q) = \left(\sum_{k=1}^n \sum_{j=1}^n g_{jk}(q) \dot{q}_j \dot{q}_k \right) / 2 + \sum_{j=1}^n f_j(q) \dot{q}_j + U(q);$$

this is a generalization not only of the results of Szebehely for 2-dimensional conservative systems but also of those of Drămbă for the planar circular restricted three-body problem.

Key words: generalized Lagrangian systems, potential function, inverse problem.

The inverse problem of Dynamics requires the determination of the potential (or force) function when an orbit or a family of orbits is known. In a natural way, the topic was considered for 2-dimensional conservative systems with $V : D \subseteq \mathbb{R}^2 \rightarrow \mathbb{R}$

$$\begin{aligned} \ddot{x} &= V_x \\ \ddot{y} &= V_y \end{aligned} \tag{1}$$

by Szebehely (1974). A similar problem was studied by Drămbă (1963) for the planar circular restricted three-body problem with $\Omega : D \subseteq \mathbb{R}^2 \rightarrow \mathbb{R}$

$$\begin{aligned} \ddot{x} - 2\dot{y} &= \Omega_x \\ \ddot{y} + 2\dot{x} &= \Omega_y. \end{aligned} \tag{2}$$

Recently, the result of Szebehely was extended by Stavinschi and Mioc (1993) for n -dimensional conservative systems with $W : D \subseteq \mathbb{R}^n \rightarrow \mathbb{R}$,

$$\ddot{x}_i = W_{x_i}, \quad i = 1, \dots, n. \tag{3}$$

The roots of the problem are to be found before the beginning of our century, in the paper of Dainelli (1880), described by Whittaker (1904) in his well-known

book *Analytical Dynamics*, for 2-dimensional systems of the type

$$\begin{aligned}\ddot{x} &= F \\ \ddot{y} &= G,\end{aligned}\tag{4}$$

where $F, G : D \subseteq \mathbb{R}^2 \rightarrow \mathbb{R}$.

The result of Szebehely (1974) follows from that of Stavinschi and Mioc (1993) for the particular value $n = 2$. The force function in these cases is a function of x, y , respectively of $x_i, i = 1, \dots, n$, so it does not depend on velocities; it means that the system (2) which contains the terms $-2\dot{y}$ and $2\dot{x}$ due to the Coriolis forces is of another kind, so Drâmbă's result cannot be obtained as a special case of (1) or (3).

In the following we consider a generalized Lagrangian system

$$\frac{d}{dt} L_{\dot{q}} = L_q,\tag{5}$$

where $L : D' \times D \subseteq \mathbb{R}^{2n} \rightarrow \mathbb{R}$, $q = \begin{pmatrix} q_1 \\ \vdots \\ q_n \end{pmatrix} \in D$, $\dot{q} = \begin{pmatrix} \dot{q}_1 \\ \vdots \\ \dot{q}_n \end{pmatrix} \in D'$ is given by

$$L(\dot{q}, q) = \langle \dot{q}, G(q)\dot{q} \rangle / 2 + \langle \dot{q}, f(q) \rangle + Uq.\tag{6}$$

By $\langle \cdot, \cdot \rangle$ we denote the inner product in $\mathbb{R}^n$, $f = \begin{pmatrix} f_1 \\ \vdots \\ f_n \end{pmatrix} : D \rightarrow \mathbb{R}^n$ and $G = (g_{ij})_1^n : D \rightarrow \mathcal{M}_n^{\text{sym}}(\mathbb{R})$, $U, f_i, g_{ij} \in C^1(D)$, $i = 1, \dots, n, j = 1, \dots, n$.
The first term in the expression of L is a "kinetic energy"

$$T(\dot{q}, q) = \langle \dot{q}, G(q)\dot{q} \rangle / 2,$$

while $V(\dot{q}, q) = \langle \dot{q}, f(q) \rangle + U(q)$ represents a "generalized force function", the first part of which contains the effect of the Coriolis forces.

LEMMA 1. *The system (5) admits an energy integral*

$$T(\dot{q}, q) - U(q) = h,\tag{7}$$

i.e. $\langle \dot{q}, G(q)\dot{q} \rangle / 2 - U(q) = h$.

Proof. Multiplying the i^{th} equation in (5) by $\dot{q}_i$ and adding we have

$$\sum_{i=1}^n \left(\frac{d}{dt} L_{\dot{q}_i} \right) \dot{q}_i = \sum_{i=1}^n L_{q_i} \dot{q}_i.$$

Hence

$$\begin{aligned} \frac{d}{dt} \left(-L + \sum_{i=1}^n L_{\dot{q}_i} \dot{q}_i \right) &= -\sum_{i=1}^n L_{q_i} \dot{q}_i - \sum_{i=1}^n L_{\dot{q}_i} \ddot{q}_i + \frac{d}{dt} \sum_{i=1}^n L_{\dot{q}_i} \dot{q}_i = \\ &= -\sum_{i=1}^n \left[\left(\frac{d}{dt} L_{\dot{q}_i} \right) \dot{q}_i + L_{\dot{q}_i} \ddot{q}_i \right] + \frac{d}{dt} \sum_{i=1}^n L_{\dot{q}_i} \dot{q}_i = 0 \end{aligned}$$

and $-L + \sum_{i=1}^n L_{\dot{q}_i} \dot{q}_i = h.$

But

$$L_{\dot{q}_i} = \sum_{k=1}^n g_{ik}(\mathbf{q}) \dot{q}_k + f_i, \quad i = 1, \dots, n, \text{ so } \sum_{i=1}^n L_{\dot{q}_i} \dot{q}_i = 2T + \langle \mathbf{f}, \dot{\mathbf{q}} \rangle$$

and the above relation gives $T(\dot{\mathbf{q}}, \mathbf{q}) - U(\mathbf{q}) = h.$ ■

The system (5) has an equivalent explicit form. In order to obtain it, we define, following Wintner (1941), p. 118, the functions

$$P_{ik}, \Gamma_{ijk}, \Gamma_{jk}^i, P_k^i: D \rightarrow \mathbb{R}$$

of the position $\mathbf{q}$ in the configuration space, in terms of the coefficients g_{ik}, f_i in (6) and of their first order partial derivatives:

$$P_{ik} = f_{iq_k} - f_{kq_i}$$

$$\Gamma_{ijk} = (g_{ikq_j} + g_{jkq_i} - g_{ijq_k})/2$$

$$\Gamma_{jk}^i = \sum_{l=1}^n g^{il} \Gamma_{jkl}$$

$$P_k^i = \sum_{l=1}^n g^{il} P_{lk},$$

where $(g^{il})_1^n = \mathbf{G}^{-1}$ is the inverse of the matrix $\mathbf{G}$.

Let us denote

$$\Gamma^i = (\Gamma_{jk}^i)_1^n, \quad \mathbf{P}^i = \begin{pmatrix} P_1^i \\ \vdots \\ P_n^i \end{pmatrix}, \quad \mathbf{g} = \begin{pmatrix} g^{11} \\ \vdots \\ g^{nn} \end{pmatrix}, \quad i = 1, \dots, n.$$

LEMMA 2. The system (5) with L given by (6) where $\mathbf{G}$ is a symmetric matrix with $\det \mathbf{G} \neq 0$ has the normal form

$$\ddot{q}_i = -\langle \dot{\mathbf{q}}, \Gamma^i(\mathbf{q}) \dot{\mathbf{q}} \rangle - \langle \mathbf{P}^i(\mathbf{q}), \dot{\mathbf{q}} \rangle + \langle \mathbf{g}^i(\mathbf{q}), U_q(\mathbf{q}) \rangle, \quad i = 1, \dots, n. \quad (8)$$

Proof. The system (5) is equivalent to

$$\sum_{k=1}^n \ddot{q}_k L_{\dot{q}_k \dot{q}_k} + \sum_{k=1}^n \dot{q}_k L_{q_k \dot{q}_k} - L_{q_i} = 0, \quad i = 1, \dots, n$$

which is in fact

$$\sum_{k=1}^n g_{ik} \ddot{q}_k + \sum_{j=1}^n \sum_{k=1}^n \Gamma_{jki} \dot{q}_j \dot{q}_k + \sum_{k=1}^n P_{ik} \dot{q}_k - U_{q_i} = 0, \quad i = 1, \dots, n.$$

The matrix G having the inverse G^{-1} , we have

$$\ddot{q}_i = - \sum_{j=1}^n \sum_{k=1}^n \Gamma_{jki}^i(q) \dot{q}_j \dot{q}_k - \sum_{k=1}^n P_k^i(q) \dot{q}_k + \sum_{k=1}^n g^{ik}(q) U_{q_k}(q),$$

which is exactly (8). ■

Let there be given the family of curves

$$F_p(q) = c_p, \quad p = 1, \dots, n-1 \quad (9)$$

where $F : D \subseteq \mathbb{R}^n \rightarrow \mathbb{R}^{n-1}$ is twice differentiable on the open set D and at least one $(n-1) \times (n-1)$ minor of $F_q = (F_{pq})$, $p = 1, \dots, n-1$, $j = 1, \dots, n$ is nonzero. We suppose that the curves of the family (9) are orbits for the system (5).

Differentiating in (9) we obtain the linear system

$$\langle F_q, \dot{q} \rangle = 0 \quad (10)$$

which has the solution

$$\dot{q} = \alpha(q)L(q), \quad (11)$$

where $L = \begin{pmatrix} L_1 \\ \vdots \\ L_n \end{pmatrix}$ contains the determinants

$$L_i = (-1)^{i+1} \begin{vmatrix} F_{1q_1} \dots F_{1q_{i-1}} & \dots & F_{1q_{i+1}} \dots F_{1q_n} \\ F_{n-1,q_1} \dots F_{n-1,q_{i-1}} & & F_{n-1,q_{i+1}} \dots F_{n-1,q_n} \end{vmatrix}. \quad (12)$$

LEMMA 3. Let G be a symmetric and positive definite matrix; then the system (10) to which the energy integral is joined has a unique (up to a sign) solution $\dot{q}$.

Proof. Introducing $\dot{q} = \alpha(q)L(q)$ in (7) one has

$$\alpha^2(q) \langle L(q), G(q)L(q) \rangle / 2 = h + U(q),$$

hence we can consider

$$\alpha(q) = \pm [2(h + U(q)) / \langle L(q), G(q)L(q) \rangle]^{1/2}. \quad (13)$$

Now we can state the main theorem.

THEOREM. *Let us suppose that the curves of the family (9) are orbits for the system (5) with $\mathbf{G}$ a symmetric and positive definite matrix, so that $U, f_i, g_{ij} \in C^1(D), i=1, \dots, n, j=1, \dots, n$. Then the function U satisfies the following system of partial differential equations:*

$$a^2 \left\langle \mathbf{L}, \left(F_{pqq} - \sum_{i=1}^n F_{pq_i} \Gamma^i \right) \mathbf{L} \right\rangle - a \sum_{i=1}^n F_{pq_i} \langle \mathbf{P}^i, \mathbf{L} \rangle + \sum_{i=1}^n F_{pq_i} \langle \mathbf{g}^i, U_q \rangle = 0, \quad p=1, \dots, n-1 \quad (14)$$

where a is given by (13), $\mathbf{L}$ by (12) and $\Gamma^i, \mathbf{P}^i, \mathbf{g}^i$ depend only on the coefficients g_{ij}, f_i in (6), all the functions involved having the argument $\mathbf{q}$.

Proof. Let us differentiate in (10). We have

$$\langle \dot{\mathbf{q}}, \mathbf{F}_{pqq} \dot{\mathbf{q}} \rangle + \langle \mathbf{F}_{pq}, \ddot{\mathbf{q}} \rangle = 0, \quad p=1, \dots, n-1$$

where $\mathbf{F}_{pqq} = (F_{pqj,q_k})_1^n$ is the Hessian of F_p .

Replacing $\mathbf{q}$ by the expression obtained in (8), we have

$$\langle \dot{\mathbf{q}}, \mathbf{F}_{pqq} \dot{\mathbf{q}} \rangle - \sum_{i=1}^n F_{pq_i} [\langle \dot{\mathbf{q}}, \Gamma^i \dot{\mathbf{q}} \rangle + \langle \mathbf{P}^i, \dot{\mathbf{q}} \rangle - \langle \mathbf{g}^i, U_q \rangle] = 0,$$

$$\left\langle \dot{\mathbf{q}}, \left(F_{pqq} - \sum_{i=1}^n F_{pq_i} \Gamma^i \right) \dot{\mathbf{q}} \right\rangle - \sum_{i=1}^n F_{pq_i} \langle \mathbf{P}^i, \dot{\mathbf{q}} \rangle + \sum_{i=1}^n F_{pq_i} \langle \mathbf{g}^i, U_q \rangle = 0.$$

Now $\mathbf{q}$ is given by (11) and (12) and we get the relations (14). ■

Remark. The functions U and its partial derivatives appear in the function a given by (13) and in the last sum in (14).

The system (3) is a Lagrangian system with $\mathbf{L}$ given by (6) for $\mathbf{G}(\mathbf{q}) = I_n$, $\mathbf{f} = 0$ and $U = W$. In this case $\Gamma^i = 0, \mathbf{P}^i = 0, i=1, \dots, n$ and (14) becomes

$$a^2 \langle \mathbf{L}, \mathbf{F}_{pqq} \mathbf{L} \rangle + \sum_{i=1}^n F_{pq_i} U_{q_i} = 0,$$

where $a = \pm 2\Omega / \sum_{i=1}^n L_i^2$ with $\Omega = W + h$, and $U_{q_i} = \Omega_{q_i}$. Here we have $\mathbf{q} = \mathbf{x}$.

We have obtained

COROLLARY 1. (Stavinschi, Mioc). *Let us suppose that a particle has a trajectory given by (9). If its motion is governed by the system (3), the function W satisfies*

$$\left(2(W + h) / \sum_{i=1}^n L_i^2 \right) \sum_{i=1}^n \left(\sum_{k=1}^n F_{p x_i x_k} L_i L_k + F_{pq_i} W_{x_i} \right) = 0,$$

h being the energy constant and L_i given by (12). ■

The system (2) has $n = 2$, $G = I_2$, $f_1(x, y) = -y$, $f_2 = x$ and the energy integral

$$(\dot{x}^2 + \dot{y}^2)/2 = \Omega(x, y) + h.$$

Consider the system

$$\begin{aligned}\ddot{x} + \lambda(x, y)\dot{y} &= \Omega_x \\ \ddot{y} + \lambda(x, y)\dot{x} &= \Omega_y,\end{aligned}\tag{15}$$

where $\lambda(x, y) = u_y(x, y) - v_x(x, y)$, which is a slight generalization of (2) considered by Birkhoff (1917).

This is a Lagrangian system with $n = 2$, $G = I_2$, $(f_1, f_2) = (u, v)$ which has the same energy integral as (2). The system being 2-dimensional, the equation of the orbit is given by

$$F(x, y) = c.\tag{16}$$

Hence $L_1 = F_y$, $L_2 = -F_x$ and $a = \pm [2(h + \Omega)/(F_x^2 + F_y^2)]^{1/2}$.

We have $\Gamma_{ijk} = 0$, $\Gamma_{jk}^i = 0$ for every $i, j, k \in \{1, 2\}$ and $P_k^i = P_{ik} = f_{iq_k} - f_{kq_i}$, so $P_k^i = 0$ for $i = k$, and $P_2^1 = -P_1^2 = \lambda$.

In this case (14) contains one equality and this is

$$a^2(F_{xx}F_y^2 - 2F_{xy}F_xF_y + F_{yy}F_x^2)\mu a\lambda(F_x^2 + F_y^2) + F_x\Omega_x + F_y\Omega_y = 0,$$

the argument of all the functions being (x, y) .

COROLLARY 2. *Let us suppose that a particle has a trajectory given by (16) in the 2-dimensional space. If its motion is governed by system (15), with $\lambda = u_y - v_x$ then the force function Ω satisfies*

$$\begin{aligned}& [2(h + \Omega)/(F_x^2 + F_y^2)](F_{xx}F_y^2 - 2F_{xy}F_xF_y + F_{yy}F_x^2) \mp \\ & \mp [2(h + \Omega)]^{1/2}\lambda(F_x^2 + F_y^2)^{1/2} + F_x\Omega_x + F_y\Omega_y = 0,\end{aligned}$$

h being the energy constant. ■

The planar circular restricted three-body problem (2) is a special case of (15) with $u(x, y) = -y$, $v(x, y) = x$, so $\lambda(x, y) = -2$.

From Corollary 2 we obtain

COROLLARY 3 (Drămbă). *Let us suppose that a particle has a trajectory in 2-dimensional space given by (16). If the motion is governed by the system (2), then the function Ω satisfies*

$$\begin{aligned}& [2(h + \Omega)/(F_x^2 + F_y^2)](F_{xx}F_y^2 - 2F_{xy}F_xF_y + F_{yy}F_x^2) \pm \\ & \pm 2[2(h + \Omega)]^{1/2}(F_x^2 + F_y^2)^{1/2} + F_x\Omega_x + F_y\Omega_y = 0,\end{aligned}\tag{17}$$

h being the energy constant. ■

The system (1) is obtained from (15) with $\lambda(x, y) \equiv 0$, or from (3) with $n = 2$, hence Corollaries 1 and 2 give

COROLLARY 4 (Szebehely). *Let us suppose that a particle has a trajectory in the 2-dimensional space given by (16). If the motion is governed by the system (1), then the function V satisfies*

$$[2(h + V)(F_x^2 + F_y^2)](F_{xx}F_y^2 - 2F_{xy}F_xF_y + F_{yy}F_x^2) + F_xV_x + F_yV_y = 0, \quad (18)$$

h being the energy constant. ■

In fact, the relation (18) appears in Whittaker (1902a), and (17) in Whittaker (1902b), where they are deduced as a consequence of a variational principle of Maupertuis' type.

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BOUNDED NONCOLLISIONAL ORBITS IN GENERALIZED MANEFF-TYPE FIELDS

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Abstract. The motion of a particle under the influence of a generalized Maneff-type potential field is considered. Using reduction at one degree of freedom and the graphic of the effective potential energy, possible orbits are given. In the case of bounded noncollisional orbits complete integration is made.

Key words: celestial mechanics-Maneff's generalized law

1. INTRODUCTION

The gravitational post-Newtonian non-relativistic law proposed by Maneff in 1924 (see Maneff 1924; see also Maneff 1925, 1930a, 1930b) proved itself able to explain with efficacy the secular motion of both inner planets perihelion and the Moon's perigee. Reconsidered in our days (see Diacu 1993, Diacu and Illner 1993, Diacu et al. 1995, Mioc and Stoica 1995), the model of Maneff in which it is assumed that the mutual gravitational attraction of two masses M , m is equal to $(-GMm/r^2)(1 + 3G(M + m)/(2c^2r))$ with G -the universal gravitational constant and c -the speed of light, shows itself as an interesting departure for future investigations and offers a classical alternative for relativity.

In this paper we shall consider a generalized version of Maneff's potential function, not necessarily purely gravitational, given by $A/r + B/(2r^2)$, where $A \in \mathbf{R}^*$, $B \in \mathbf{R}$. As it can be easily seen, such a version can model various situations, including Coulombian potentials and radiation pressure potentials in the neighbourhood of a star. Our goal is to find all possible orbits which can appear fixing the constant angular momentum and the level of energy.

For this purpose, a first step is the reduction of motion at one degree of freedom and the introduction of the effective potential energy $V(r)$ (see Arnold 1980). Settled the constant angular momentum and plotting $V(r)$'s graphic, the particle's orbit will be found in the domain $V(r) \leq h$, where h is a fixed energy level.

Although all situations can be completely integrated, we stop ourselves at the bounded noncollisional case. Here, Kepler's laws can find themselves a natural generalization. The exact formula of trajectory is given and, after defining the period, the frequency of motion and the eccentric anomaly, the equivalents for Kepler's third law and Kepler's equation are given.

2. BASIC EQUATIONS

Let a particle of unit mass be moving under the influence of a potential field of the form

$$U(\mathbf{r}) = -\frac{1}{r} \left(A + \frac{B}{2r} \right) \quad (1)$$

where $\mathbf{r}$ is the position vector in a frame originated in field's source, r is $\mathbf{r}$'s absolute value and $A \in \mathbb{R}^*$, $B \in \mathbb{R}$. The motion is described by the initial value problem:

$$\ddot{\mathbf{r}} = -\frac{1}{r^2} \left(A + \frac{B}{r} \right) \frac{\mathbf{r}}{r}; \quad \mathbf{r}(t_0) = \mathbf{r}_0, \quad \dot{\mathbf{r}}(t_0) = \dot{\mathbf{r}}_0. \quad (2)$$

Since the force acting on the particle is a central one, the motion takes place in a fixed plane passing through the origin of the frame and in polar coordinates (r, θ) equation (2) becomes:

$$\begin{aligned} \ddot{r} - r\dot{\theta}^2 &= -\frac{1}{r^2} \left(A + \frac{B}{r} \right), \\ \frac{d}{dt}(r^2\dot{\theta}) &= 0. \end{aligned} \quad (3)$$

Let C be the constant angular momentum supplied by the second equation of system (3) and let h be the constant of energy obtained by applying the energy conservation law:

$$h = \frac{\dot{r}^2}{2} + \frac{C^2}{2r^2} - \frac{1}{r} \left(A + \frac{B}{2r} \right). \quad (4)$$

Again, since the force in question is a central one, the study of the motion can be reduced at the study of the one degree of freedom problem (see Arnold 1980):

$$\ddot{r} = -\frac{\partial V}{\partial r}, \quad (5)$$

where $V(r)$ is the effective potential energy defined by

$$V(r) = -\frac{1}{r} \left(A - \frac{C^2 - B}{2r} \right) \quad (6)$$

and the total energy is the same as the one in the initial problem (2).

3. ORBIT BEHAVIOUR

Settled the constant angular momentum C and plotting $V(r)$'s graphic, the orbit will be found in the domain $V(r) \leq h$, where h is a fixed total energy level. Let $r_{cr} = (C^2 - B)/A$ be the point where $V(r)$ vanishes and $h_{cr} = V(r_{cr}) = (-A^2)/[2(C^2 - B)]$. The following cases can be distinguished:

1) $(C^2 - B) > 0, A > 0$; motion is possible for

$$\begin{cases} h \in [h_{cr}, 0) \Rightarrow r \in [r_{min}, r_{max}] \\ h \in [0, \infty) \Rightarrow r \in [r_{min}, \infty) \end{cases}$$

allowing bounded noncollisional orbits (see Fig. 1);

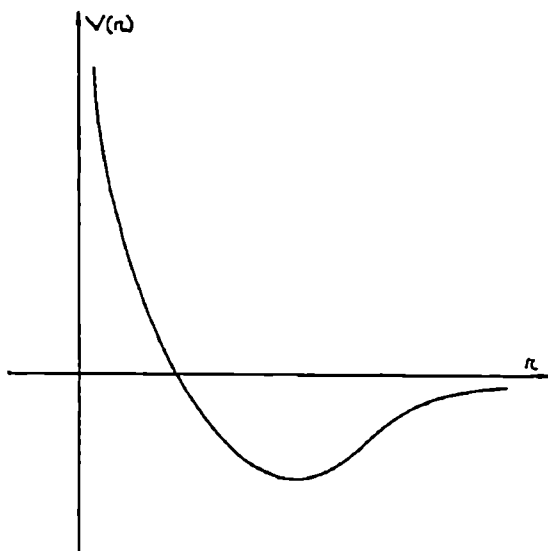


Fig. 1

2) $(C^2 - B) \geq 0, A < 0$; motion is possible for $h > 0 \Rightarrow r \in [r_{min}, \infty)$, so the orbits are unbounded (see Fig. 2);

3) $(C^2 - B) \leq 0, A > 0$; motion is possible for $h < 0 \Rightarrow r \in (0, r_{max}]$, the orbits being bounded but of collisional type (see Fig. 3);

4) $(C^2 - B) < 0, A < 0$; motion is possible for

$$\begin{cases} h \in (-\infty, 0] \Rightarrow r \in (0, r_{max}] \\ h \in (0, h_{cr}) \Rightarrow r \in (0, r_{max}] \cup [r_{min}, \infty). \end{cases}$$

The orbits are unbounded or bounded of collisional type (see Fig. 4).

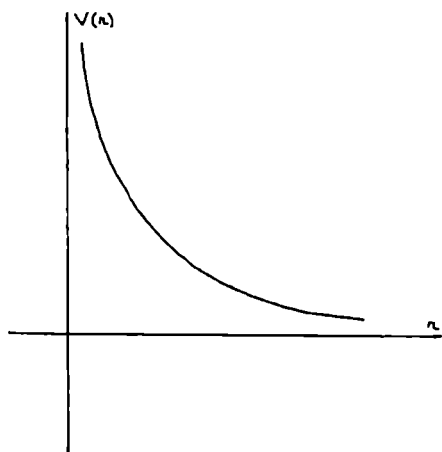


Fig. 2

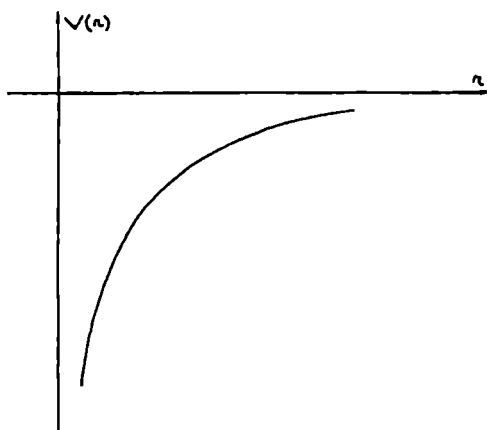


Fig. 3

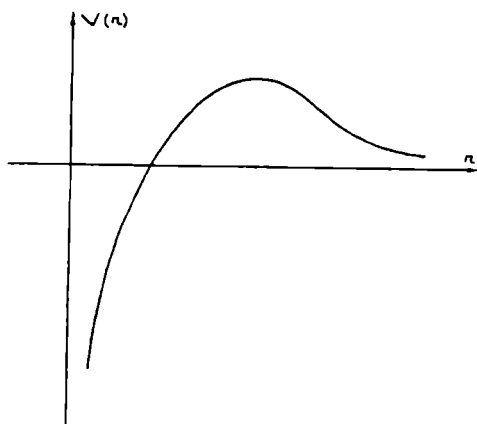


Fig. 4

4. BOUNDED NONCOLLISIONAL ORBITS. COMPLETE DESCRIPTION

The bounded noncollisional orbits are possible only in the case $(C^2 - B) > 0$, $A > 0$. For integration, we use the energy relation (4), obtaining an equation of Binet's type. From (4) it follows

$$C \frac{d}{dr} \left(\frac{1}{r} \right) = \pm \sqrt{\left(2h + \frac{A}{(C^2 - B)} \right) - \left(\sqrt{C^2 - B} \left(\frac{1}{r} \right) - \frac{A}{\sqrt{C^2 - B}} \right)^2}. \quad (7)$$

The calculus offers the equation of trajectory:

$$r(\theta) = \frac{\frac{(C^2 - B)}{A}}{1 + \sqrt{1 + 2 \frac{(C^2 - B)}{A^2} h \cos \frac{\sqrt{C^2 - B}}{C} (\theta - \theta_0)}}. \quad (8)$$

The trajectory may be seen as a pseudoconic whose elements can be identified with the elements of a classical one and the condition of boundedness $h \in [-A^2/2(C^2 - B), 0)$ revert to $0 \leq \sqrt{1 + 2(C^2 - B)h/A^2} < 1$. Let us denote

$$\begin{aligned} p &:= \frac{C^2 - B}{A}, \\ e &:= \sqrt{1 + 2 \frac{(C^2 - B)}{A^2} h}, \\ \alpha &:= \frac{\sqrt{C^2 - B}}{C}, \quad f := (\theta - \theta_0), \end{aligned} \quad (9)$$

then (8) becomes

$$r(f) = \frac{p}{1 + e \cos(\alpha f)} \quad (10)$$

with $0 < \alpha \leq 1$ and $0 \leq e < 1$. The angle between a pericentre $r_m = p/(1 + e)$ and a successive apocentre $r_M = p/(1 - e)$ is given by:

$$\Theta = \frac{C}{\sqrt{C^2 - B}} \pi = \frac{\pi}{\alpha}. \quad (11)$$

The orbit is closed if the angle Θ is commensurable with 2π , otherwise it covers densely the annulus $r_m \leq r \leq r_M$.

Let us define the anomalistic period of motion T as the particle's necessary time to cover the trajectory between two successive pericentres. In a whole period,

the radius vector sweeps the angle 2Θ and the corresponding area $\mathcal{A}$ can be calculated with the formula:

$$\mathcal{A} = \int_0^{\Theta} \left(\frac{p}{1 + e \cos(\alpha s)} \right)^2 ds. \quad (12)$$

The area's law is applied in the classical way

$$\frac{d\mathcal{A}}{dt} = \frac{r^2 \dot{\Theta}}{2} = \frac{C}{2}, \quad (13)$$

and using (9), we obtain:

$$\frac{p^2}{\alpha(1 - e^2)^{3/2}} \pi = T \frac{C}{2}. \quad (14)$$

The period T can be expressed as function of the field's parameters and the two constants C and h by replacing e by (9). Thus

$$T = \frac{A(C^2 - B)}{C^2(-h)\sqrt{-2h}} \pi. \quad (15)$$

Let be $a := (r_m + r_M)/2$. Once again, using (9) it follows that $a = p/(1 - e^2)$ and, as it can be easily seen, in the particular case of Newtonian potential, a is the semi-major axis of the classical elliptic orbit. The Kepler's third law gets an equivalent applying area's law to a complete period $T = (2\Theta)/n$ (n is the mean motion) or (see (9) and (11)) $T = (2\pi/\alpha)/n$. Taking into account the definitions of p and a , we have

$$C = \sqrt{Aa(1 - e^2) + B} \quad (16)$$

and the generalization of Kepler's third law is:

$$n^2 a^3 = A + \frac{1}{a(1 - e^2)} B. \quad (17)$$

With the purpose to find the correspondent of Kepler's equation, we define the eccentric anomaly in a natural way using the next applications: $\gamma : [0, 2\pi/\alpha] \rightarrow [0, 2\pi]$, $\gamma(f) := \alpha f$ and $\rho : [0, 2\pi] \rightarrow [r_m, r_M]$, $\rho(v) = p/(1 + e \cos v)$. ρ represents an ellipse of semi-latus rectum p and eccentricity e . For $\rho(\cdot)$, the passing from true anomaly v to eccentric anomaly E is made by $E : [0, 2\pi] \rightarrow [-1, 1]$

$$E(v) = \begin{cases} \arccos [(e + \cos v)(1 + e \cos v)^{-1}], & v \in [0, \pi] \\ \arccos [(e - \cos(v - \pi))(1 - e \cos(v - \pi))^{-1}], & v \in (\pi, 2\pi]. \end{cases} \quad (18)$$

It's easy to guess that for the motion in question we shall consider as eccentric anomaly $\tilde{E} : [0, 2\pi/\alpha] \rightarrow [-1, 1]$, $\tilde{E}(f) := (E \circ \gamma)(f)$. The classical formulae relating eccentric anomaly and true current polar angle keep the same form:

$$\cos(\alpha f) = \frac{\cos \tilde{E} - e}{1 - e \cos \tilde{E}}, \quad \tan \frac{\alpha f}{2} = \sqrt{\frac{1+e}{1-e}} \tan \frac{\tilde{E}}{2} \quad (19)$$

and the trajectory can be described by $r(\tilde{E}) = a(1 - e \cos \tilde{E})$. The correspondent of Kepler's equation is obtained by integrating $r^2 \dot{f} = C$ (the relation of the conservation of angular momentum) between 0 and f , where from

$$\int_0^f p^2 ds / (1 + e \cos(\alpha f))^2 = C(t - t_0).$$

Using (16)–(17) and (19) we have:

$$n(t - t_0) = \tilde{E} - e \sin \tilde{E}. \quad (20)$$

Some of these results can be recovered in the classical literature. I've pointed out the boundedness noncollisional conditions of the orbits in generalized Maneff-type fields, the equation of trajectory and the equivalents of Kepler's third law, eccentric anomaly and Kepler's equation for motion in question. The classical laws can be recovered for $A = G(M + m)$, $B = 0$.

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SOLUTION OF REGULARIZED EQUATIONS FOR THE TWO-BODY PROBLEM IN MANEFF-TYPE FIELDS

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Abstract. The two-body problem governed by a Maneff-type interaction law (combination of an inverse square law and an inverse cubic law) is being considered. The equations of motion and the integral of energy are regularized. The problem is completely solved for the whole allowed interplay between the parameters of the field and the initial conditions.

Key words: celestial mechanics — two-body problem — Maneff-type fields — regularization.

1. INTRODUCTION

We shall call *Maneff-type field* a force field featured by a potential function of the form $\tilde{A}/r + \tilde{B}/r^2$, where r = distance between two particles, and A, B = constants. This potential function generalizes that proposed by Maneff (1924, 1925, 1930 a, b) and reconsidered recently by Diacu (1993), Diacu and Illner (1993), Diacu et al. (1995), Mioc and Stoica (1995). Observe that the nature of this field is not necessarily purely gravitational, and may remain unspecified.

If one assigns certain expressions to $\tilde{A}$ and $\tilde{B}$, different concrete astronomical (and not only) situations are recovered. Among such situations modellable in this way, we mention: the motion in certain post-Newtonian nonrelativistic fields (obviously, in Maneff's one, too), the motion in certain relativistic gravitational fields (e. g. in Fock's one with a truncated expression for the relativistic "force"; see Mioc 1994), the motion in the photogravitational field (see e.g. Saslaw 1978; Mioc and Radu 1992; Șelaru et al. 1993) generated by a source of constant luminosity (whose gravitational action is not necessarily Newtonian), the two-body problem with constant equivalent gravitational parameter (see Mioc et al. 1988; Mioc 1989; Șelaru et al. 1992), and so forth. The implications of such a model can be extended to astrodynamics, cosmogony, astrophysics, etc.

In this paper we shall tackle the two-body problem in a Maneff-type field. We shall regularize the equations of motion and the integral of energy, then we shall point out the corresponding solutions. In such a way we generalize the results obtained by Mioc and Stoica (1995) for Maneff's gravitational field.

2. REGULARIZED EQUATIONS OF MOTION

Consider two (eventually pointlike) particles of masses m_1, m_2 in a Maneff-type field, and let us denote $k = (m_1 + m_2)/(m_1 m_2)$, $A = k\tilde{A}$, $B = k\tilde{B}$. In this case, using polar coordinates (r, u) , the equations which describe the relative motion of m_2 , say, with respect to m_1 , will be obtained in the form

$$\ddot{r} - r\dot{u}^2 + A/r^2 + 2B/r^3 = 0, \quad (1)$$

$$r\ddot{u} + 2\dot{r}\dot{u} = 0. \quad (2)$$

Equation (2) leads immediately to the first integral of angular momentum, but we prefer for the needs of this paper keep the equations of motion (1)–(2) as such. As to the first integral of energy, this one reads

$$\dot{r}^2 + (r\dot{u})^2 - 2A/r - 2B/r^2 = h, \quad (3)$$

with $h = \text{constant of energy}$.

Let us regularize equations (1)–(3) by means of the transformation $dt = r^n ds$, $n > 0$ (e.g. Mangad 1967). It is easy to see that equations (1)–(2) will respectively become

$$r'' + (n-1)ru'^2 - (2n-1)Ar^{2n-2} - 2(n-1)Br^{2n-3} - nhr^{2n-1} = 0, \quad (4)$$

$$u'' - (n-2)r'u'/r = 0, \quad (5)$$

where primes denote differentiation with respect to s , and (3) was also used to obtain (4). The integral of energy acquires the form

$$r'^2 + r^2u'^2 = 2Ar^{2n-1} + 2Br^{2n-2} + hr^{2n}. \quad (6)$$

The concrete value $n = 2$ is very suitable to make simpler the above equations. We shall hence put $n = 2$ and ensure ourselves that in this case equations (4)–(6) are regular everywhere, collision ($r \rightarrow 0$) included. More exactly, we shall check the boundedness of every term in (4)–(6) for $n = 2$ as $r \rightarrow 0$. Actually only

the terms containing ru'^2 and $r'u'/r$ interest us, because for the other terms all is clear.

So, put $n = 2$ in (4)–(6); these equations become respectively

$$r'' + ru'^2 - 3Ar^2 - 2Br - 2hr^3 = 0, \quad (7)$$

$$u'' = 0, \quad (8)$$

$$r'^2 + r^2u'^2 = 2Ar^3 + 2Br^2 + hr^4. \quad (9)$$

By (9) it is easy to see that

$$\begin{aligned} |ru'^2| &= |r^2u'^2|/r \leq |r'^2 + r^2u'^2|/r = |2Ar^3 + 2Br^2 + hr^4|/r = \\ &= |2Ar^2 + 2Br + hr^3| \rightarrow 0 \quad \text{for } r \rightarrow 0. \end{aligned} \quad (10)$$

Also

$$\begin{aligned} |r'u'/r|^2 &= |r'|^2 |ru'|^2/r^4 \leq |r'^2 + r^2u'^2|^2/r^4 = \\ &= |(2Ar^3 + 2Br^2 + hr^4)/r^2|^2 = |2Ar + 2B + hr^2|^2 \rightarrow |2B|^2 \quad \text{for } r \rightarrow 0. \end{aligned} \quad (11)$$

Thus the two above mentioned terms are bounded for $n = 2$ and $r \rightarrow 0$. This means that equations (7) and (8) are regular.

3. SOLUTIONS

Attaching to the coupled equations (7)–(8) the initial conditions

$$(r, r', u, u')(s_0) = (r_0, r'_0, u_0, u'_0), \quad (12)$$

equation (8) gives immediately

$$u(s) = u_0 + u'_0(s - s_0). \quad (13)$$

By (13) $u' = u'_0$, so (9) becomes (denoting $u_0'^2 = w$)

$$r'^2/r + wr - 2Ar^2 - 2Br - hr^3 = 0, \quad (14)$$

remaining a first integral for the equation in $r(s)$. Since $ru'^2 (=rw)$ and r'^2/r are bounded (see above) for $r \rightarrow 0$, equation (14) is regular, too.

In these conditions, (14) leads to the integral form

$$\int_{r_0}^r \frac{dr}{r\sqrt{hr^2 + 2Ar + 2B - w}} = s - s_0, \quad (15)$$

where the integrand is defined on a real interval I .

To solve (15) in order to obtain $r = r(s)$, we shall distinguish three cases which, by a somewhat forced analogy with the Keplerian two-body problem, will be denoted by E ("elliptic", $h < 0$), P ("parabolic", $h = 0$), and H ("hyperbolic", $h > 0$). Due to the interplay between A , B , h and w , each of the three cases will have its own subcases (with different intervals I). Let us denote

$$r_1 = \frac{-A - \sqrt{A^2 + h(w - 2B)}}{h}, \quad r_2 = \frac{-A + \sqrt{A^2 + h(w - 2B)}}{h}, \quad (16)$$

and specify all subcases for which the integral in (15) is meaningful, each subcase with its I . Recalling that w is nonnegative, we have

E1: $w < 2B, \forall A:$	$I = (0, r_1);$	
E2: $w = 2B, A > 0:$	$I = (0, -2A/h);$	
E3: $2B < w < 2B - A^2/h, A > 0:$	$I = (r_2, r_1);$	
P1: $w < 2B, A < 0:$	$I = (0, (w - 2B)/(2A));$	
P2: $w < 2B, A = 0:$	$I = (0, \infty);$	
P3: $w < 2B, A > 0:$	$I = (0, \infty);$	
P4: $w = 2B, A > 0:$	$I = (0, \infty);$	
P5: $w > 2B, A > 0:$	$I = ((w - 2B)/(2A), \infty);$	
H1: $w < 2B - A^2/h, \forall A:$	$I = (0, \infty);$	(17)
H2: $w = 2B - A^2/h, A < 0:$	$I = (0, -A/h) \cup (-A/h, \infty);$	
H3: $w = 2B - A^2/h, A > 0:$	$I = (0, \infty);$	
H4: $2B - A^2/h < w < 2B, A < 0:$	$I = (0, r_1) \cup (r_2, \infty);$	
H5: $2B - A^2/h < w < 2B, A > 0:$	$I = (0, \infty);$	
H6: $w = 2B, A < 0:$	$I = (-2A/h, \infty);$	
H7: $w = 2B, A = 0:$	$I = (0, \infty);$	
H8: $w = 2B, A > 0:$	$I = (0, \infty);$	
H9: $w > 2B, \forall A:$	$I = (r_2, \infty).$	

For all other cases the integral is meaningless, because $I = \emptyset$.

Now, performing the integration in (15), by observing (17), then expliciting $r = r(s)$, and abridging

$$L = w - 2B, K = A^2 + hL, \quad (18)$$

we get

$$\text{E1, H4, H5: } r(s) = L / \{A - \sqrt{K} \cosh [C_1 - \sqrt{-L}(s - s_0)]\}, \quad (19)$$

$$C_1 = \arg \cosh [(Ar_0 - L) / (r_0 \sqrt{K})];$$

$$\text{H1: } r(s) = L / \{A - \sqrt{-K} \sinh [C_2 - \sqrt{-L}(s - s_0)]\}, \quad (20)$$

$$C_2 = \arg \sinh [(Ar_0 - L) / (r_0 \sqrt{-K})];$$

$$\text{E3, H9: } r(s) = L / \{A - \sqrt{K} \sin [C_3 + \sqrt{L}(s - s_0)]\}, \quad (21)$$

$$C_3 = \arcsin [(Ar_0 - L) / (r_0 \sqrt{K})];$$

$$\text{E2, H6, H8: } r(s) = 2A / \{A^2 [C_4 - (s - s_0)]^2 - h\}, \quad (22)$$

$$C_4 = \sqrt{2A / r_0 + h} / A;$$

$$\text{H7: } r(s) = r_0 / [1 - r_0 \sqrt{h}(s - s_0)]; \quad (23)$$

$$\text{H2, H3: } r(s) = A \{ \tanh [C_5 - A(s - s_0) / (2\sqrt{h})] - 1 \} / (2h), \quad (24)$$

$$C_5 = \arg \tanh (2hr_0 / A + 1);$$

$$\text{P1: } r(s) = -L \{ \tanh^2 [C_6 - \sqrt{-L}(s - s_0) / 2] - 1 \} / (2A), \quad (25)$$

$$C_6 = \arg \tanh \sqrt{1 - 2Ar_0 / L};$$

$$\text{P2: } r(s) = r_0 \exp [\sqrt{-L}(s - s_0)]; \quad (26)$$

$$\text{P3: } r(s) = -2(L / A) C_7 \exp [\sqrt{-L}(s - s_0)] / \{1 - C_7 \exp [\sqrt{-L}(s - s_0)]\}^2, \quad (27)$$

$$C_7 = (\sqrt{2Ar_0 - L} - \sqrt{L}) / (\sqrt{2Ar_0 - L} + \sqrt{-L});$$

$$\text{P4: } r(s) = 2r_0 / [\sqrt{2} - A\sqrt{r_0}(s - s_0)]^2; \quad (28)$$

$$\text{P5: } r(s) = L / \{2A \cos^2 [C_8 + \sqrt{L}(s - s_0) / 2]\}, \quad (29)$$

$$C_8 = \arctan \sqrt{2Ar_0 / L - 1}.$$

The couples consisting of (13) and each of the equations (19)–(29) constitute the solutions of the regularized equations of the two-body problem for a Maneff-type field.

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CONSIDERATIONS CONCERNING THE DEPENDENCE OF THE ORBITAL ENERGY OF AN ARTIFICIAL SATELLITE ON THE SOLAR AND GEOMAGNETIC ACTIVITY

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Abstract. A numerical method for the determination of the orbital energy variation $\Delta E = \Delta E(F_{10.7}, K_p)$ for a given orbit is proposed and tested by means of the TD88 and C atmospheric models; a quasilinear variation with both $F_{10.7}$ and K_p is found for a theoretical (ideal) orbit similar to Intercosmos 19 rocket's orbit. Numerical examples are given.

Key words: artificial satellites, orbital energy.

1. BASIC EQUATIONS

The orbital energy (per unit mass) of an artificial satellite is determined by the relation

$$E = \frac{v^2}{2} - U, \quad (1)$$

where v = satellite velocity and U = gravitational potential function. To evaluate E at each moment of time, we need $v(t)$ and $U(t)$; these can be deduced from the orbital elements of the satellite.

We consider that the trajectory of the satellite is characterized by the following set of orbital elements

$$\{p, \Omega, i, q, k, t\}, \quad (2)$$

where p = semilatus rectum; Ω = longitude of the ascending node; i = inclination with respect to equator; $q = e \cdot \cos \omega$; $k = e \cdot \sin \omega$; e = eccentricity, ω = argument of perigee.

As independent variable we use the argument of latitude (u). The connection between time and the argument of latitude is presented below. In this case the sixth element of (2) represents the value of time corresponding to angle u . We consider

that the orbital elements are functions of u and their variations are given by the equations:

$$\begin{aligned}
 \frac{dp}{du} &= 2 \frac{z}{\mu} r^3 T, \\
 \frac{d\Omega}{du} &= \frac{z}{\mu p} r^3 \frac{\sin u}{\sin i} W, \\
 \frac{di}{du} &= \frac{z}{\mu p} r^3 \cos u W, \\
 \frac{dq}{du} &= \frac{z}{\mu p} r^3 k \frac{\sin u \cos i}{\sin i} W + \\
 &\quad + \frac{z}{\mu} r^2 \left[\frac{r(q + \cos u)}{p} + \cos u \right] T + \\
 &\quad + \frac{z}{\mu} r^2 \sin u S, \\
 \frac{dk}{du} &= - \frac{z}{\mu p} r^3 q \frac{\sin u \cos i}{\sin i} W + \\
 &\quad + \frac{z}{\mu} r^2 \left[\frac{r(k + \sin u)}{p} + \sin u \right] T - \\
 &\quad - \frac{z}{\mu} r^2 \cos u S, \\
 \frac{dt}{du} &= \frac{r^2}{\sqrt{\mu p}} z,
 \end{aligned} \tag{3}$$

where

$$z = \left[1 - \frac{r^2}{\sqrt{\mu p}} \cos i \frac{d\Omega}{dt} \right]^{-1}, \tag{4}$$

and S , T , W represent the components of the perturbing force (per unit mass), respectively in the direction of the radius vector, perpendicular to the radius vector (in the plane of the osculating orbit) and normal on the plane of osculating orbit. Parameter μ is the product of the gravitational constant and the mass of the Earth (gravitational parameter).

Note. It can be seen that the last equation (3) shows the dependence $t = t(u)$; for this reason the time parameter is considered to be the sixth orbital element of

the set (2). In general the solution of the system (3) can be found through numerical methods if the initial conditions (Cauchy problem) are given.

$$\begin{aligned} u &= u_0, \quad p_0 = p(u_0), \quad \Omega_0 = \Omega(u_0), \quad i_0 = i(u_0), \\ q_0 &= q(u_0), \quad k_0 = k(u_0), \quad t_0 = t(u_0). \end{aligned} \quad (5)$$

2. THE COMPONENTS OF THE PERTURBING FORCE

As perturbing force in the motion of the satellite we consider the noncentrality of the gravitational field (only the influence of the second harmonic J_2) and the atmospheric drag.

The expressions of the components S , T , W depending on J_2 have the form (Zhongolovich, 1960)

$$\begin{aligned} S_2 &= +3J\mu a_e^2 r^{-4} \left(\sin^2 u \sin^2 i - \frac{1}{3} \right), \\ T_2 &= -2J\mu a_e^2 r^{-4} \sin u \cos u \sin^2 i, \\ W_2 &= -2J\mu a_e^2 r^{-4} \sin u \sin i \cos i, \end{aligned} \quad (6)$$

where a_e is the mean terrestrial equatorial radius, and

$$J = \frac{3}{2} J_2. \quad (7)$$

The expressions of S , T , W due to the atmospheric drag have the form

$$\begin{aligned} S_a &= -\frac{1}{2} \rho \delta V v_r, \\ T_a &= -\frac{1}{2} \rho \delta V (v_n - r \omega_a \cos i), \\ W_a &= -\frac{1}{2} \rho \delta V r \omega_a \sin i \cos u. \end{aligned} \quad (8)$$

In eqs. (8) ρ = air density at the flight height of the satellite; V = velocity of the satellite with respect to the air; ω_a = angular velocity of rotation of the atmosphere (we assume that it is constant and equal to that of the Earth); v_r , v_n = the radial and transversal components of the satellite velocity V with respect to the

centre of the Earth. Parameter δ represents the ballistic coefficient of the artificial satellite and is given by the relationship (King-Hele, 1964)

$$\delta = C_D \frac{A}{m}, \quad (9)$$

where C_D = dimensionless drag coefficient (usually considered equal to 2.2); A = cross-sectional area of the satellite and m = satellite mass.

3. THE EVALUATION OF THE ORBITAL ENERGY

The variation of the satellite orbital energy is given by the relation (Cârșmaru and Oproiu, 1994)

$$\frac{dE}{du} = -\frac{1}{2} \delta \rho \frac{r^2}{\sqrt{\mu p}} V^3 Z. \quad (10)$$

At the initial moment $u_0 = u(t_0)$ we have

$$E_0 = E(u_0) = -\frac{\mu}{2p_0} (1 - q_0^2 - k_0^2). \quad (11)$$

The differential equation (10) together with the initial condition (11) is attached to system (3) and is integrated simultaneously. By performing the integration on a nodal period ($u \in [0, 2\pi]$) we obtain both the variation of the orbital energy ($\Delta E = E(2\pi) - E(0)$) and the allure of the curves $E(u)$ and dE/du . The sixth differential equation of (3) gives just the nodal period: $T_\Omega = \pi(2\pi) - t(0)$.

4. CONSIDERATIONS CONCERNING THE NUMERICAL INTEGRATION

System (3) together with (10) are integrated numerically by using the algorithm Runge-Kutta (Gill's variant). In this integration an important stage is the calculation of the values of the components S , T , W of the perturbing acceleration. A significant step in this calculation also is the evaluation of the air density at the current position of the satellite by means of an atmospheric model.

Among the reference models used by us are the TD88 model (Sehnal and Pospíšilová, 1988) and the C model (Köhnlein, 1980). By means of these models the air density can be evaluated as a function of the solar and geomagnetic activity, local time, day of the year, altitude and latitude.

5. NUMERICAL APPLICATION

For the practical application we consider the orbital elements of Intercosmos-19 rocket (1979 20 B) taken from NASA lists (two line orbital elements) in the following way: epoch (t_0) = 49360.20744566 MJD (January 8, 1994), $n = 14.91248555$ rev/day, $e = 0.200403$, $\Omega = 2^\circ.784$, $\omega = 231^\circ.222$, $M_0 = 127^\circ.091$, $i = 73^\circ.9755$.

The rocket has the form of a cylinder with the diameter (D) of 2.4 m, length (L) of 7.4 m and mass of 2200 kg. The ballistic coefficient δ has been considered equal to $0.015 \text{ m}^2/\text{kg}$ (Cârşmaru and Oproiu, 1993).

The angular velocity of the atmosphere rotation has been considered equal to that of the Earth, namely $\omega_a = \omega_{\text{Earth}} = 1.002737812$ rev/day.

The integration step has been assumed to be equal to $1^\circ.0$ and the integration has been made on the interval $[0, 2\pi]$, that is, on a nodal period.

System (3) together with equation (10) are integrated numerically under the hypotheses:

— parameter “day” from the calculation of the density according to the model TD88 is considered equal to 180 (around this value the semiannual effect in the atmosphere density is minimum);

— values between $F_{10.7} = 60$ (the minimum of the solar activity) and $F_{10.7} = 300$ (the maximum of the solar activity), with the step $\Delta F_{10.7} = 20$, are used;

— for the geomagnetic index K_p we use the values 0, 1, 2, ..., 9;

— for every pair ($F_{10.7}$, K_p) we determined the values $\Delta E = E(2\pi) - E(0)$.

6. RESULTS

Table 1 lists the values ΔE obtained by using the TD88 and C atmospheric models for three levels of solar activity: low ($F_{10.7} = 60$), mean ($F_{10.7} = 180$), high ($F_{10.7} = 300$) and for geomagnetic index $K_p = 0, 1, \dots, 9$.

Table 1

($-\Delta E \times 10^8 \text{ km}^2/\text{sec}^2$); $\Delta E = E(2\pi) - E(0)$

$K_p \backslash F_{10.7}$	0	1	2	3	4	5	6	7	8	9	Model
60	47	54	61	70	81	92	105	120	135	152	C
60	120	127	134	140	147	154	160	167	174	180	TD
180	822	902	986	1072	1162	1253	1347	1442	1539	1638	C
180	500	528	555	583	611	639	667	694	722	750	TD
300	7325	7686	8048	8411	8776	9143	9514	9890	10270	10658	C
300	974	1028	1083	1137	1191	1245	1299	1353	1407	1461	TD

In Figure 1 (a and b) are plotted $(\Delta E)^{TD88}$ and $(\Delta E)^C$ versus K_p for three levels of solar activity, function of the atmospheric model TD88, respectively C.

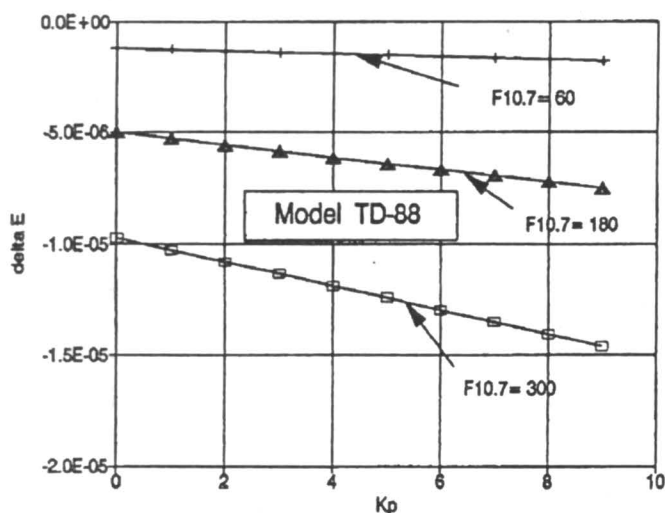


Fig. 1a. — The values of ΔE , versus K_p , obtained by TD88 model for three levels of solar activity.

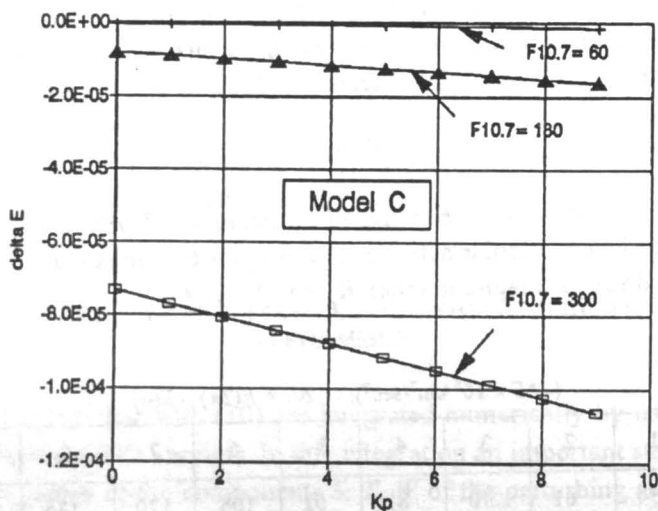


Fig. 1b. — The values of ΔE , versus K_p , obtained by C model for three levels of solar activity.

In Figure 2 (a and b) there are plotted the orbital energy variation, $-\Delta E$, of the satellite Intercosmos 19 rocket, in comparison with the values of $F_{10.7}$, during the period 1982–1994.

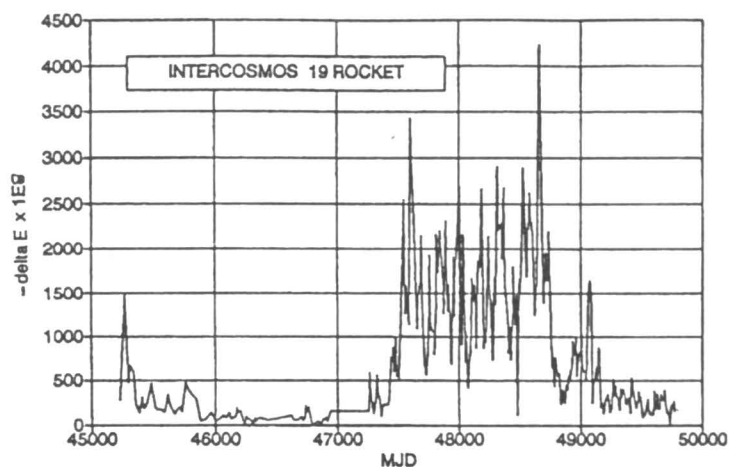


Fig. 2a. — The variation of orbital energy, $-\Delta E$, of the Intercosmos 19 rocket, in period 1982–1994.

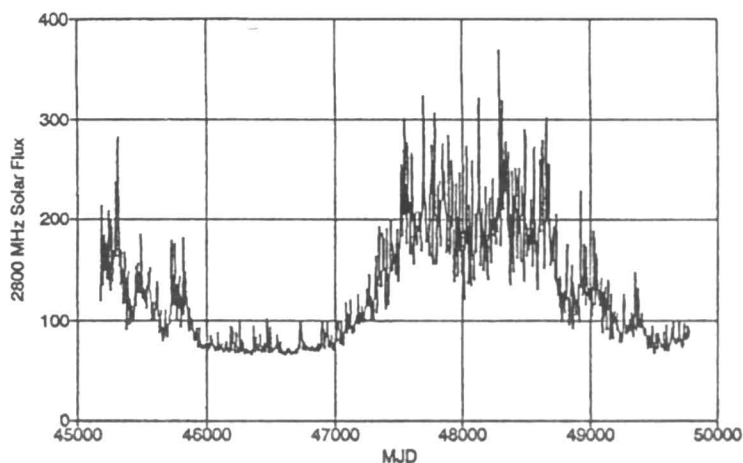


Fig. 2b. — The values of solar flux $F_{10.7}$ in period 1982–1994.

7. CONCLUSION

1. About the applications taken into consideration we may say, as a first approximation, that the variation of the orbital energy on a nodal period has a quasi-linear variation with both $F_{10.7}$ and K_p ; this fact can be seen very well in both Table 1 and Figure 1 (a and b).
2. The results obtained by numerical method agree with the real case of Figure 2, where at linear increase of $F_{10.7}$ (Fig. 2b) there corresponds a linear decrease of ΔE (Fig. 2a).

3. The ΔE values obtained by using the C model are smaller than those obtained by the TD88 model, for high levels of solar activity.

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ANALYSIS OF DOUBLE STARS CATALOGUES OBSERVED IN BELGRADE AND BUCHAREST

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Abstract. The analysis of the systematic errors $\Delta\alpha_\alpha$ and $\Delta\delta_\alpha$ was made by comparing stars positions from Double Stars Catalogues observed in Belgrade and Bucharest.

Key words: double stars catalogues, systematic errors.

Double Stars Catalogues were observed at Bucharest Observatory in two different periods:

— 1966 Aug.–1971 May, declination zone ($+0^\circ$, $+21^\circ$) and

— 1985 July–1989 Aug., declination zone (-5° , $+5^\circ$).

The observations were performed with the Gauthier-Prin Meridian Circle having a Merz-Steinheil objective of $d = 19$ cm and $f = 235$ cm. The list observed in the period, contains 330 close double stars with magnitude comprised in the interval ($6^m.0$, $9^m.0$). The accuracies of these stars positions are (Liculescu and Popescu, 1991):

$$\varepsilon_\alpha \cos \delta = \pm 0''.025 \quad \text{and} \quad \varepsilon_\delta = 0''.29$$

The same programme of Double Stars was observed at Belgrade Observatory in the period: 1981 Feb.–1987 Apr., with the Meridian Circle $d = 19$ cm/ $f = 257.8$ cm. 1610 double stars were observed in the declination zone (-30° , $+60^\circ$), with magnitude interval ($6^m.0$, $9^m.0$) and mutual distance of the components comprised within the limits $2''.0$ and $29''.0$. The final accuracy of star positions in Belgrade Catalogue was (Sadjakov and Dačić, 1990):

$$\varepsilon_\alpha \cos \delta = \pm 0''.022 \quad \text{and} \quad \varepsilon_\delta = 0''.32$$

The reduction methods were similar for both catalogues in right ascension and in declination and the same number of fundamental reference stars were used for obtaining the instrumental parameters.

All the double stars observed in Bucharest are included in Belgrade catalogue. For these stars we have computed individual star positions differences $\Delta\alpha \cos \delta$ and $\Delta\delta$ in the form Bucharest–Belgrade, in the system FK5. The differences at a medium epoch of observation have been computed by means of the proper motions comprised in Scott's final list.

We have analysed the gaussian distribution of these differences — figures 1 and 2 — which shows a shifting of the peak.

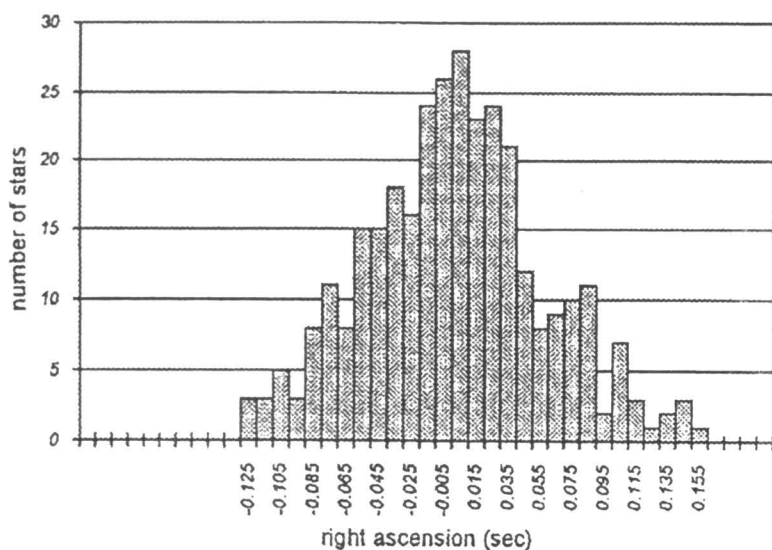


Figure 1. — Distribution of differences in right ascension.

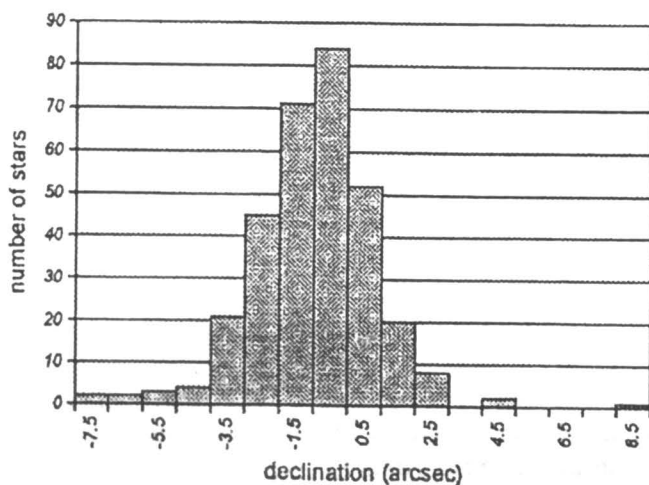


Figure 2. — Distribution of differences in declination.

The distributions are gaussian, so we have performed a harmonic analysis (Sadjakov et al., 1983), using the formulae:

$$\Delta\alpha \cos \delta = A_0 + \sum_{i=1}^3 (A_i \sin i\alpha + B_i \cos i\alpha)$$

$$\Delta\delta = A'_0 + \sum_{i=1}^3 (A'_i \sin i\alpha + B'_i \cos i\alpha)$$

The analysis of $\Delta\alpha \cos \delta$ differences has been performed by the least square method, using (n_e) equations of condition. The values of the coefficients A_i , B_i , their errors (in units of $0''.001$), and the error of the weight unit ϵ_0 , are listed in Table 1. The second variant has been obtained after analysing the residuals and the elimination of erroneous stars.

Table 1

	n_e	ϵ_0	A_0	A_1	B_1	A_2	B_2	A_3	B_3
$\Delta\alpha \cos \delta$	328	.060	-11 ± 4	29 ± 5	-3 ± 6	9 ± 5	5 ± 5	6 ± 5	0 ± 5
$\Delta\alpha \cos \delta$	318	.058	-9 ± 3	22 ± 4	-2 ± 6	12 ± 5	4 ± 5	2 ± 5	0 ± 5

Table 2

	n_e	ϵ_0	A'_0	A'_1	B'_1	A'_2	B'_2	A'_3	B'_3
$\Delta\delta$	321	.24	-97 ± 11	-21 ± 15	-22 ± 19	-60 ± 17	-5 ± 17	-24 ± 17	0 ± 10
$\Delta\delta$	253	.31	-52 ± 7	-5 ± 10	3 ± 13	-23 ± 11	-2 ± 11	-7 ± 11	0 ± 6

The analysis of $\Delta\delta$ differences is shown in Table 2 (coefficients in units of $0''.01$). By means of the coefficients A_1 , B_1 and A'_1 , B'_1 we have computed the amplitudes K , K' and the phases π , π' with the formulae:

$$\Delta\alpha_\alpha \cos \delta = K \sin (\alpha - \pi)$$

$$\Delta\delta_\alpha = K' \sin (\alpha - \pi')$$

In figures 3 and 4 there are the individual differences $\Delta\alpha \cos \delta$ and $\Delta\delta$, the curves representing their general form.

The figures show a good correlation of the results in the analysis of $\Delta\alpha_\alpha$ and a systematic difference of about $-0''.010$; in $\Delta\delta_\alpha$, the values of the coefficients and

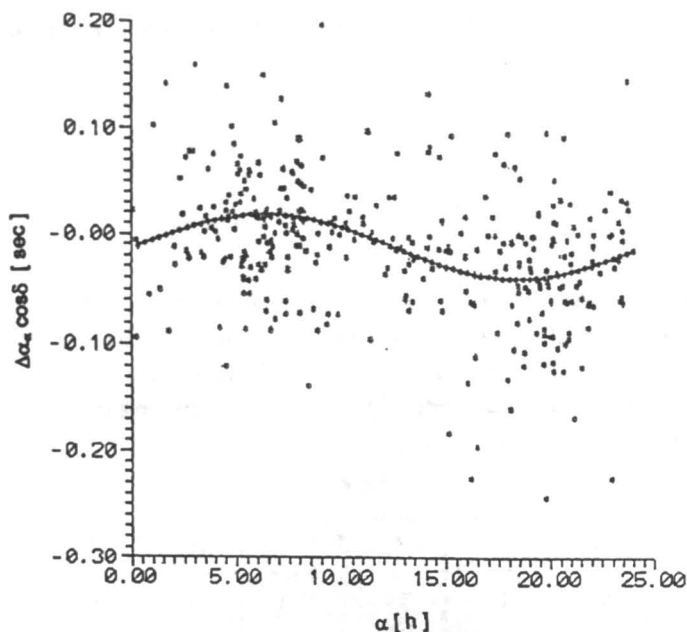


Figure 3. — The correlation $\Delta\alpha_\alpha$.
The graph has the form $\Delta\alpha_\alpha = 0.029 \sin(\alpha - 5^\circ.9)$.

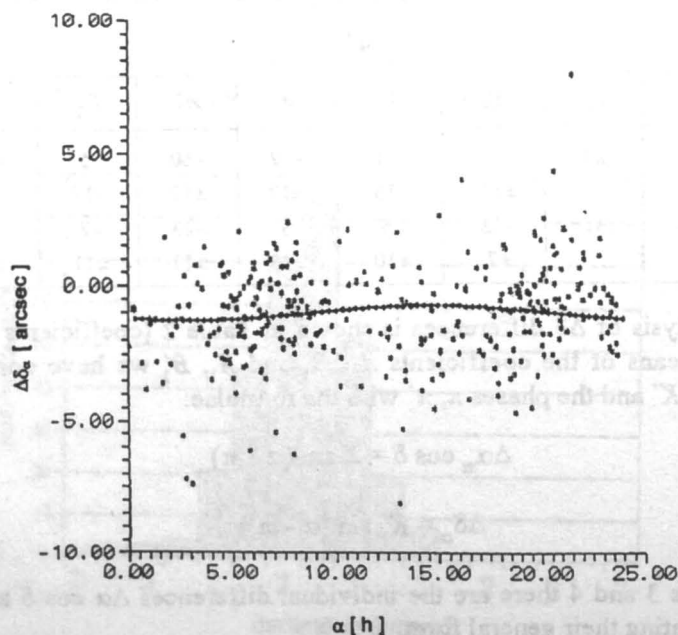


Figure 4. — The correlation $\Delta\delta_\alpha$.
The graph has the form $\Delta\delta_\alpha = 0.30 \sin(\alpha - 133^\circ.7)$.

the errors show a bigger dispersion, the systematic shift is also bigger: $-0''.50$. The dispersion may be due to the identification of the components observed in Bucharest and Belgrade.

The individual analysis of positions, including also the final results of the second Double Stars Catalogue 1985–1989, observed at Bucharest, will provide better information and will permit to detect erroneous observations of the components.

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OBSERVATIONS OF MINOR PLANETS IN 1991–1994 AT THE BUCHAREST ASTRONOMICAL OBSERVATORY

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Abstract. The precise positions of asteroids observed at Bucharest Observatory in 1991–1994 period are presented.

Key words. Astrometry, minor planets.

This paper presents the observations of minor planets, obtained in 1991–1994 with a 380/6000 mm astrograph. Astrophotographic plates with a $2^\circ \times 2^\circ$ field were used. The measurements were carried out by means of an ASCORECORD measuring machine. Both Turner's constants and Schlesinger's dependence methods were used for the computation of the normal coordinates of the object.

In Table 1 the observations of minor planets are presented: the date of the observation and the UT values, the right ascension and the declination, the (O–C) values on the ascension respectively the declination and the observer's initials.

Table 2 presents the reference stars used to compute the values for the 1 Ceres, 2 Pallas, 4 Vesta, 6 Hebe, 7 Iris, 11 Parthenope, 39 Laetitia, 148 Gallia, 389 Industria, 532 Herculina asteroids which are selected in the "Establish the fundamental plane of an inertial reference system" program. Table 2 contains the index number (the same as that in Table 1), the BD number from the catalogue for reference stars, the ascension (s), the declination (') and the dependences.

Table 1

No	Date and UT			$\alpha_{1950.0}$		$\delta_{1950.0}$		$(O-C)_\alpha$	$(O-C)_\delta$	O.C.	
1991											
3 JUNO											
			h	m	s	°	'	''	s	''	
1	9.76072	Sep.	19	00	31.67	-10	27	52.6	-0.062	+0.64	GB
2	9.77457	"	19	00	31.72	10	27	57.7	-0.068	+0.69	GB
3	12.77408	"	19	00	52.74	10	46	23.9	-0.040	+0.45	MB
4	12.77784	"	19	00	52.85	-10	46	28.7	-0.042	+0.58	MB
6 HEBE											
5	9.84659	Sep.	22	21	21.52	-20	48	37.2	-0.039	-0.50	MB
6	9.85491	"	22	21	21.20	20	48	44.3	-0.009	-0.91	MB
7	10.81858	"	22	20	43.99	21	01	20.6	-0.058	-0.11	MB
8	10.82689	"	22	20	43.66	-21	01	26.8	-0.050	+0.27	MB
7 IRIS											
9	26.76347	Sep.	22	33	56.01	+2	29	36.0	-0.079	-0.75	GB
10	26.769	"	22	33	55.74	2	29	33.6	-0.095	-0.82	GB
11	27.78740	"	22	33	16.47	2	23	09.6	-0.061	-0.60	GB
12	27.79295	"	22	33	16.24	2	23	07.1	-0.071	-0.45	GB
13	30.81141	"	22	31	29.80	2	04	13.1	-0.045	-0.50	GB
14	30.81834	"	22	31	29.53	2	04	10.7	-0.065	-0.35	GB
15	7.82727	"	22	28	27.07	1	22	13.8	+0.022	+0.18	MB
16	7.83628	"	22	28	26.88	+1	22	10.5	+0.024	-0.13	MB
39 LAETITIA											
17	9.73786	Sep.	18	24	26.58	-15	02	31.3	+0.023	-0.40	GB
18	9.74895	"	18	24	26.86	15	02	34.6	+0.010	-0.30	GB
19	10.74205	"	18	24	55.98	15	07	31.6	+0.026	+0.17	GB
20	10.75314	"	18	24	56.30	-15	07	35.2	+0.028	-0.11	GB
1992											
532 HERCULINA											
21	24.82830	Jun.	16	11	35.32	-6	32	37.7	-0.023	-0.03	GB
22	24.83869	"	16	11	34.93	-6	32	42.3	-0.015	+0.14	GB

Table 1 (continued)

1993

No	Date and UT			$\alpha_{2000.0}$		$\delta_{2000.0}$		$(O-C)_a$	$(O-C)_\delta$	O.C.	
1 CERES											
23	11.85165	Oct.	h 2	m 16	s 44.40	° +0	' 00	'' 19.7	s -0.068	'' +0.33	GB
24	11.86065	„	2	16	44.00	+0	00	18.8	-0.026	+0.14	GB
2 PALLAS											
25	3.76738	Sep.	21	40	12.08	+6	15	07.5	-0.008	+0.15	GB
26	3.77915	„	21	40	11.56	6	14	58.9	-0.028	+0.23	GB
27	10.75450	„	21	35	37.41	4	47	59.4	-0.041	-0.68	GB
28	10.76835	„	21	35	36.88	4	47	48.9	-0.052	-0.73	GB
29	13.73385	„	21	33	51.86	4	10	15.2	-0.065	-0.65	GB
30	13.74770	„	21	33	51.38	+4	10	5.6	-0.061	-0.28	GB
4 VESTA											
31	7.82329	Sep.	22	32	37.56	-20	00	39.4	+0.058	-0.64	GB
32	7.83022	„	22	32	37.20	20	00	40.7	+0.074	+0.44	GB
33	10.83310	„	22	30	3.79	20	16	32.8	+0.014	-0.30	GB
34	10.84003	„	22	30	3.50	20	16	34.1	+0.080	+0.53	GB
35	13.81280	„	22	27	39.41	20	30	16.1	+0.058	-0.94	GB
36	13.82388	„	22	27	38.82	20	30	18.4	+0.010	-0.42	GB
37	11.79071	„	22	15	20.08	20	56	42.5	+0.044	-0.34	GB
38	11.80040	„	22	15	20.02	20	56	41.0	+0.025	-0.13	GB
39	13.73227	„	22	15	17.97	20	52	13.3	+0.050	-0.63	GB
40	14.74196	„	22	15	17.97	-20	52	11.9	+0.032	-0.71	GB
11 PARTHENOPE											
41	11.80906	Oct.	1	1	47.47	-1	42	23.9	-0.049	-0.13	GB
42	11.82083		1	1	46.83	-1	42	27.6	-0.040	+0.20	GB

Table 1 (continued)

No	Date and UT			$\alpha_{2000.0}$		$\delta_{2000.0}$		$(O-C)_\alpha$		$(O-C)_\delta$		O.C.
80 SAPHO												
43	26.80669	Jul.	18	33	56.84	-7	39	46.2	-0.070	-0.40	GB	
44	26.81708	„	18	33	56.36	-7	39	47.4	-0.070	-0.40	GB	
148 GALLIA												
45	27.76779	Apr.	11	56	10.15	+23	40	22.3	-0.045	+0.15	GB	
46	27.77957	„	11	56	9.91	+23	40	23.0	-0.014	+0.20	GB	
389 INDUSTRIA												
47	11.83087	Oct.	1	32	42.19	+22	53	36.5	+0.037	-0.30	GB	
48	11.84126	„	1	32	41.61	22	53	33.5	+0.032	-0.43	GB	
49	13.75270	„	1	30	57.41	22	44	23.8	-0.004	-0.09	GB	
50	13.75962	„	1	30	57.07	+22	44	21.6	+0.042	-0.27	GB	
409 ASPASIA												
51	26.83474	Jul.	19	43	34.67	-5	13	34.7	-0.070	+0.00	GB	
52	26.84617	„	19	43	34.02	-5	13	35.3	-0.070	+0.00	GB	
1994												
48 DORIS												
53	1.77282	Sep.	20	56	55.65	-10	44	7.1	+0.040	+0.10	GB	
54	1.78667	„	20	56	55.16	-10	44	11.2	+0.040	-0.10	GB	
179 KLYTAEMNESTRA												
55	1.80329	Sep.	21	24	19.03	-2	38	50.4	-0.010	+0.60	GB	
56	1.81714	„	21	24	18.39	-2	38	54.1	-0.010	+0.60	GB	
216 KLEOPATRA												
57	1.83376	Sep.	23	20	37.52	+14	23	46.4	+0.190	+0.00	GB	
58	1.84484	„	23	20	37.13	+14	23	41.7	+0.170	+0.10	GB	

Table 2

No	BD	$(\alpha)_i^*$	$(\delta)_i^{**}$	Dependences	
1-2	-10-4904	22.629	44.35	0.010349	0.010110
	-10-4912	20.512	36.44	.186698	.185447
	-10-4918	49.884	34.91	.043847	.045221
	-10-4930	52.496	34.59	.271566	.272689
	-10-4934	23.503	12.25	.487540	.486534
3-4	-10-4918	49.968	33.99	0.283145	0.282661
	-11-4849	19.500	62.81	.236471	.237827
	-10-4931	14.138	17.74	.200104	.198219
	-11-4869	4.865	55.11	.143360	.144196
	-10-4946	14.344	37.91	.136919	.137099
5-6	-22-5897	50.581	61.78	-0.196843	-0.194913
	-21-6207	12.816	52.11	+0.057731	+0.058379
	-21-6213	48.267	0.59	.454236	.453187
	-21-6214	1.017	14.05	.421114	.419989
	-2215881	31.378	3.91	.263761	.263359
7-8	-22-5897	50.581	51.78	0.187275	0.188238
	-21-6207	12.816	52.11	.281005	.281265
	-21-6213	46.267	0.59	.300064	.299629
	-2215889	52.994	11.33	.053406	.053671
	-21-6226	23.206	25.32	.178250	.177197
9-10	2 4520	23.721	10.93	0.196177	0.196768
	1 4629	36.930	6.09	.051275	.052065
	2 4528	20.439	14.09	.383204	.382658
	1 4634	43.549	55.11	.072350	.072533
	1 4637	24.537	42.35	.296993	.295977
11-12	2 4520	23.721	10.93	0.279808	0.280326
	1 4629	36.930	6.09	.169692	.170267
	2 4528	20.439	14.09	.297213	.296874
	1 4634	43.549	55.11	.104583	.104658
	1 4637	24.537	42.5	.148703	.147875
13-14	1 4623	49.130	43.84	0.220136	0.221198
	1 4626	14.520	4.89	.250585	.251687
	2 4520	23.721	10.93	.172965	.172518
	1 4629	36.930	6.09	.201374	.301047
	1 4631	57.031	9.28	.154939	.153551
15-16	0 4884	47.386	38.82	0.256246	0.258078
	1 4620	48.130	6.67	.472409	.471684
	0 4891	6.204	57.41	.171442	.170693
	0 4892	8.054	46.22	.075199	.075462
	0 4894	7.811	4.71	.024704	.024084

Table 2 (continued)

No	BD	$(\alpha_i)^*$	$(\delta_i)^{**}$	Dependences	
17-18	-15-4965	45.795	55.77	0.157403	0.156880
	-14-5044	16.029	25.32	.274611	.273365
	-15-4972	39.240	19.78	.114593	.115391
	-14-5065	15.320	11.26	.287454	.287138
	-15-4982	25.879	59.14	.165938	.167226
19-20	-15-4965	45.795	55.77	0.098672	0.098071
	-14-5044	16.029	25.32	.155637	.154273
	-15-4972	39.240	19.79	.187286	.188182
	-14-5065	15.320	11.26	.264060	.263700
	-15-4982	25.879	59.14	.294345	.295774
21-22	-6-4377	20.089	4.11	0.213139	0.213789
	-5-4254	15.359	56.64	.239434	.239337
	-7-4230	32.437	53.85	.162543	.163534
	-6-4393	47.032	20.50	.168198	.167909
	-5-4267	21.444	1.00	.216686	.215431
23-23	-0-0336	43.050	35.55	0.152210	0.153101
	-1-0307	13.152	52.85	.128012	.128848
	-0-0338	38.335	40.87	.222278	.222466
	-1-0311	19.190	31.11	.163099	.162848
	-0-0345	41.514	54.02	.334401	.332737
25-26	5 4831	48.182	10.95	0.106449	0.108678
	6 4868	58.932	25.96	.077545	.077900
	6 4878	48.969	13.71	.197881	.196569
	5 4841	5.705	5.66	.269397	.270066
	5 4847	43.702	15.08	.348729	.346786
27-28	3 4575	11.067	47.37	0.168670	0.170501
	4 4701	35.900	5.01	.172569	.171682
	3 4581	47.409	29.97	.205539	.207403
	4 4705	18.515	36.77	.202240	.200457
	3 4588	5.909	59.06	.250982	.249958
29-30	3 4570	38.579	6.23	0.039674	0.040874
	3 4572	8.549	3.37	.050991	.053487
	3 4577	22.048	13.94	.287993	.285402
	3 4578	36.746	10.05	.217563	.218668
	3 4581	47.409	29.97	.403779	.401569
31-32	-20-6439	42.516	4.40	0.098737	0.099416
	-20-6441	36.985	21.80	.244490	.245254
	-20-6442	11.737	8.22	.082042	.081994
	-21-6248	58.268	40.15	.361169	.360881
	-20-6453	33.015	56.72	.213561	.212455

Table 2 (continued)

No	BD	(α_p) [*]	(δ_p) ^{**}	Dependences	
33-34	-21-6221	0.931	40.25	0.211939	0.212382
	-20-6431	20.222	43.69	.167310	.167487
	-21-6241	22.974	13.13	.260651	.260842
	-20-6445	46.430	6.17	.147861	.147865
	-21-6248	58.268	40.15	.212239	.211865
35-36	-21-6221	0.931	40.25	0.349610	0.351567
	-20-6431	20.222	43.69	.214223	.214659
	-21-6230	30.650	51.30	.274254	.274819
	-20-6439	42.516	4.40	.046092	.044422
	-21-6241	22.974	13.13	.115820	.114532
37-38	-21-6175	46.699	25.41	0.004560	0.004746
	-21-6178	38.952	19.72	.096919	.097141
	-22-5867	56.126	18.06	.250922	.250593
	-21-6185	11.976	28.78	.265527	.265809
	-21-6188	9.021	18.42	.382072	.381712
39-40	-21-6175	46.699	25.41	-0.005056	-0.005045
	-21-6178	38.952	19.72	+0.128226	+0.128444
	-22-5867	56.126	18.06	.185342	.185013
	-21-6185	11.976	28.78	.337970	.338266
	-21-6188	9.021	18.42	.353519	.353322
41-42	-2-0136	3.672	8.43	0.160997	0.163225
	-2-0137	5.470	29.44	.315737	.316281
	-2-0148	25.442	28.05	.111079	.111662
	-2-0154	25.756	43.18	.271700	.269537
	-2-0155	55.475	16.35	.140486	.139294
45-46	24 2405	34.277	39.60	0.349011	0.349742
	24 2407	2.883	24.80	.287224	.287466
	25 2439	54.988	9.86	.153410	.153520
	24 2414	16.544	19.21	.169672	.169207
	24 2416	5.939	21.56	.040684	.040064
47-48	22 0236	41.178	34.06	+0.099327	+0.100311
	22 0238	48.684	26.09	-0.011457	-0.010875
	21 0200	6.816	53.63	+0.198115	+0.199065
	22 0246	28.705	25.80	.276933	.275641
	21 0211	0.128	1.76	.437082	.435858
49-50	22 0236	41.178	34.06	0.277585	0.278170
	22 0238	48.684	26.09	.089981	.090235
	21 0200	6.816	53.63	.372071	.372704
	22 0246	28.705	25.80	.043505	.042735
	21 0211	0.128	1.76	.216859	.216155

Received on 10 April, 1995

INTERNATIONAL CONFERENCE OF ARCHAEOASTRONOMY AND ETHNOASTRONOMY

Sibiu, 1–3 September 1995

Between 1 and 3 September the proceedings of the 3rd Annual International Conference of the European Society "Astronomy in Culture" took place at "Lucian Blaga" University in Sibiu. There were present the President of the Society, Dr. Clive Ruggles, a professor at Leicester University, Dr. Wolfhard Schlosser of Bochum University, as well as participants from Bulgaria, Hungary, Italy, Lithuania, Poland, Romania and Spain.

Numerous scientific papers on archaeoastronomy and ethnoastronomy were presented. The day of 3 September was dedicated to the visitation of the Dacian monuments of Sarmizegetusa Regia and Costești-Blidaru. Besides their political, military and economic roles, these two strongholds played also an important religious part. This explains the existence of some evidence of the astronomical knowledge of the Dacian priests.

The conclusions of the participants in the conference, historians, archaeologists, astronomers and other specialists were that these millennial vestiges of a truly civilized society must be placed under UNESCO's protection in order to be repaired and preserved and the superb zone in which they are situated should be declared historical reservation, so that it can be protected from any voluntary or involuntary damages.

Magdalena Stavinschi

THE INTERNATIONAL SYMPOSIUM "SYMBOLIC COMPUTATION AND ITS APPLICATIONS IN FUNDAMENTAL RESEARCH"

Bucharest, 6–8 September 1995

The Astronomical Institute of the Romanian Academy does its best to be in the van of modern computation methodology and technique. We should like to mention only our last acquisition — a SCI Power Challenge M supercomputer, with a speed of 300 M flops, completed with an INDY (8 bytes) graphics station.

This was the context in which the proceedings of the international symposium "Symbolic Computation And Its Applications In Fundamental Researches", organized by the Astronomical Institute and the Cybernetics Institute of the Academy of Sciences in Russia, took place between 6 and 8 September 1995 in Bucharest.

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Besides the Romanian specialists there also participated Russian specialists from the Lomonosov University in Moscow and the Institute of Theoretical Astronomy in St. Petersburg, Pulkovo Astronomical Observatory, Institute of Mechanics, Department of Mathematics, Institute of Applied Mathematics, these latter institutes being all part of the Russian Academy of Sciences.

From among the main topics presented we mention: General Formulation of Some Problems of Celestial Mechanics for Constructing Solutions Using Computers, Implementation of Some Constructive Methods of Theory Perturbations into the Symbolic Computation System "Maple", An Application of Symbolic Computations in Investigations of the Equilibrium Solutions Stability Problem, for the Conservative Hamiltonian Systems, Mission Oriented Mathematical Informational Network in the CIS Countries, REFAL — the Language for Symbolic Transformation, Associative Switching Interconnection Network, On the Generators of the Perturbation Methods, About Valuations of Efficiency of Numerical Methods of Integration of Dynamic Models in Celestial Mechanics Problems, About the Software Package for Fulfilment of Operations with Multiple Fourier Series, The Use of Computer Symbolical Manipulations in Mechanics, Astronomical Data Bank and Applications in Fundamental Researches.

Magdalena Stavinschi

Chr. Sterken, J. Manfroid, *Astronomical Photometry, a Guide*, Astrophysics and Space Science Library, Vol. 175, Kluwer Academic Publishers, Dordrecht, Boston, London, 1992, 272 pp., ISBN 0-7923-1776-9.

The publication of this book constitutes an important editorial event, with great and positive implications in both astronomical research and teaching. Written by two reputed scientists owners of a remarkable experience in observational astrophysics as well as in organizing of coordinated observing campaigns, authors and co-authors of numerous papers in the above mentioned field, this guide represents an authoritative introduction to practical astronomical photometry.

The authors develop a systematic and unitary treatment of the tackled subject having in view the important goal of improving the quality of photometric observations, and "to extract the maximum of information out of these" ones. The book has a logical structure consisting in the following sixteen chapters: (1) *Introduction*, (2) *The telescope*, (3) *Photoelectric photometers*, (4) *The photomultiplier*, (5) *Photometric filters*, (6) *Atmospheric extinction*, (7) *Atmospheric turbulence: scintillation and seeing*, (8) *Color transformation*, (9) *Interstellar extinction*, (10) *Principles of data analysis*, (11) *Homogenization*, (12), *Infrared photometry*, (13) *Charged-coupled devices*, (14) *Photographic photometry*, (15) *The observations*, (16) *Photometric systems*. The manner in which the topics is presented achieves an optimum equilibrium between: complexity and strictness of the exposition, richness of information, accessibility and attractiveness. By its high quality, this textbook makes easier the assimilation of both the fundamentals of the astronomical photometry and the up-to-date topics. We also notice the author's permanent care to provide operational knowledge. The novice readers (especially undergraduate students) will acquire a wide spectrum of essential key words and key problems about the astronomical photometry, allowing for a subsequent understanding of high level papers in the domain. The reference list (Appendix A), containing about 244 titles, offers a solid basis for next studies in this field. The following appendices — *Glossary* (B), *Symbols and notations* (C), and *Index* (D) — uphold the clarity and the rigour of this book.

The above characteristics recommend this remarkable guide to astronomical photometry to all those interested in this field, especially astronomy students, and professional astronomers. Ambitious amateur astronomers will get an important scientific profit, too.

Alexandru Pop

Tipărit: **Semne**

NOTICE TO AUTHORS

ROMANIAN ASTRONOMICAL JOURNAL is a journal which appears twice a year and is open to original contributions in Astronomy and related disciplines. The contributions — in English or French — can be accepted only if they were neither published before nor destined to any other publication.

Manuscripts should be submitted in duplicate; they must be typewritten on white A4-sized paper, onesided and doublespaced (enclosing abstract, references, footnotes and figure captions). The first page should contain: the article's title (brief and informative), author's name and affiliation, followed by an Abstract in English and Key words. The text should be clear and concise (it is recommended not to exceed 10 pages). *The Abstract* will present clearly the principal conclusions of the work, in no more than 10–15 lines.

Chapters and Paragraphs. Papers, except short notes, should be divided into Chapters, numbered by Arabic numerals. Chapters may be divided into Paragraphs denoted by the number of the Chapter and the number of the Paragraph; each Chapter and each Paragraph should have a short descriptive title (e.g. "3.2. Results").

Formulae have to be numbered consecutively in Arabic numerals, too, but included in parenthesis on the right side of the manuscript; all formulae should be written in legible form.

Tables should be numbered consecutively in Arabic numerals; each should be typed on a separate sheet.

Figures and Illustrations should be submitted separately in such a form as to permit reproduction without retouching. Any lettering should be large enough to be legible after the figure has been reduced in size for printing. Captions should be given on a separate sheet and labelled to show which illustration each is accompanying. All the figures should be numbered consecutively in Arabic numerals and referred to in the text as e.g. Fig. 2 or Figs. 2–5. Photographs should be given only if essential and should be enlarged enough to permit clear reproduction.

The Places of tables and figures within the text have to be marked with lead pencil on the left margin of the manuscript.

References are indicated in the text by the author's name and year of publication. They should be listed in alphabetic and chronologic order at the end of the paper, as follows: name and the initials of the author(s), the year of publication, suitable abbreviation of the journal (or title of the book and editing house), its volume and page.

Please pay attention to these recommendations; it will contribute to a faster publishing of the manuscript.

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