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Institutul Astronomic,

Str. Cușitul de Argint 5,

Cod: 75212, București 28,

Telephone: 401-335 6892

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ROMANIAN ASTRONOMICAL JOURNAL

Vol. 8, No. 2, 1998

CONTENTS

<i>IN MEMORIAM</i> - Alexandru Dinescu (1929 – 1998)	61
D. M. ȘURAN, Nedelia Antonia POPESCU. The Model of Two Points Correlation Function for the Determination of the Large Scale Structure of the Universe (II).....	63
Iordana AȘTEFĂNOAEL, G. MAFTEI, The Primordial Gravitational Waves with Cylindrical Symmetries.....	79
A. HORVATH, O. GHEREGA, L. FARKAȘ, Period Changes of BW Vulpeculae : the Influence of Motion in a Binary System.....	89
G. D. CHIȘ. About the System VX Lacertae.....	101
A. ONCICA, The Structures of Quasi-Periodic Patterns in Solar Radio Bursts	105
V. URECHE, Gravitational Redshift in Maneff's Field.	119
V. MIOC, Magda STAVINSCHI, The Schwarzschild-de Sitter Problem (I): Behaviour at Collision and Infinity.....	125
Rodica ROMAN, The Elliptic Restricted Three-Body Problem. Constant Equipotential Surfaces.....	139

MISCELLANEA

National Seminar "The Total Solar Eclipse of August 11, 1999 – Interdisciplinary Approach", Bucharest, October 15, 1998 (Magda Stavinschi).....	147
The First Romanian Astronomer: The Monk Dionysius Exiguus (G. Drăgulin)	148

ALEXANDRU DINESCU (1929 – 1998)

On January 19, 1998, our distinguished colleague Dr. Eng. ALEXANDRU DINESCU died in the town Santé in California (USA) where he was living with his family since 1991.

He was born in 1929, August 19 in the town Slobozia, being the sixth child of a teacher family. He attended primary school in the native town and in the period 1941 – 1948 he was a school boy in a military secondary school in Breaza and Timișoara. But in 1948 military schools were cancelled as a result of an education reform, so he finished the secondary school in the town Călărași (1950).

In autumn of 1950 he took courses at the Faculty of Mathematics of the Bucharest University, but because of financial difficulties he decided to follow the Faculty of Geodesy of the *Military Technical Academy*, becoming a geodetic engineer in 1954. In the same year he began his activity as an assistant at the Geodesic Astronomy chair led by Professor Călin Popovici. After a short "stay" of about two years at *Land Survey Office*, Alexandru Dinescu became a scientific researcher at the *Astronomical Observatory of the Romanian Academy* in 1961 and Prof. Călin Popovici, the chief of the Astrophysical Section at that time, has offered him the leadership of the newly created *Department of Satellite Researches*.

In 1964, Alexandru Dinescu obtained his doctoral degree in geodetic astronomy with a thesis concerned with the geocentric coordinate determination of a station by crossing spatial directions obtained by means of satellite observations.

Dr. Alexandru Dinescu continued his scientific activity dedicated to space research at the Bucharest Astronomical Observatory till 1976, when he decided to leave for the *Institute of Geodesy, Fotogrammetry and Land Organisation*. He retired in 1991.

Throughout his activity, Dr. Alexandru Dinescu demonstrated a great interest in optical observations of artificial satellites and in their use for geodetic purposes. Thus, he elaborated and published many scientific papers in Romanian and foreign journals. His researches were much appreciated and were used as references in the studies of many experts in this domain.

Dr. Alexandru Dinescu participated in many international scientific meetings and symposia concerning spatial geodesy and geophysics, where he presented his original results.

Dr. Alexandru Dinescu took active part in some scientific international programmes such as *FRAROM*, *INTEROBS*, *EURAFRICA*, *ISAGEX*, collaborating with well known researchers as F. Barlier (France), L. Cihowicz and W. Dobaczewska (Poland), A. Zhongolovich and A. Massevitch (Russia), R. Sage (USA), et, to interpret the observational data.

In Romania Dr. Alexandru Dinescu has collaborated with Prof. Călin Popovici, particularly in the application of the *simultaneous circle method* at the construction of the space triangulation by determinations of the spatial directions between tracking stations.

They determined the *first spatial direction in Europe* (Potsdam – Bucharest) in 1964, using this method; the high accuracy of their determination was confirmed, six month later, by K. Arnold et al.

In 1970 Dr. Alexandru Dinescu received a one-year grant at the *Astronomical Council of the Sciences Academy* in Moscow, where he continued his research on geodetic applications of the satellite observations.

Secretary of the Organizing Committee of the *INTERCOSMOS Symposium on Results of Satellite Observations*, held in Bucharest in 1970, he was also the editor of the *Proceedings* of the similar Symposium held in 1974 in Budapest (Hungary).

Dr. Alexandru Dinescu published at the *Technical Publishing House* (1980) a reference book with a suggestive title: *Introduction to Space Geometrical Geodesy*, which was the first book in this field in our country. The book presented approaches to the theoretical and practical problems in this research domain, regarding our planet's shape and dimensions by means of artificial satellites. Romanian research, that of the author included, was also presented.

In the town Santé (California), where he lived in the last time, Dr. Alexandru Dinescu sporadically worked in geodesy and he finished the book *The Visibility of Artificial Satellites* to be published.

Dr. Alexandru Dinescu was a member of the *International Astronomical Union (IAU)* and a member of the *Romanian National Committee for Astronomy*. Also, he was a member of *COSPAR* in the *Group of geodesy and satellite dynamics* and a secretary of the *National Committee COSPAR*.

Enthusiast, sincere, unassuming, he was endowed with an unusual memory and had an astonishing knowledge of history, geography, literature and evidently of astronomy and geodesy.

Dr. Alexandru Dinescu was an excellent colleague, ready any time to help everybody. He was a very pleasant partner of discussion with a vast knowledge in any domain. Many of us are missing him.

THE MODEL OF TWO POINTS CORRELATION FUNCTION FOR THE DETERMINATION OF THE LARGE SCALE STRUCTURE OF THE UNIVERSE (II)

DORU MARIAN ȘURAN, NEDELIA ANTONIA POPESCU

Astronomical Institute of the Romanian Academy

Str. Cuștilor de Argint 5, 75212 Bucharest 28, Romania

Abstract. In this paper we investigate the inhomogeneities on large scale in the Universe. Our results indicate that the signals of the inhomogeneities in different 3D catalogs (galaxies, clusters, superclusters) are correlated, suggesting the presence of phase fluctuations in the long tail of the correlation function $\xi(r)$. The power spectrum $P(k)$, the corresponding transfer-function $T_r(k)$, the density fluctuation spectrum and peculiar velocity field of the LSU, for $H_0 = 100$, $\Omega_0 = 1$, $b = 1$ model, have been determined and plotted. In order to analyse the topological texture of the Universe at small, intermediate and large scale ($r < 1000 \text{ h}^{-1} \text{ Mpc}$) we made different simulations.

Key words: cosmology- density fluctuation spectrum, peculiar velocity field; large scale structure of the Universe- topological texture.

1. INTRODUCTION

In this paper we present a new method for obtaining the values of the fundamental cosmological parameters (H_0, Ω_0, Λ_0), using the *observed correlation function*.

We try to calculate the form of the correlation function $\xi(r)$ at different scales and to decelate different kinds of signals in this function at small ($\leq 15 \text{ h}^{-1} \text{ Mpc}$), intermediate ($15 \text{ h}^{-1} \text{ Mpc} - 100 \text{ h}^{-1} \text{ Mpc}$) and large distances ($\sim 100 \text{ h}^{-1} - 1000 \text{ h}^{-1} \text{ Mpc}$):

$$\xi(r) = \begin{cases} (r/r_0)^\gamma \text{ (linear function) with } \gamma \cong -1.77 & \text{for } r \leq r_G = 15 \text{ h}^{-1} \text{ Mpc,} \\ f_p(r) \text{ (periodic function)} & \text{for } r_G \leq r \leq r_L = 1000 \text{ h}^{-1} \text{ Mpc,} \\ f_G(r) \text{ (Gaussian fluctuations)} & \text{for } r \geq r_L \end{cases}$$

where r_0 is the correlation scale length.

Using power spectrum calculations we are able to determine the precise form of:

- the spectrum of density and mass fluctuations;
- the velocity field;
- the topology

of the local and very distant Universe.

2. THE LINEAR PART OF THE CORRELATION FUNCTION
(SMALL SCALE, $r \leq r_G = 15 \text{ h}^{-1} \text{ Mpc}$)

The estimations are based on our own automatic algorithm for the detection of high and low densities in catalogs of galaxies. This algorithm has been used for the determination of voids, clusters and superclusters in the Shane-Wirtanen 2D- (α, δ) catalog of galaxies (SW).

The estimations of matter distribution at low and high densities are based on a threshold parameter, δ_c . The algorithm is described in Şuran et al. (1991). From the SW catalog, we use in our calculations a zone (Southern Galactic Pole zone, for the galactic latitude $b^{\text{II}} < -70^\circ$) defined by:

$$SP: \begin{cases} 0^{\text{h}} \leq \alpha \leq 2^{\text{h}} \\ -25^\circ \leq \delta \leq 10^\circ \\ m_{\text{ph}} < 19^{\text{m}} \\ N(\text{Abell cluster}) = 183 \end{cases}$$

with the following determined statistics of voids and clusters (Table 1). In Table 1, R represents the richness classification (which is related to the number of members of the cluster brighter than $m_3 + 2$ and placed within a radius $R < R_A = 1.5 \text{ h}^{-1} \text{ Mpc}$ around the cluster center) and D represents the distance group (estimated from m_{10}). We have to mention that m_3 , m_{10} , represent the magnitude of the third brightest galaxy, respective the tenth brightest galaxy of the cluster.

The mean estimated threshold is: $\langle \delta_c \rangle = 44 \text{ galaxies}/(1^\circ \times 1^\circ)$.

Table 1
Statistic of voids and clusters

Our estimations				Abell catalog (N=183)					
Voids		Clusters		Richness class (R)		Distance class (D)		Composite	
δ_c	N_v	δ_c	N_{cl}	type	N_{cl}	type	N_{cl}	type	N_{cl}
15	14	100	16	$R \geq 3$	9	$D \leq 2 \text{ s}$	1	$D \leq 4, R \geq 3$	1
-	-	90	29	-	-	-	-	-	-
20	56	80	48	$R \geq 2$	45	$D \leq 2$	23	$D \leq 4, R \geq 2$	2
24	102	70	90	$R \geq 1$	136	-	-	$D \leq 4, R \geq 1$	10
28	171	60	155	$R \geq 0$	183	$D \leq 2$	183	-	-

We estimate different forms of the correlation function:

$$w(\theta) = w_0 \theta^\Gamma \text{ (angular correlation function),}$$

$$\xi(r) = (r/r_0)^\gamma \text{ (spatial correlation function),}$$

corresponding to high and low densities (Table 2).

Table 2

Forms of the correlation function

Voids					Clusters				
δ_c	angular		spatial		δ_c	angular		spatial	
	w_0	Γ	r_0	γ		w_0	Γ	r_0	γ
15	2.80	0.68	13.00	-0.32	90	2.67	-0.74	22.09	-1.74
20	1.55	-0.47	15.85	-1.47	80	1.88	-0.87	17.56	-1.87
28	1.50	-0.73	16.89	-1.73	60	1.82	-0.78	17.74	-1.78

As calibration data we used:

a) **for high densities** - the two-point correlation functions for:

- galaxies: $\xi_{g,g}(r) \equiv (r/5)^{-1.8}$ (Bahcall 1988)

- clusters: $\xi_{cc}(r) = \begin{cases} (r/24)^{-1.8} (R=1) \\ (r/48)^{-1.8} (R \geq 2) \\ (r/26)^{-1.8} (R \geq 1) \end{cases}$ (Bahcall 1988)

$\xi_{cc}(r) \equiv (r/25)^{-1.6}$ for $R \geq 0, D \leq 4$ (Klypin, Kopylov 1983)

$\xi_{cc}(r) \equiv (r/19)^{-1.8}$ for $R \geq 0$ (Batuski, Burns 1985)

$\xi_{cc}(r) \equiv (r/20)^{-1.8}$ for $R \geq 0$ (Huchra 1988)

- superclusters: $\xi_{sc,sc}(r) \equiv (r/60)^{-1.8}$ (Bahcall 1988)

b) **for low densities** - the correlation functions estimated by us, using the nearest neighbor distribution (Popescu et al. 1997).

Our results are presented in Figure 1, in which the symbols are as follows (from right to left):

- the squares represent the correlation functions for superclusters, clusters and galaxies (Bahcall 1988);
- (x) represents the correlation function for voids (Popescu et al. 1997);
- (+) and diamonds represent the correlation functions calculated by us for Shane-Wirtanen Catalog, with different thresholds $\delta_c = 90, 80, 44, 28, 15$ galaxies/ $(1^0 \times 1^0)$.

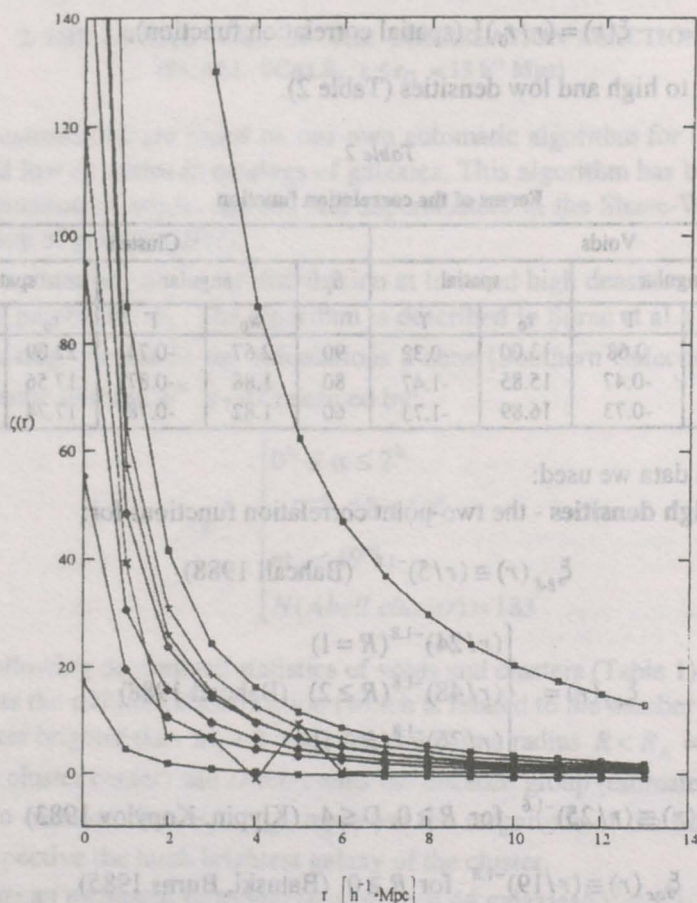


Fig. 1 – Small scale correlation function.

3. PERIODIC SIGNALS IN THE CORRELATION FUNCTION (INTERMEDIATE AND LARGE SCALES, $r_G \leq r \leq r_L = 1000 \text{ h}^{-1} \text{ Mpc}$)

These kinds of signals were derived using different 3D (α, δ, z) catalogs (see also Șurân, Popescu 1993) for:

- galaxies ($r \leq 750 \text{ h}^{-1} \text{ Mpc}$) - *LCRS catalog*;
- clusters ($r \leq 500 \text{ h}^{-1} \text{ Mpc}$) - *The Northern Cone of the Metagalaxy*;
- superclusters ($r \leq 500 \text{ h}^{-1} \text{ Mpc}$) - *The Northern Cone Superclusters*.

3.1. OBSERVATIONAL DATA (3D CATALOGS)

- **galaxies:** *The Las Campanas Deep Redshift Survey in the two Polar Caps (LCRS)*;

$$NP: \begin{cases} 10^h \leq \alpha \leq 15^h \\ -18^0 \leq \delta \leq 0^0 \\ 0.03 \leq z \leq 0.2 \end{cases} \quad SP: \begin{cases} 21^h \leq \alpha \leq 4^h \\ -48^0 \leq \delta \leq -30^0 \\ 0.03 \leq z \leq 0.2 \end{cases}$$

$N = 10^4$ galaxies;

- **clusters:** *The Northern Cone of the Metagalaxy (Kopylov et al. 1987):*

$$NP: \begin{cases} 11^h \leq \alpha \leq 15^h \\ -18^0 \leq \delta \leq -50^0 \\ 0.05 \leq z \leq 0.28 \end{cases}$$

$N = 58 (R \geq 2)$;

- **superclusters:** *The Northern Cone Superclusters (Lebedev, Lebedeva 1988):*

$$NP: \begin{cases} 8^h \leq \alpha \leq 18^h \\ -20^0 \leq \delta \leq -90^0 \\ 0.05 \leq z \leq 0.15 \end{cases}$$

$N = 49 (R \geq 0, D \leq 4)$;

- **with a common area of all these catalogs:**

$$NP: \begin{cases} 11^h \leq \alpha \leq 15^h \\ -18^0 \leq \delta \leq 0^0 \\ 0.05 \leq z \leq 0.15 \end{cases}$$

3.2. THEORETICAL MODELS

We found that the form of the correlation function depends on distance, as follows:

a) **Two points correlation function** (on small distances, as usual):

$$\xi^i(r) = \left(\frac{r}{r_0^i}\right)^\gamma \text{ for } r \leq r_G = 15 \text{ h}^{-1} \text{ Mpc}$$

where $i = \{g \text{ (galaxy), } c \text{ (cluster), } sc \text{ (supercluster)}\}$.

b) **Long tail of the correlation function $f(r)$:**

- on medium distances (phase correlated at higher orders):

$$\xi(r) = f_p(r) \text{ (periodic function) for } r_G \leq r \leq r_L = 1000 \text{ h}^{-1} \text{ Mpc};$$

- on large distances (random, stationary):

$$\xi(r) = f_G(r) \text{ (Gaussian fluctuations) for } r \geq r_L.$$

So, the processed signals in $\xi(r)$ have the form:

$$\xi(r) = \alpha(r)[\xi_S(r) + \xi_N(r)] = \alpha(r)[\xi_L(r) + \xi_P(r) + \xi_G(r) + \xi_N(r)],$$

where: $\xi_S(r)$ - the signals (fluctuations) in $\xi(r)$;

$\xi_N(r)$ - the background noise;

$\xi_L(r), \xi_P(r), \xi_G(r)$ - the linear, respective periodic, gaussian form of the correlation function;

$\alpha(r)$ - the normalization parameter (including sampling, smoothing and applied window functions).

As tests for periodic fluctuations in $\xi(r)$ we used two different methods for revealing periodical events in $\xi(r)$:

- FOURIER analysis;
- Statistical WHITTAKER-ROBINSON method (Şuran 1991).

3.3. RESULTS

The correlation function and the corresponding periodograms reveal (see Figure 1, 2, Şuran, Popescu 1993):

1) Correlated fluctuations in the long tail of $\xi(r)$ (see also Table 1, Şuran, Popescu 1993):

- sharp maxima at $\Delta r \sim 130 \text{ h}^{-1} \text{ Mpc}$ scale (fundamental L mode, i.e. "large" mode);
- other maxima ($\Delta r > 200 \text{ h}^{-1} \text{ Mpc}$) (possible harmonics of L mode);
- a second type of maxima ($\Delta r \sim 25-30 \text{ h}^{-1} \text{ Mpc}$) (only for LCRS catalog, fundamental G mode, i.e. "galactic" mode);
- a lot of maxima ($\Delta r < 100 \text{ h}^{-1} \text{ Mpc}$) (possible harmonics of G mode or a new class of M modes, i.e. "medium scale" modes).

Δr represents the distance between two peaks in the periodograms of the correlation function.

2) The presence of a biasing effect: $\xi_{g,cl} \equiv \xi_{g,cl} \equiv 5-7$.

3) The presence of a smoothing effect: $\xi_{g,10\text{Mpc}} \equiv \xi_{g,30\text{Mpc}}$.

4) A very unsmoothed shape of the function

$$\Xi = \frac{\partial^2 \xi(r)}{\partial r^2}$$

on large scale $100 \text{ h}^{-1} \text{ Mpc} < r < 1000 \text{ h}^{-1} \text{ Mpc}$.

The comparisons with *Deep Pencil Beam Survey* (BEKS 1991) and *Transverse Survey* (Vettolani et al. 1992) show us the presence of:

- coherent 1D signals;
- periodicity with main period $P \cong 130 \text{ h}^{-1} \text{ Mpc}$;
- splitting of maxima (possible superposition of G + M + L signals).

So, our results fit very well BEKS periodograms (see Figure 2a,b, Şuran, Popescu 1993), indicating the existence of phase correlation on large scale and/or the presence of superlarge scale features.

The results obtained by us, using the linear correlation function and the corresponding 1D power spectrum $P_1(k)$ (Şuran 1991; Şuran, Popescu 1993, 1997), are confirmed by Doroshkevich et al. (1996). They realised a detailed analysis on 3D galactic distribution in *LCRS* and confirmed the existence of many periodicities in the same range as our predictions.

4. POWER SPECTRUM CALCULATIONS (INTERMEDIATE, LARGE AND SUPERLARGE SCALES)

4.1. POWER SPECTRUM FORM

From the correlation function we can obtain the *power spectrum* $P(k)$ (Şuran, Popescu 1993):

$$P(k) = \begin{cases} Ak^n & k \leq k_L \\ F_P(k) & k_L \leq k \leq k_G \\ F_G(k) & k \geq k_G \end{cases}$$

where $k = \frac{2\pi}{\lambda}$ denotes the comoving wavenumber and the indices have the same meaning as in the form of $\xi(r)$.

We have to notice that one determines the correlation function $\xi(r)$ in the real space, and the power spectrum $P(k)$ in the Fourier space. Accordingly, one defines the transition scale to homogeneity in the real space (the correlation transition scale), r_t , and in the Fourier space (the spectral transition scale), λ_t , using the correlation function and spectrum. The correlation transition scale is determined by the minimum of the second derivative of the correlation function,

$$\Xi = \frac{\partial^2 \xi(r)}{\partial r^2}, \text{ while the spectral scale is determined by the transition from negative}$$

to a positive spectral index.

For F_P , we have used the form:

$$F_P(k) = A \frac{(k/k_s)^{n_s}}{1 + (k/k_l)^{n_l - n_s}},$$

where A - the amplitude of the spectrum; k_s, k_l - the wavenumbers of the small, respective large transition scale; n_s, n_l - the spectral indices at small, respective large wavelengths.

The derived *transfer function*:

$$T_f(k) = \frac{P_f(k)}{P_i(k)},$$

which relates the initial fluctuations, from CBR-Cosmic Background Radiation, to the final ones (at large scale), was obtained (see Şuran, Popescu 1993).

As numerical results, for the best fit between the observational data for LCRS catalog of galaxies and the synthetic spectrum, we obtained (see Figure 2b, Şuran, Popescu 1993):

$$A_g = 0.818, \lambda_s = 10 \text{ h}^{-1} \text{ Mpc}, \lambda_l = 135 \text{ h}^{-1} \text{ Mpc}, n_s = -1.2, n_l = 1.$$

We compare these results with other estimations:

- Einasto 1992 - for clusters of galaxies:

$$A_c = 6.5, \lambda_s = 10 \text{ h}^{-1} \text{ Mpc}, \lambda_l = 135 \text{ h}^{-1} \text{ Mpc}, n_s = -2.1, n_l = 1.$$

- Einasto 1992 - for matter distribution:

$$A_m = 0.75, \lambda_s = 1 \text{ h}^{-1} \text{ Mpc}, \lambda_l = 135 \text{ h}^{-1} \text{ Mpc}, n_s = -1.2, n_l = -1.8;$$

- Einasto 1993 - for transition scale to homogeneity:

$$\lambda_l = 175 \text{ h}^{-1} \text{ Mpc}, n_s = -1.6, n_l = 1.$$

- Peacock 1992 - for COBE results:

$$\lambda_l = 190 \text{ h}^{-1} \text{ Mpc}, n_s = -1.5, n_l = 1.$$

4.2. POWER SPECTRUM CALCULATIONS

For our calculations we use the following input parameters:

- the cosmological parameters (H_0, Ω_0, Λ_0);
- the scale parameter (a_0);
- the *observed correlation function* $\xi^{\text{obs}}(r)$ (described in the last section).

1) For $z \equiv 0$:

- The 3D *power spectrum* and the correlation function $\xi(r)$ form a Fourier transform pair, being directly related:

$$\xi(r) = 4\pi \int_0^{\infty} P(k) \frac{\sin(kr)}{kr} k^2 dk,$$

$$P(k) = \frac{1}{2\pi^2} \int_0^{\infty} \xi(r) \frac{\sin(kr)}{kr} r^2 dr,$$

where $k = \frac{2\pi}{a_0 r} = \frac{2\pi}{\lambda}$ (we consider $a_0 = 1$).

- The *density fluctuations* are considered to be:

$$\langle |\delta_k|^2 \rangle = \left(\frac{2\pi}{L} \right)^3 P(k)$$

where L is the size of the cubic box of volume V under study.

- The *transfer function* has the form:

$$T_f^2(k) = \frac{|\delta_k|^2}{Ak^n}$$

($n = 1$ for Harrison - Zeldovich spectrum).

- The *peculiar velocity* filtered over a spherical region of scale r_a is:

$$\langle v^2(r_a) \rangle = \frac{2}{\pi^2} \int_0^{\infty} P(k) V_s(kr_a) dk,$$

where the Fourier transform of the window function can be as follows:

$$V_s(kr_a) = \begin{cases} \frac{\sin(kr_a)}{kr_a} \\ e^{-k^2 r_a^2} \end{cases}$$

- The *rms mass fluctuations* on a scale r_b has the form:

$$\langle (\delta M/M)^2 \rangle_{r_b}^{tot} \sigma^2(r_b) = \int_0^{\infty} \frac{k^2}{2\pi^2} |\delta_k|^2 W_s^2(kr_b) dk,$$

where the Fourier transform of the spherical window function is:

$$W_s(kr_b) = \left[\frac{3\sin(kr_b) - 3kr_b \cos(kr_b)}{(kr_b)^3} \right],$$

- *Bias parameter* b :

$$\xi_{\kappa} = b^2 \xi_m,$$

where $\xi_{\kappa} = \xi_{\kappa, \kappa}$ represents the galactic correlation function and ξ_m represents the matter correlation function.

The *bias parameter* b , usually defined as the inverse of the *rms* mass fluctuation on a scale $r = 8 \text{ h}^{-1} \text{ Mpc}$:

$$b^2 = \frac{1}{\sigma^2(r_8)} = 2\pi^2 \int_0^\infty k^2 P(k) W_s^2(kr_8) dk,$$

can be used to parametrize the primordial perturbations amplitude.

In our numerical calculations we used the following formulas:

$$V^2(r_m) = \left[\frac{a_0 \Omega_0^{1/2}}{\pi} \right] H_0^2 \sum_m k_m^2 |P(k_m)| \Delta r V_s(k_m r_m),$$

$$\sigma^2(r_h) = 2a_0 \sum_m k_m^4 |P(k_m)| \Delta r W_s^2(k_m r_h),$$

$$\text{with } k_m = \frac{2\pi}{a_0 r_m} \text{ and } P(k_m) = \frac{1}{2\pi^2} \int_{r_{\min}}^{r_{\max}} \frac{\xi_{\text{obs}}}{b} \cdot \frac{\sin(k_m r)}{k_m r} r^2 dr,$$

II) For $z \rightarrow z_H$ at the horizon limit ($r \geq 1000 \text{ h}^{-1} \text{ Mpc}$):

The *temperature fluctuations* can be determined as follows:

$$(\delta T/T)_{\text{rms}}^2 = \frac{1}{4\pi} \sum_{l=2}^{l_{\max}} (2l+1) C_l e^{-\frac{\theta_l^2}{2}},$$

with

$$C_l^2 = 32\pi a_0^2 \Omega_0^{1.54} \left[\frac{H_0}{2c} \right]^4 \sum_m |P(k_m)| \Delta r j_l^2(k_m r_l),$$

where $j_l(k_m r_l)$ represents the spherical Bessel function of order l and

$\theta_l \cong 0.425 \times 7'' \cong 0.05129 \text{ rad}$ is the "smoothing angle" due to the beamsize of COBE (this corresponds to the temperature fluctuations on an angular scale of $\sim 7''$).

The calculated quadrupole component is:

$$(\delta T/T)_Q^2 = \frac{5}{4\pi} C_2 e^{-2\theta_l^2}.$$

4.3. RESULTS

The set of input parameters used for our cosmological calculations is:

a) $n_s = -1.6$, $n_f = 2.4$, $\lambda_s = 10 \text{ h}^{-1} \text{ Mpc}$, $\lambda_f = 175 \text{ h}^{-1} \text{ Mpc}$, for the power spectrum form (LSU - COBE fit);

b) $A = 24^4$, $\epsilon_0 = 3 \times 10^{-5}$, $\theta_s = 0.05129$, the constants for the transfer function normalization (COBE);

c) $a_0 = 1$, $b = \{1; 1.5\}$, for the *bias* parameters;

d) $r_a = 50 \text{ h}^{-1} \text{ Mpc}$, $r_b = 10 \text{ h}^{-1} \text{ Mpc}$, $r_c = 25 \text{ h}^{-1} \text{ Mpc}$, different scales used for the calculation of the peculiar velocity field, for the *rms* mass fluctuations and temperature fluctuations;

e) $l_{\text{max}} = 15$, maximum number of harmonics used in the temperature fluctuations calculus, in order to obtain the comparison with COBE results.

The results obtained for $H_0 = \{50; 100\}$, $b = \{1; 1.5\}$, $\Omega = \{0.1; 0.2; 0.5; 0.7; 1\}$ concerning the temperature fluctuations $(\delta T/T)_Q$, $(\delta T/T)_{\text{rms}}$, peculiar velocity $v(50)$, *rms* mass fluctuations $\sigma(10)$ are presented in Table 3.

Table 3

The results of our cosmological calculations

H_0	Ω	$(\delta T/T)_Q$	$(\delta T/T)_{\text{rms}}$	$V(50)$	$\sigma(10)$
100: $b = 1$	0.1	8.35 E-7	1.90 E-6	74	2.36
	0.2	1.42 E-6	3.30 E-6	112	1.91
	0.5	2.88 E-6	6.71 E-6	194	1.45
	0.7	3.73 E-6	8.69 E-6	237	1.31
	1.0	4.91 E-6	1.14 E-5	294	1.18
50: $b = 1$	0.1	8.35 E-7	1.94 E-6	93	2.36
	0.2	1.42 E-6	3.31 E-6	141	1.91
	0.5	2.88 E-6	6.71 E-6	244	1.45
	0.7	3.73 E-6	8.69 E-6	299	1.31
	1.0	4.91 E-6	1.14 E-5	371	1.18
50: $b = 1.5$	0.1	6.82 E-7	1.58 E-6	76	1.92
	0.2	1.16 E-6	2.70 E-6	115	1.56
	0.5	2.35 E-6	5.48 E-6	199	1.19
	0.7	3.05 E-6	7.10 E-6	244	1.07
	1.0	4.01 E-6	9.34 E-6	303	0.97

In Figures 2–5 we present the power spectrum, respective the transfer function, mass fluctuation spectrum and peculiar velocity field for galaxies, clusters and superclusters (for the model $H_0 = 100$, $\Omega_0 = 1$, $b = 1$). The symbols in these figures are as follows: (x) - galaxies, (+) - clusters, squares - superclusters. In Figures 2 and 3 the continuous lines represent the calculated power spectrum, respective the calculated transfer function.

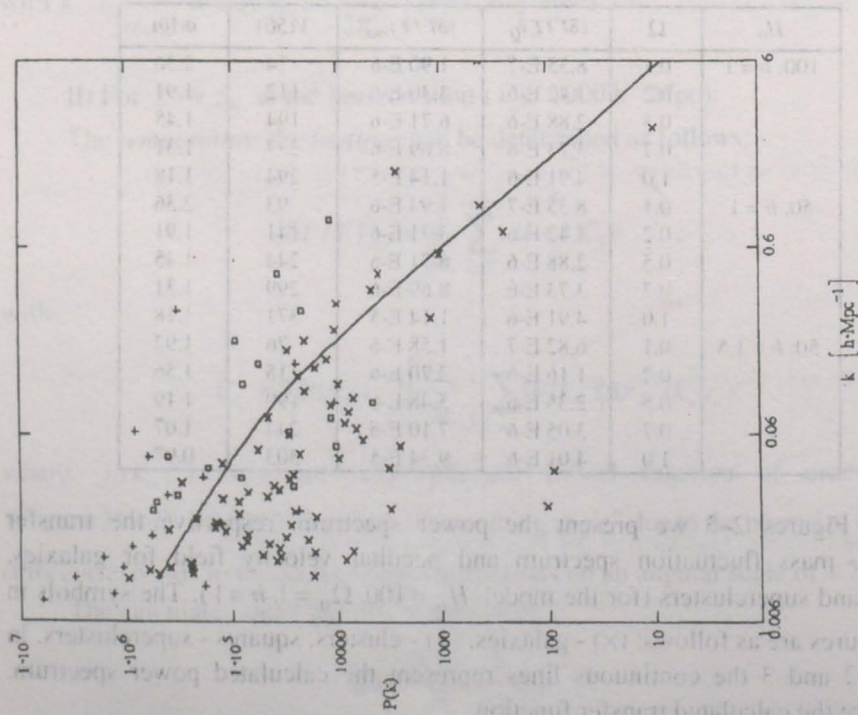


Fig. 2 – The calculated power spectrum.

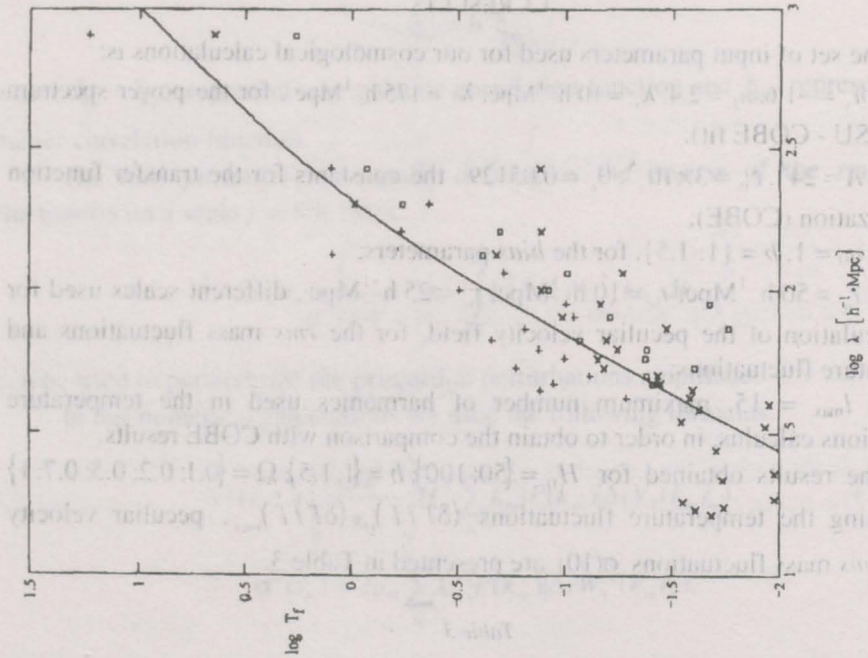


Fig. 3 – The calculated transfer function.

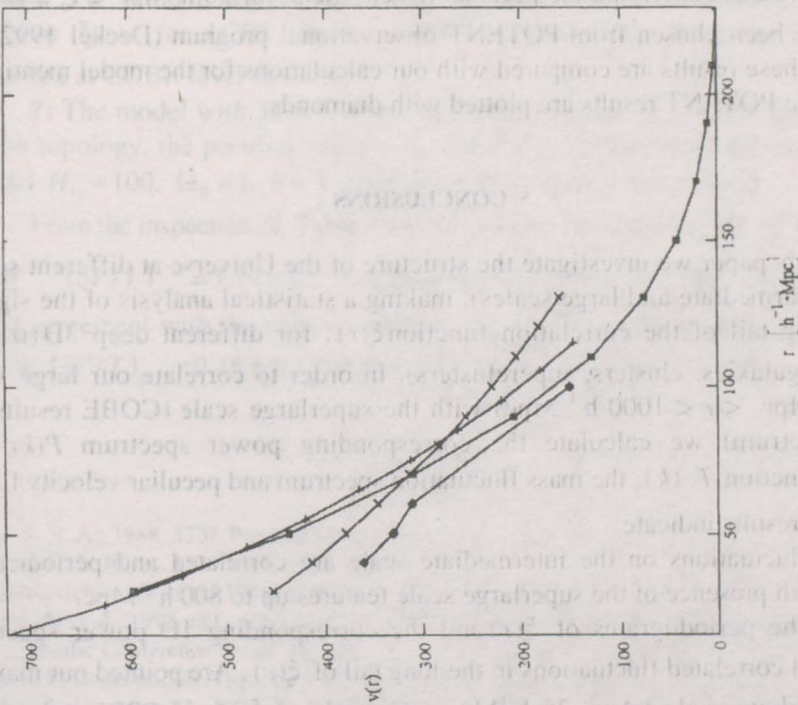


Fig. 5 – The peculiar velocity field.

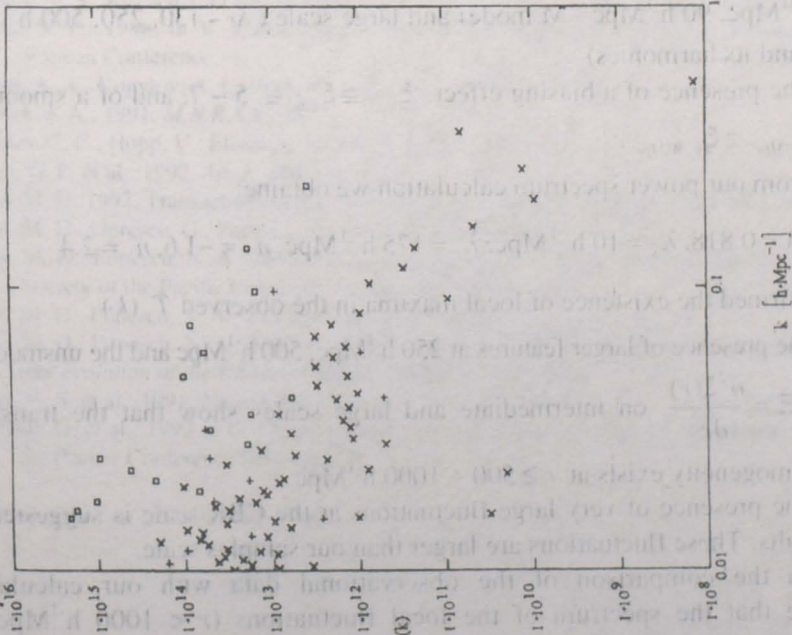


Fig. 4 – The density fluctuation spectrum.

The results concerning the peculiar velocity field, for $20 \text{ h}^{-1} \text{Mpc} \leq r_i \leq 100 \text{ h}^{-1} \text{Mpc}$, have been chosen from POTENT observational program (Deckel 1992). In Figure 5, these results are compared with our calculations for the model mentioned before. The POTENT results are plotted with diamonds.

5. CONCLUSIONS

In this paper we investigate the structure of the Universe at different scales (small, intermediate and large scales), making a statistical analysis of the signals in the long tail of the correlation function $\xi(r)$, for different deep 3D(α, δ, z) catalogs (galaxies, clusters, superclusters). In order to correlate our large scale ($100 \text{ h}^{-1} \text{Mpc} < r < 1000 \text{ h}^{-1} \text{Mpc}$) with the superlarge scale (COBE results for CBR spectrum) we calculate the corresponding power spectrum $P(k)$, the transfer function $T_i(k)$, the mass fluctuation spectrum and peculiar velocity field.

Our results indicate:

- 1) Fluctuations on the intermediate scale are correlated and periodical in phases, with presence of the superlarge scale features up to $800 \text{ h}^{-1} \text{Mpc}$.
- 2) The periodograms of $\xi(r)$ and the corresponding 1D power spectrum $P(k)$ reveal correlated fluctuations in the long tail of $\xi(r)$. Are pointed out maxima at intermediate scale ($\Delta r = 25 \text{ h}^{-1} \text{Mpc}$ - G mode, only for LCRS catalog, and $\Delta r = 60 \text{ h}^{-1} \text{Mpc}$, $90 \text{ h}^{-1} \text{Mpc}$ - M mode) and large scale ($\Delta r \sim 130, 250, 500 \text{ h}^{-1} \text{Mpc}$ - L mode and its harmonics).
- 3) The presence of a biasing effect, $\xi_{v,rel} \equiv \xi_{v,sc} \equiv 5 - 7$, and of a smoothing effect $\xi_{v,10\text{Mpc}} \equiv \xi_{v,30\text{Mpc}}$.
- 4) From our power spectrum calculation we obtain:

$$A = 0.818, \lambda_s = 10 \text{ h}^{-1} \text{Mpc}, \lambda_l = 175 \text{ h}^{-1} \text{Mpc}, n_s = -1.6, n_l = 2.4$$

and is confirmed the existence of local maxima in the observed $T_i(k)$.

- 5) The presence of larger features at $250 \text{ h}^{-1} \text{Mpc}$, $500 \text{ h}^{-1} \text{Mpc}$ and the unsmoothed shape of $\Xi = \frac{\partial^2 \xi(r)}{\partial r^2}$ on intermediate and large scales show that the transition scale to homogeneity exists at $r \geq 500 + 1000 \text{ h}^{-1} \text{Mpc}$.

- 6) The presence of very large fluctuations at the CBR scale is suggested by COBE results. These fluctuations are larger than our samples scale.

From the comparison of the observational data with our calculation we observe that the spectrum of the local fluctuations ($r < 1000 \text{ h}^{-1} \text{Mpc}$) is coupled with the fluctuations spectrum at CBR level (the level of observable

horizon $r \equiv 6000 \text{ h}^{-1} \text{Mpc}$). This thing suggests that the observable correlation function $\xi(r)$ (for *LCRS* catalog) contains informations about the fluctuations observed at COBE level ($k \ll 1$).

7) The model with $\Omega = 1$ is the most plausible model from the point of view of the topology, the peculiar velocity field and the normalization factors. Also, the model $H_0 = 100$, $\Omega_0 = 1$, $b = 1$ simulates very well the velocity field.

From the inspection of Table 3 one can see that the values of $(\delta T/T)_Q = 0.491 \times 10^{-5}$, $(\delta T/T)_{rms} = 1.14 \times 10^{-5}$, obtained for $H_0 = \{50; 100\}$, $\Omega_0 = 1$, $b = 1$, are in good agreement with the values obtained from COBE measurements (Smoot et al. 1992): $(\delta T/T)_Q = (0.48 \pm 0.15) \times 10^{-5}$, $(\delta T/T)_{rms} = (1.1 \pm 0.2) \times 10^{-5}$.

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THE PRIMORDIAL GRAVITATIONAL WAVES WITH CYLINDRICAL SYMMETRIES

IODANA AȘTEFĂNOAEI, GHEORGHE MAFTEI

Department of Theoretical Physics, "Al. I. Cuza" University,
Bd. Copou 11, 6600 Iași, Romania

E-mail: iordana@uaic.ro

Abstract. We choose the cosmological model based on the Einstein-Rosen solution and we analyze the behavior of gravitational waves in the primordial stages of Universe ($t \rightarrow \infty$). The space-time defined by the Einstein-Rosen metric travels such as cylindrical pp gravitational waves when these ones are generated by the pure radiation field. Their evolution in the Einstein-Rosen space-time is given by the function $p(z, t)$. The components of Riemann R_{abcd} and Ricci R_{ab} tensors, and the scalar curvature invariant R are calculated in the null-tetradic base $\{e_a\} = \{e_a\} = \{m, \bar{m}, l, k\}$ using the Newman-Penrose formalism. From the Einstein field equation we obtain that the function ϕ^2 describes the potential of gravitational field. The space-time travels such as pp gravitational waves when $k_{a,b} = 0$. In this case the function $b(z, t)$ becomes zero, while the potential function becomes $\Phi^2 = 1/(t^2\chi_0)$. This function is dependent on the time variable and on the cosmological constant χ_0 . In the primordial stages of the Einstein-Rosen Universe ($t \rightarrow \infty$), the potential function ϕ^2 becomes zero, thus the relation (25) defines the wave equation. In this case there are gravitational perturbations travelling as gravitational waves. The Einstein-Rosen metric describes the Petrov-type gravitational waves.

Key words: gravitational wave, Einstein-Rosen solution, Newman-Penrose formalism, energy-momentum pseudotensor, gravitational field.

1. INTRODUCTION

In the last time there have been made important discoveries that give us a crucial relation between Einstein's theory of gravitation and other branches of physics. They have important applications in relativistic astrophysics.

Rosen achieved a deep investigation upon the cylindrical gravitational waves considered by him and Einstein. The cylindrical gravitational waves or Einstein-Rosen waves are the solutions of the field equations of general relativity and occupied a special position in the theory of gravitational radiation.

Rosen found an unexpected result. The pseudotensor that measures the density of gravitational energy and momentum in the cylindrical wave is everywhere zero.

It does not follow that the energy of the gravitational wave is zero. Rather, this energy is undefined because the wave is unbounded. Many of the otherwise apparently paradoxical properties of this cylindrical wave can be understood by taking into account the analogy between gravitational waves and electromagnetic waves, as well as the special demands of the equivalence principle which rules out a special role for any particular frame of reference.

The curvature tensor of Riemann R_{ijkl} does not vanish in the region occupied by the wave; nor do the invariants formed from the components of this curvature tensor vanish. The anomalous behavior of Einstein-Rosen waves is completely consistent with the equivalence principle of the general relativity.

Weber and Wheeler (1957) analysed the influence of cylindrical gravitational wave upon at rest before the arrival of a pulse.

Feinstein (1988) analysed the gravitational waves in the primordial stages of the Expanding Universe. He chose the cosmological model based on the Einstein-Rosen metric because this one describes the isotropic space-time. The Einstein-Rosen space-times provide an effective tool for studying the early as well as the late stages in development of the Universe. A possible consequence of the initially chaotic structure of the early Universe is the emergence of a cosmological gravitational radiation background.

During the early stages in the history of the Universe these gravitational waves appear as large amplitude shear and rotational inhomogeneities. Using the asymptotic expansion in time, Feinstein found that the primordial chaotic behavior transfers itself into a radiative gravitational field of Petrov type N , as $t \rightarrow \infty$. On the other hand, to complete the physical picture he studied the high-frequency behavior of the field variables in the Newman-Penrose formalism. He showed that in this case the space-time is of general Petrov type I and then calculated the tetrad components of the stress-energy tensor induced by the "disappeared" radiation by means of Newman-Penrose equations, too.

Because Feinstein (1988) supposed that in the primordial stages of the Universe there were strong gravitational perturbations travelling as gravitational waves we choose the cosmological model based on the Einstein-Rosen solution and we analyse the behavior of the gravitational waves in the primordial stages of the Universe, $t \rightarrow \infty$.

We use the Newman-Penrose formalism. We calculate in the null-tetradic base $\{\bar{e}_a\} = \{e_a\} = \{m, \bar{m}, l, k\}$ the spin coefficients, the directional derivatives, the commutation relations, the Bianchi identities, and we obtain the Newman-Penrose equations, the tetradic components of Ricci and Weyl tensors.

The gravitational waves are solutions of the system

$$R_{mn} = 0, R_{ambn} k^n = 0, k^n k_n = 0,$$

but the conditions

$$R_{mn} = 0, k_{a;n} = 0, k^n k_n = 0,$$

lead us to the solutions of planar gravitational waves.

The requirement that $k_{a;n} = 0, k^n k_n = 0$, generalizes the property of planar waves in flat space that they possess parallel rays with which a null vector k^n is associated. These waves are therefore called plane-fronted waves with parallel rays (pp gravitational waves) (Stephani, 1985).

In this paper we propose to analyse the pp gravitational waves of Einstein-Rosen Universe in the primordial stages using the Newman-Penrose formalism.

The space-time described by the Einstein-Rosen metric has cylindrical symmetries. The function $p(z, t)$ describes the gravitational waves in the primordial stages of the Einstein-Rosen Universe.

2. THE TETRADIC COMPONENTS OF RICCI AND WEYL TENSORS IN EINSTEIN - ROSEN UNIVERSE

In the cosmological model based on Einstein-Rosen solution (Kramer and Stephani, 1980), the metric is given by

$$ds^2 = t \cdot (e^p \cdot dx^2 + e^{-p} \cdot dy^2) + e^b \cdot (dz^2 - dt^2), \quad (1)$$

where $b = b(z, t)$ and $p = p(z, t)$ are real functions of the variables z and t .

We choose the coordinates $x^i = \{x, y, z, ct\}$, $c = 1$ (c = the light velocity) and consider the null-tetradic base $\{\bar{e}_a\} = \{e_a\} = \{m, \bar{m}, l, k\}$ with the properties:

$$\begin{cases} m \cdot m = \bar{m} \cdot \bar{m} = l \cdot l = k \cdot k = 0, m \cdot \bar{m} = -l \cdot k = 1 \\ m \cdot l = m \cdot k = \bar{m} \cdot l = \bar{m} \cdot k = 0 \end{cases} \quad (2)$$

The bar means complex conjugacy, while the dot means the scalar product defined by

$$e_a \cdot e_b = g_{ab} = \eta_{ij} e_a^i \cdot e_b^j \quad \eta_{ij} = \text{diag}(1, 1, 1, -1) \quad (3)$$

We found the dual base $\{\omega^a\} = \{\bar{m}, m, -k, -l\}$ and the metric can be written:

$$ds^2 = g_{ab} \omega^a \omega^b = 2 \cdot (\omega^1 \omega^2 - \omega^3 \omega^4). \quad (4)$$

The components of the dual base are:

$$\omega^1 = \omega^1_i dx^i = \bar{m}_i dx^i = \frac{1}{\sqrt{2}} \cdot (t^{1/2} \cdot e^{p/2} \cdot dx + i \cdot t^{1/2} \cdot e^{-p/2} \cdot dy), \quad (5)$$

$$\omega^2 = \omega^2_i dx^i = m_i dx^i = \frac{1}{\sqrt{2}} \cdot (t^{1/2} \cdot e^{p/2} \cdot dx - i \cdot t^{1/2} \cdot e^{-p/2} \cdot dy), \quad (6)$$

$$\omega^3 = \omega^3_i dx^i = -k_i dx^i = \frac{1}{\sqrt{2}} \cdot (e^{b/2} \cdot dt - e^{b/2} \cdot dz), \quad (7)$$

$$\omega^4 = \omega^4_i dx^i = -l_i dx^i = \frac{1}{\sqrt{2}} \cdot (e^{b/2} \cdot dt + e^{b/2} \cdot dz), \quad (8)$$

g_{ab} is the metric tensor in the null-tetradic base $\{e_a\}$.

$$g_{ab} = g^{ab} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

The contravariant components of the null-tetradic base $\{e^a_i\}$ are

$$e^i_1 = m^i = \left(\frac{1}{t^{1/2} \cdot e^{p/2} \cdot \sqrt{2}}, \frac{1}{i \cdot t^{1/2} \cdot e^{-p/2} \cdot \sqrt{2}}, 0, 0 \right) \quad (9)$$

$$e^i_2 = \bar{m}^i = \left(\frac{1}{t^{1/2} \cdot e^{p/2} \cdot \sqrt{2}}, -\frac{1}{i \cdot t^{1/2} \cdot e^{-p/2} \cdot \sqrt{2}}, 0, 0 \right) \quad (10)$$

$$e^i_3 = k^i = \left(0, 0, -\frac{1}{e^{b/2} \cdot \sqrt{2}}, \frac{1}{e^{b/2} \cdot \sqrt{2}} \right) \quad (11)$$

$$e^i_4 = l^i = \left(0, 0, \frac{1}{e^{b/2} \cdot \sqrt{2}}, \frac{1}{e^{b/2} \cdot \sqrt{2}} \right) \quad (12)$$

Using the first Cartan equation,

$$d\omega^a = g^{ad} \cdot \Gamma_{dbc} \cdot \omega^b \wedge \omega^c \quad (13)$$

we found the following nonzero spin coefficients:

$$\rho \equiv -\Gamma_{142} = -\frac{t^{-1}}{2 \cdot e^{b/2} \sqrt{2}}, \quad (14)$$

$$\sigma \equiv -\Gamma_{141} = -\frac{\dot{p} + p'}{2 \cdot e^{b/2} \sqrt{2}} \quad (15)$$

$$\mu \equiv \Gamma_{231} = \frac{t^{-1}}{2 \cdot e^{b/2} \sqrt{2}}, \quad (16)$$

$$\lambda \equiv \Gamma_{232} = \frac{\dot{p} - p'}{2 \cdot e^{b/2} \sqrt{2}}, \quad (17)$$

$$\varepsilon \equiv -(\Gamma_{344} - \Gamma_{214}) = \frac{\dot{b} + b'}{4 \cdot e^{b/2} \sqrt{2}}, \quad (18)$$

$$\gamma \equiv \frac{1}{2} \cdot (\Gamma_{433} - \Gamma_{123}) = \frac{b' - \dot{b}}{4 \cdot e^{b/2} \sqrt{2}}, \quad (19)$$

“.” means $\frac{\partial}{\partial T}$ and “'” means $\frac{\partial}{\partial R}$.

Directional derivatives are given by the following expressions:

$$\delta \equiv m' \cdot \frac{\partial}{\partial x^i} = \left(\frac{1}{t^{1/2} \cdot e^{p/2} \cdot \sqrt{2}} \cdot \frac{\partial}{\partial x} + \frac{1}{i \cdot t^{1/2} \cdot e^{-p/2} \cdot \sqrt{2}} \cdot \frac{\partial}{\partial y} \right), \quad (20)$$

$$\bar{\delta} \equiv \bar{m}' \cdot \frac{\partial}{\partial x^i} = \left(\frac{1}{t^{1/2} \cdot e^{p/2} \cdot \sqrt{2}} \cdot \frac{\partial}{\partial x} - \frac{1}{i \cdot t^{1/2} \cdot e^{-p/2} \cdot \sqrt{2}} \cdot \frac{\partial}{\partial y} \right), \quad (21)$$

$$\Delta \equiv l' \cdot \frac{\partial}{\partial x^i} = \left(-\frac{1}{e^{b/2} \cdot \sqrt{2}} \cdot \frac{\partial}{\partial z} + \frac{1}{e^{b/2} \cdot \sqrt{2}} \cdot \frac{\partial}{\partial t} \right), \quad (22)$$

$$D \equiv k' \cdot \frac{\partial}{\partial x^i} = \left(\frac{1}{e^{b/2} \cdot \sqrt{2}} \cdot \frac{\partial}{\partial z} + \frac{1}{e^{b/2} \cdot \sqrt{2}} \cdot \frac{\partial}{\partial t} \right). \quad (23)$$

The commutation relations are verified and from the Newman-Penrose equations we obtain the following tetradic components for the Ricci ($\Phi_{\alpha\beta}$, $\alpha, \beta = 0, 1, 2$) and Weyl (Ψ_{α} , $\alpha = 0, 1, 2, 3, 4$) tensors:

$$\Phi_{00} \equiv \frac{1}{2} \cdot R_{44} = \frac{1}{8 \cdot e^b} \left[\frac{2}{t} \cdot (b' + \dot{b}) + \frac{1}{t^2} - (p' + \dot{p})^2 \right] \quad (24)$$

$$\Phi_{22} = \bar{\Phi}_{22} \equiv \frac{1}{2} \cdot R_{33} = \frac{1}{8 \cdot e^b} \left[\frac{1}{t^2} + \frac{2}{t} \cdot (\dot{b} - b') - (\dot{p} - p')^2 \right] \quad (25)$$

$$\Phi_{11} = \bar{\Phi}_{11} \equiv \frac{1}{4} \cdot (R_{43} + R_{12}) = \frac{1}{16 \cdot e^h} \left[2 \cdot (b'' - \ddot{b}) + \frac{1}{t^2} + p'^2 - \dot{p}^2 \right] \quad (26)$$

$$\Phi_{20} = \bar{\Phi}_{02} \equiv \frac{1}{2} \cdot R_{11} = \frac{1}{4 \cdot e^h} \cdot (\ddot{p} - p'' + \frac{\dot{p}}{t}) \quad (27)$$

$$R = \frac{1}{2 \cdot e^h} \cdot \left[\dot{p}^2 - p'^2 - \frac{1}{t^2} + 2 \cdot (\ddot{b} - b'') \right] \quad (28)$$

$$\Psi_1 \equiv C_{4341} = C_{abcd} \cdot k^a \cdot l^b \cdot k^c \cdot m^d = 0, \quad (29)$$

defines a longitudinal wave in the l direction,

$$\Psi_3 \equiv C_{3432} = C_{abcd} \cdot l^a \cdot k^b \cdot l^c \cdot \bar{m}^d = 0, \quad (30)$$

defines a longitudinal wave in the k direction

$$\begin{aligned} \Psi_0 &= C_{4141} = C_{abcd} \cdot k^a \cdot m^b \cdot k^c \cdot m^d = \\ &= \frac{1}{4 \cdot e^h} \left[(\dot{b} - b') \cdot (p' + \dot{p}) - \ddot{p} - p'' - 2\dot{p}' - \frac{1}{t} \cdot (p' + \dot{p}) \right] \end{aligned} \quad (31)$$

defines a transversal wave in the l direction.

$$\begin{aligned} \Psi_4 &= C_{3232} = C_{abcd} \cdot k^a \cdot \bar{m}^b \cdot k^c \cdot \bar{m}^d = \\ &= \frac{1}{4 \cdot e^h} \cdot \left[\frac{1}{t} \cdot (\dot{p} - p') + (b' - \dot{b}) \cdot (\dot{p} - p') + \ddot{p} + p'' - 2 \cdot \dot{p}' \right] \end{aligned} \quad (32)$$

defines a transversal wave in the k direction.

$$\Psi_2 \equiv \frac{1}{2} \cdot C_{abcd} \cdot k^a \cdot l^b \cdot (k^c \cdot l^d - m^c \cdot \bar{m}^d) = \frac{1}{16 \cdot e^h} \cdot \left[\dot{p}^2 - p'^2 - \frac{1}{t^2} + 2 \cdot (b'' - \ddot{b}) \right] \quad (33)$$

represents the "Coulomb" component.

3. THE PURE RADIATION FIELD IN EINSTEIN - ROSEN UNIVERSE

The energy-momentum tensor for the radiation field is given by (see Kramer and Stephani, 1980)

$$T_{ij} = \Phi^2 \cdot k_i \cdot k_j, \quad k_i \cdot k^i = 0 \quad (34)$$

The null associated vector k^n ($k^n k_n = 0$), has the components,

$$k_i = \frac{1}{\sqrt{2}} (0, 0, e^h, -e^h), \quad k^i = \frac{1}{\sqrt{2}} (0, 0, e^{-h}, -e^{-h}). \quad (35)$$

The nonzero components of energy-momentum tensor are:

$$T_{33}=T_{44}=\frac{1}{2}\cdot e^{2b}\cdot\Phi^2, \quad T_{34}=-\frac{1}{2}\cdot e^{2b}\cdot\Phi^2, \quad (36)$$

If f is a scalar function, the wave equation can be written as:

$$g^{ij}\cdot\nabla_i\nabla_j f = g^{ij}f_{,ij} = g^{ij}f_{;ij} = 0 \quad (37)$$

Because

$$g^{ij} = \text{diag}\left(\frac{e^{-p}}{t}, \frac{e^p}{t}, e^{-b}, -e^{-b}\right)$$

the expression (36) becomes

$$\frac{e^{-p}}{t}\cdot f_{,\xi\xi} + \frac{e^p}{t}\cdot f_{,\eta\eta} + e^{-b}f_{,RR} - e^{-b}f_{,TT} = 0. \quad (38)$$

For the function $b(R, T)$ the equation (36) reads:

$$b'' - \ddot{b} = 0 \quad (39)$$

and for $p(R, T)$

$$p'' - \ddot{p} = 0.$$

Using the Einstein field equations,

$$R_{ij} - \frac{1}{2}g_{ij}R = \chi_0 T_{ij}$$

we obtain for Φ^2 the following expression:

$$\Phi^2 = \frac{4}{\sqrt{t}\cdot\chi_0}\cdot e^{-2b}\cdot\left[\frac{1}{2\sqrt{t}}\cdot(\dot{b}-b') - \frac{1}{4t\sqrt{t}} - \dot{b}^2\cdot\sqrt{t}\right] \quad (40)$$

If the null vector k^n verifies the relation $k_{a,b} = 0$, then the space-time describes the pp gravitational wave. We obtain that $b = \text{const.}$ and we choose $b = 0$. The metric becomes:

$$ds^2 = t(e^p dx^2 + e^{-p} dy^2) + dz^2 - dt^2, \quad (41)$$

$$\Phi^2 = \frac{1}{t^2\cdot\chi_0}. \quad (42)$$

In this context the components of the Ricci tensor and the curvature scalar invariant are zero. The components of the Weyl tensor are:

$$\Psi_2 = \Psi_0 = 0 \quad (43)$$

$$\Psi_4 = \frac{1}{4} \cdot \left(\frac{1}{t} \cdot (\dot{p} - p') + \ddot{p} + p'' - 2\dot{p}' \right).$$

The metric (41) describes the gravitational field of the algebraical type N (Petrov N type), that is, the space-time (41) with (42) and (43) propagates in the form of pp gravitational wave.

4. THE ENERGETIC CONDITIONS

If u^i is the 4-velocity, then the first energetic condition

$$T_{ij} \cdot u^i \cdot u^j \geq 0 \quad ; \quad u^i = (0, 0, 0, \frac{1}{e^c}); \quad u_i = (0, 0, 0, -e^c)$$

$$q^i \cdot q_i \leq 0$$

requests that

$$\Phi^2 \geq 0. \quad (44)$$

The second energetic condition is $q^a q_a \leq 0$, $q^a \equiv T_b^a u^b$.

In our case, this condition reduces to $q^a q_a = 0$.

Therefore, the energetic conditions are satisfied.

The nonzero contravariant components of the energy-momentum tensor are:

$$T^{33} = T^{44} = T^{34} = \frac{1}{2} \cdot \Phi^2. \quad (45)$$

The conservative law $T^{ij}_{;j} = 0$ is equivalent to the result:

$$\frac{1}{2} \cdot \left(\frac{\partial \Phi^2}{\partial R} + \frac{\partial \Phi^2}{\partial T} \right) = 0. \quad (46)$$

From the Einstein field equations we obtain the following equation

$$\ddot{p} - p'' = \mp \frac{1}{t^2} \quad \text{or} \quad \ddot{p} + p'' = \mp \chi_0 \Phi^2 \quad (47)$$

In the primordial stages of the Universe (41) ($t \rightarrow \infty$) the function $p(z, t)$ defines a pp gravitational wave (in agreement with (38) and (42)).

5. CONCLUSIONS

In the primordial stages of a model based on the Einstein-Rosen solution there exist gravitational perturbations that propagate in the form of a cylindrical gravitational wave.

The function $p(z, t)$ describes a pp cylindrical gravitational wave in the case in which the Einstein-Rosen Universe is radiative. In this case, the function $b(z, t)$ is zero and the Einstein-Rosen metric is of type Petrov N.

The energetic conditions request that the function Φ^2 verifies the inequality $\Phi^2 \geq 0$.

The metric (1) has a cylindrical symmetry.

When the "potential function" Φ^2 verifies the condition $\Phi^2 \geq 0$, $p(z, t)$ defines a gravitational wave that propagates in the z direction in the space-time described by the metric (1).

The Einstein field equations and the relation $k_{a,b} = 0$ show that the space-time described by the Einstein-Rosen metric travels as pp gravitational waves if the function $b(z, t)$ becomes zero.

Only the component Ψ_4 is nonzero. The space-time (1) is curved, but the metric is defined by

$$ds^2 = t(e^p dx^2 + e^{-p} dy^2) + dz^2 - dr^2.$$

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PERIOD CHANGES OF BW VULPECULAE : THE INFLUENCE OF MOTION IN A BINARY SYSTEM

A. HORVATH, O. GHEREGA, L. FARKAS

*Astronomical Institute of Romanian Academy,
Timișoara Astronomical Observatory
P-ța Axente Sever 1, 1900 Timișoara, Romania*

Abstract. This paper presents the reduced photoelectric observational data on the variable star BW Vulpeculae obtained at Timișoara Astronomical Observatory. Using the moments of the observed minima and others from references, we computed a quadratic ephemeris describing a secular evolution. Removing the evolutionary term, from the *O-C* diagram, it still remains a periodicity. Assuming this due to the light-time effect in a binary system, these residuals allowed us to compute the orbital elements. Finally we concluded that the period variation is described by the superposition of two separate physical effects: the evolution of the pulsating star and the orbital motion in a binary system.

Key words: β Cephei stars - BW Vulpeculae - photometry - period variation - light time effect.

1. INTRODUCTION

The variable star BW Vulpeculae (HD 199140 = HR 8007 = DB + 27° 3909) is a β Cephei (β Canis Majoris) type pulsating star. The spectral type of this variable is B2III and the mean magnitude is 6^m.55 in V light. The period is about 5 hours and is increasing by several seconds per century. The period variation of this variable is not established yet: some authors, Valtier (1976), Odell (1984) and Jiang (1985) provided a secular variation, about 2 seconds/century, others like Tunca (1978), Chapellier (1985) and Chapellier & Garrido (1990), are talking about rapid period variations, followed by intervals of constant periods. Pigulski and Boratyn (1992) speak about period changes due to the motion in a binary system.

BW Vulpeculae emphasizes the largest amplitude in light and radial velocity variation among β Cephei stars. The amplitude of light variation is about 0^m.2 in V and 1^m.2 in UV.

An important characteristic of the light curve is the so-called stillstand phase, which begins about 0.^d05 before the moment of maximum and lasts about 0.^d03.

1. OBSERVATIONS

The photoelectric measurements were carried out at Timișoara Astronomical Observatory between July 1995 and September 1997. The observatory is placed in the town, at $45^{\circ}44'14''.6$ N latitude, $1^{\text{h}}44^{\text{m}}55^{\text{s}}$ E longitude and 88 m altitude.

The main instrument is a Cassegrain telescope with aluminized primary mirror ($D = 300$ mm, $F = 1860$ mm). The photoelectric device is endowed with an unrefrigerated EMI 9502 B type photomultiplier operating DC mode. The output signal is recorded using a pen-writer. One can use the following filters: R: RG1 / 2 mm, V: OG4 / 2 mm, B: BG12 / 2 mm + GG3 / 1 mm, U: UG1 / 2.5 mm.

Observations were made using B filter, $\lambda = 445$ nm. The observing scheme was C, V, C, V, ... and sky background was measured after each star measurement. The comparison star was HD 198527.

3. DATA REDUCTION

The first step was the determination of atmospheric extinction coefficient for each night applying Hardie's method (Hardie, 1962). After the reduction for the atmospheric extinction effect we obtained the light curves. Observations from the thirteen nights contained well bordered minima, which permitted the computation of the moments of minima using the Kwee-Van Woerden method (Kwee, van Woerden, 1956). The results of these computations are shown in Table 1.

Table 1

Heliocentric times of photometric minima for BW Vul
observed at Timișoara Astronomical Observatory

No.	Date	HJD _{min}	σ	Observer	Filter
1	9 July 1995	2449908.4734	0.0011	Horvath	B
2	10 July 1995	2449909.4771	0.0021	"	B
3	13 Oct. 1995	2450004.3681	0.0014	"	B
4	14 Oct. 1995	2450005.3763	0.0009	"	B
5	11 Nov. 1996	2450399.2208	0.0013	"	B
6	12 Nov. 1996	2450400.2273	0.0023	Gherega	B
7	13 Nov. 1996	2450401.2291	0.0013	Farkas	B
8	11 Sept. 1997	2450703.4021	0.0005	Gherega	B
9	17 Sept. 1997	2450709.4362	0.0015	"	B
10	19 Sept. 1997	2450711.4438	0.0013	Horvath	B
11	20 Sept. 1997	2450712.4520	0.0029	Farkas	B
12	21 Sept. 1997	2450713.4576	0.0032	Gherega	B
13	28 Sept. 1997	2450720.4924	0.0016	"	B

4. THE PERIOD VARIATIONS OF BW VUL

All available observed minima are contained in Table 2. A number of 40 observational data in the table are spectroscopic. Before using these points together with photometric data, we had to take into account the phase lag between radial velocity curve and light curve minima. Fortunately, we could find 7 common minima of the two data sets, and so, we were able to compute the mean phase lag. Assuming that the phase lag is constant, we determined this as $0^d.0533$, so we increased the heliocentric times of radial velocity curves minima with this value.

Because the observational data cover a great number of cycles (more than a hundred thousand) and due to the gaps between the series of measurements, we had some problems in correct cycle count. One of the authors (A. H.) suggested a simple method to get over this. One can compute the uncorrected cycle number simply as:

$$E_0 = \text{int}[(HJD_{\min} - HJD0_{\min})/P] \quad (1)$$

and the corresponding value of $(O-C)$. Representing $(O-C)$ versus E_0 we got a kind of $(O-C)$ diagram but portions of it are shifted up or down, every break of the diagram represents an error in cycle count. To get the correct number of cycles E , one have to add or subtract one unit (or in some cases more units) to from the preliminary number of cycles, E_0 . Generally the number added or subtracted is one, only in the vicinity of gaps this number is two or more.

Using the ephemeris:

$$HJD_{\min} = 2428802^d.5487 + 0^d.201038 \cdot E \quad (2)$$

given by Sterken (1993), we computed the $(O-C)$ residuals. These values are plotted against the number of cycles E in Figure 1.

Using the least squares method, the plots were first fitted with a parabola, which permitted us to compute the quadratic ephemeris which would fit best the observations:

$$HJD_{\min} = 2428802^d.5668 + 0^d.20102919 \cdot E + 0^d.775 \cdot 10^{-12} \cdot E^2 \pm 0^d.0018 \pm 0^d.00000008 \pm 0^d.007 \cdot 10^{-12} \quad (3)$$

Figure 1 represents this parabolic fit of the plots, while Figure 2 shows the residuals after subtracting the parabola from the observations. These residuals showed a periodicity which led us to conclude that an orbital motion of the pulsator, in a binary system, caused the staggers of the observed minima, due to the light-time effect.

Table 2

All available heliocentric times of minima for BW Vul. The reference codes (ref. column) stand for:
 S – Sterken (1993), P – Pigulski (1993), O – Odell (1994), T – present paper

HJD _{min}	E	Ref.	HJD _{min}	E	Ref.
2400000+			2400000+		
25444.1893	-16706	P	34879.9633	30231	P
26548.0192	-11215	P	34880.9708	30236	P
27365.6090	-7148	P	34881.9742	30241	P
27988.9915	-4047	P	34882.9801	30246	P
28359.8860	-2202	P	34883.9872	30251	P
28392.8559	-2038	P	34912.9340	30395	P
28745.8634	-282	P	34937.8636	30519	P
28763.7561	-193	P	34965.8075	30658	P
28802.5487	0	S	34966.8101	30663	P
29130.8359	1633	P	34975.8612	30708	P
29138.8767	1673	P	35003.8039	30847	P
29141.8878	1688	P	35009.8308	30877	P
29477.8161	3359	P	35015.8634	30907	P
29478.8145	3364	P	36025.4587	35929	S
33504.8836	23391	P	36032.2989	35963	S
33853.2860	25124	S	36032.5001	35964	S
33859.3140	25154	S	36052.3953	36063	S
33868.3583	25199	S	36052.4015	36063	S
33870.3629	25209	S	36067.4654	36138	S
33892.2760	25318	S	36067.4812	36138	S
33896.9050	25341	S	37134.7710	41447	S
33896.9080	25341	P	37136.7860	41457	S
33898.3109	25348	S	37971.0760	45607	S
33914.1950	25427	S	37973.0830	45617	S
33916.1992	25437	S	37974.0790	45622	S
33940.3238	25557	S	38726.9630	49367	S
34202.8729	26863	P	39828.4440	54846	S
34207.8989	26888	P	39829.4380	54851	S
34221.7810	26957	S	40432.7529	57852	S
34232.8260	27012	S	40433.7644	57857	S
34232.8309	27012	P	40449.8493	57937	S
34233.8307	27017	S	40474.7814	58061	S
34233.8356	27017	P	40475.7834	58066	S
34233.8361	27017	S	40841.6606	59886	S
34245.8959	27077	S	41537.8627	63349	S
34247.7090	27086	S	41538.8734	63354	P
34250.7210	27101	S	41868.7771	64995	S
34251.7330	27106	S	41871.7903	65010	S
34257.7594	27136	S	41891.6908	65109	S
34257.7608	27136	S	42350.6706	67392	S
34262.7909	27161	S	42356.6959	67422	S
34263.7860	27166	S	42678.7693	69024	S
34602.9367	28853	P	43002.8477	70636	S

Table 2 (continued)

HJD _{min}	E	Ref.	HJD _{min}	E	Ref.
2400000+			2400000+		
43012.7015	70685	S	45160.8248	81370	S
43060.5422	70923	S	45161.0287	81371	S
43371.3540	72469	S	45161.8263	81375	S
43382.2095	72523	S	45162.8311	81380	S
43385.2208	72538	S	45163.4346	81383	S
43422.2177	72722	S	45163.6368	81384	S
43724.7769	74227	S	45163.8378	81385	S
43729.8094	74252	S	45164.0422	81386	S
43745.2839	74329	S	45164.6396	81389	S
43750.3144	74354	S	45164.8413	81390	S
43753.3296	74369	S	45165.8468	81395	S
43755.7400	74381	S	45166.4526	81398	S
43764.1856	74423	S	45166.8529	81400	S
43770.2155	74453	S	45167.4581	81403	S
43776.2470	74483	S	45167.8567	81405	S
44047.6495	75833	P	45168.4641	81408	S
44047.6563	75833	S	45170.4732	81418	S
44065.7440	75923	S	45172.2792	81427	S
44065.9433	75924	P	45173.4879	81433	S
44081.8291	76003	S	45175.2999	81442	S
44117.8103	76182	S	45175.4954	81443	S
44142.7499	76306	S	45176.3009	81447	S
44143.5440	76310	S	45176.5040	81448	S
44144.7554	76316	S	45177.3159	81452	S
44146.5653	76325	S	45177.5105	81453	S
44149.5738	76340	S	45186.7566	81499	S
44153.5972	76360	S	45188.7678	81509	S
44155.6081	76370	S	45189.7716	81514	S
44157.6223	76380	S	45190.7765	81519	S
44159.6495	76390	S	45192.7878	81529	S
44163.4495	76409	S	45194.7977	81539	S
44401.6890	77594	S	45195.8070	81544	S
44402.6920	77599	S	45196.6072	81548	S
44403.6990	77604	S	45196.8070	81549	S
44475.6662	77962	S	45197.4142	81552	S
44482.7091	77997	S	45197.6110	81553	S
44483.7124	78002	S	45198.4190	81557	S
44484.7178	78007	S	45199.4240	81562	S
44500.6040	78086	S	45217.7182	81653	S
44504.6206	78106	S	45223.7483	81683	S
44505.6279	78111	S	45224.5543	81687	S
44723.3520	79194	S	45224.7533	81688	S
44872.7330	79937	S	45225.5585	81692	S
45157.8103	81355	S	45225.7597	81693	S
45158.8126	81360	S	45226.3678	81696	S
45160.6222	81369	S	45226.7642	81698	S

Table 2 (continued)

HJD _{min} 2400000+	E	Ref.	HJD _{min} 2400000+	E	Ref.
45226.9702	81699	S	47371.5011	92366	S
45227.3669	81701	S	47424.3724	92629	S
45227.7668	81703	S	47425.3806	92634	S
45228.3756	81706	S	47426.3865	92639	S
45228.5741	81707	S	47432.4180	92669	S
45229.3771	81711	S	47695.3799	93977	S
45230.3832	81716	S	47704.8300	94024	O
45230.5825	81717	S	47729.3576	94146	S
45230.7838	81718	S	47769.5647	94346	S
45231.5857	81722	S	47776.4017	94380	S
45231.7885	81723	S	47791.4807	94455	S
45232.1904	81725	S	47798.1145	94488	S
45232.3917	81726	S	47799.9207	94497	S
45232.7951	81728	S	47801.1304	94503	S
45233.1977	81730	S	47826.6639	94630	O
45233.3970	81731	S	47833.7020	94665	O
45233.8022	81733	S	47840.7373	94700	O
45234.8048	81738	S	47848.1737	94737	S
45235.4127	81741	S	48062.8885	95805	O
45235.6083	81742	S	48084.8152	95914	O
45235.8118	81743	S	48133.4573	96156	S
45236.4156	81746	S	48174.2720	96359	S
45236.8148	81748	S	48193.7676	96456	O
45237.0235	81749	S	48403.8630	97501	O
45237.2204	81750	S	48421.7744	97590	O
45237.6166	81752	S	48497.5467	97967	S
45240.4365	81766	S	48505.1900	98005	S
45552.0523	83316	S	48506.3898	98011	S
45604.7253	83578	S	48507.3993	98016	S
45604.9223	83579	P	48508.4064	98021	S
45605.5292	83582	S	49662.6000	103762	O
45605.7307	83583	S	49663.6025	103767	O
45658.0005	83843	S	49908.4734	104985	T
45902.0698	85057	S	49909.4771	104990	T
45919.7658	85145	S	50004.3679	105462	T
46721.9203	89135	P	50005.3763	105467	T
46722.9265	89140	P	50399.2208	107426	T
46724.9380	89150	P	50400.2273	107431	T
47000.3711	90520	S	50401.2291	107436	T
47115.1714	91091	S	50703.4021	108939	T
47117.1814	91101	S	50709.4362	108969	T
47328.4729	92152	S	50711.4438	108979	T
47331.4893	92167	S	50712.4520	108984	T
47366.4717	92341	S	50713.4576	108989	T
47368.4829	92351	S	50720.4924	109024	T
47369.4868	92356	S			

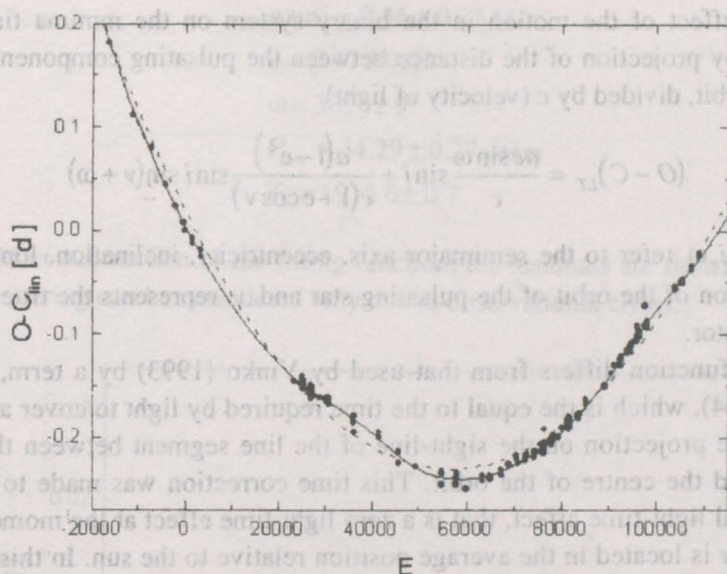


Fig. 1 – $(O - C)_{lin}$ diagram for BW Vul, obtained using linear ephemeris. Dashed line represents the quadratic fit, while the full line is superposition of quadratic term and the light-time effect.

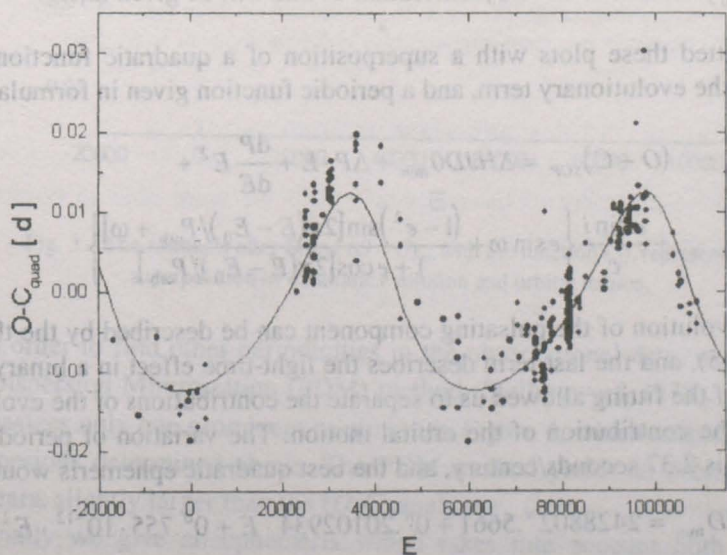


Fig. 2 – $(O - C)_{quad}$ diagram for BW Vul, obtained from $(O - C)_{lin}$ by subtracting the quadratic ephemeris given in eq. (2). The continuous line represents the best fit due to light-time effect.

The effect of the motion in the binary system on the minima timings, is described by projection of the distance between the pulsating component and the centre of orbit, divided by c (velocity of light).

$$(O - C)_{LT} = \frac{ae \sin \omega}{c} \sin i + \frac{a(1 - e^2)}{c(1 + e \cos v)} \sin i \sin(v + \omega) \quad (4)$$

where a, e, i, ω refer to the semimajor axis, eccentricity, inclination, longitude of the periastron of the orbit of the pulsating star and v represents the true anomaly of the pulsator.

This function differs from that used by Vinko (1993) by a term, the first one in eq. (4), which is the equal to the time required by light to cover a distance equal to the projection on the sight-line of the line segment between the centre of mass and the centre of the orbit. This time correction was made to obtain a symmetrical light-time effect, that is a zero light-time effect at the moment when the pulsator is located in the average position relative to the sun. In this case the $(O - C)$ residuals due to the light-time effect have an equally probable distribution around the zero value.

We assumed that replacing true anomaly by mean anomaly in eq. (4) yields no significant differences in the light-time effect, so we shall consider

$$\frac{2\pi}{P_{orb}}(E - E_0) \text{ instead of } v. \text{ The justification of this will be given later.}$$

We fitted these plots with a superposition of a quadratic function, corresponding to the evolutionary term, and a periodic function given in formula (4):

$$(O - C)_{SUP} = \Delta HJD0_{min} + \Delta P \cdot E + \frac{dP}{dE} E^2 + \frac{a \sin i}{c} \left\{ e \sin \omega + \frac{(1 - e^2) \sin[2\pi(E - E_0)/P_{orb} + \omega]}{1 + e \cos[2\pi(E - E_0)/P_{orb}]} \right\} \quad (5)$$

The evolution of the pulsating component can be described by the third term in formula (5), and the last term describes the light-time effect in a binary system. The result of the fitting allowed us to separate the contributions of the evolutionary term from the contribution of the orbital motion. The variation of period for BW Vulpeculae is 2.37 seconds/century, and the best quadratic ephemeris would be:

$$HJD_{min} = 2428802.^d.5661 + 0.^d.20102934 \cdot E + 0.^d.755 \cdot 10^{-12} \cdot E^2 \quad (6)$$

$$\pm 0.^d.0008 \pm 0.^d.00000003 \quad \pm 0.^d.003 \cdot 10^{-12}$$

The parameters describing the orbital motion are:

$$a \sin i = 2.36 \pm 0.13 \text{ AU}$$

$$e = 0.47 \pm 0.05$$

$$\omega = 302^\circ \pm 9^\circ$$

$$P_{orb} = 34.29 \pm 0.27 \text{ years}$$

$$T_0 = 1976.6 \pm 0.7.$$

(7)

After subtraction of the fitting function, the residuals are shown in Figure 3. The scattering can be considered only due to observational errors.

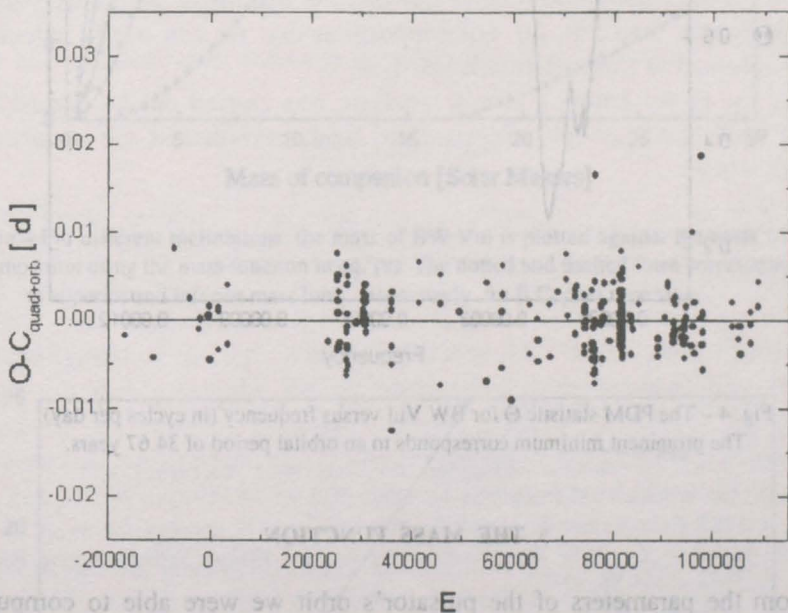


Fig. 3 – The residuals after fitting $(O - C)_{lin}$ with the function (5), representing superposition of quadratic evolution and orbital motion.

In order to find other periodicities in the observational data, we applied the Phase Dispersion Minimization (PDM) method (Stellingwerf, 1978). The statistic Θ emphasizes only one prominent minimum in Figure 4, which corresponds to the orbital periods determined above. The PDM method yields an orbital period of 34.67 years, slightly larger than the $(O - C)$ analyses.

Finally we give an ephemeris which takes into account both the period variation caused by evolution and that due to the light-time effect:

$$HJD_{min} = 2428802^d \cdot 5661 + 0^d \cdot 20102934 \cdot E + 0^d 759 \cdot 10^{-12} \cdot E^2 +$$

$$+ 0^d.013630223 \cdot \left\{ -0.3986 + \frac{0.7791 \sin[0.00010084 (E - 7934) + 5.274]}{1 + 0.47 \cos[0.00010084 (E - 7934)]} \right\}. \quad (8)$$

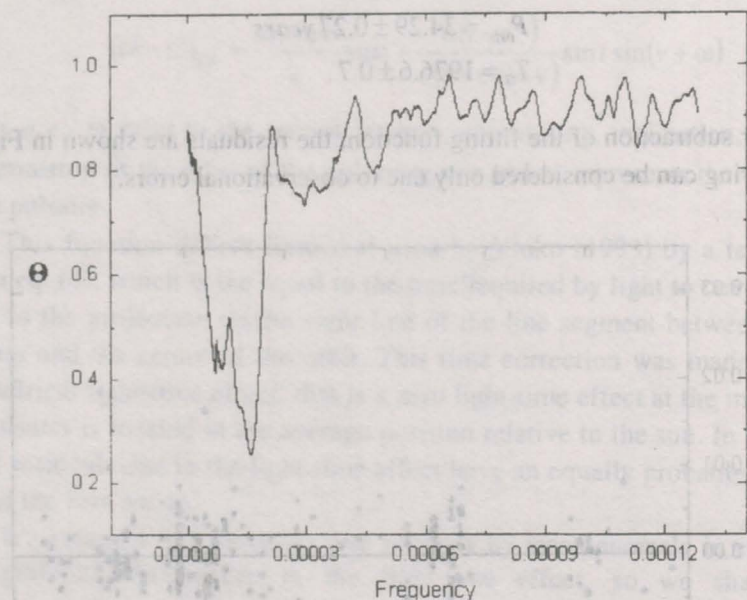


Fig. 4 – The PDM statistic Θ for BW Vul versus frequency (in cycles per day). The prominent minimum corresponds to an orbital period of 34.67 years.

5. THE MASS FUNCTION

From the parameters of the pulsator's orbit we were able to compute the mass function of this component:

$$f_{\text{puls}} = 0.0113 \pm 0.0019 M_{\odot} \quad (9)$$

a value in good agreement with the one given by Pigulski (1993). The pulsator being of β Cephei type, its mass should be in the range of 11.8–18.2 solar masses, Ledoux and Walraven (1958). We tried to get some information about the mass of the companion. In Figure 5a one can see the mass of the pulsator versus the mass of its companion, for several values of the orbit's inclination i . We noticed that even if the inclination is 90° the mass of the companion has a lower limit at 1.2 solar masses, see Figure 5b. For inclinations lower than this value, the mass of the companion must be larger. This limit is given by the mass function (computed with the orbital elements) and the mass limits of a β Cephei type pulsator.

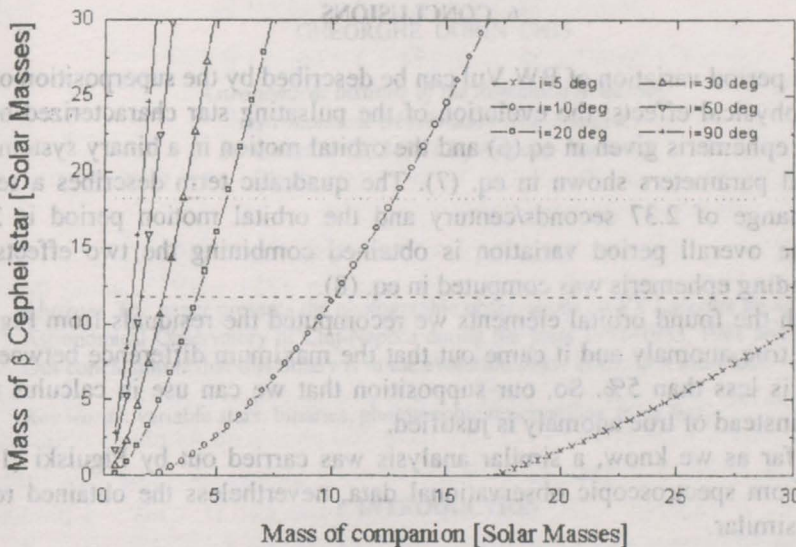


Fig. 5a – For different inclinations, the mass of BW Vul is plotted against the mass of the companion using the mass function in eq. (9). The dotted and dashed lines correspond to superior and inferior mass limit, respectively, for β Cephei type stars.

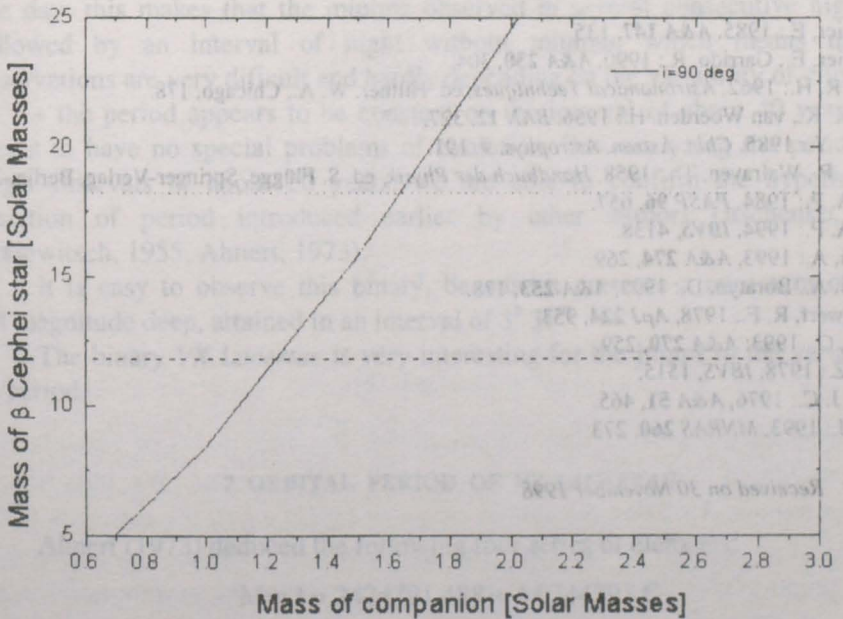


Fig. 5b – Relationship between pulsator and companion masses for $i = 90^\circ$.

6. CONCLUSIONS

The period variation of BW Vul can be described by the superposition of two separate physical effects: the evolution of the pulsating star characterized by the quadratic ephemeris given in eq.(6) and the orbital motion in a binary system with the orbital parameters shown in eq. (7). The quadratic term describes a secular period change of 2.37 seconds/century and the orbital motion period is 34.67 years. The overall period variation is obtained combining the two effects, the corresponding ephemeris was computed in eq. (8).

With the found orbital elements we recomputed the residuals from Figure 4 using the true anomaly and it came out that the maximum difference between the residuals is less than 5%. So, our supposition that we can use in calculus mean anomaly instead of true anomaly is justified.

As far as we know, a similar analysis was carried out by Pigulski (1993), starting from spectroscopic observational data, nevertheless the obtained results are quite similar.

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GHEORGHE DORIN CHIȘ

Astronomical Institute of the Romanian Academy,

Astronomical Observatory Cluj-Napoca

Str. Cîreșilor 19, 3400 Cluj-Napoca, Romania

Abstract. The paper presents the photographic observations on VX *Lacertae* obtained at Astronomical Observatory of Cluj-Napoca during the years 1952–1953, 1961 and 1972.

Our conclusion is that this binary is in the evolution phase of the first mass loss.

Key words: variable stars, binaries, photographic observations, mass loss.

1. INTRODUCTION

The binary system VX *Lacertae* was discovered in 1927 on the photographic plates at Berlin-Babelsberg by Gutnik and Prager (1927), and was observed by several observers more than 60 years ago (Zinner, 1933, Itschenko, 1948, Szafraniec, 1966, Zessewitsch, 1955) in sporadic mode due to following motives:

- the period of the light variation is about 1.07 days, hence approximately one day; this makes that the minima observed in several consecutive nights are followed by an interval of night without minima, which means that the observations are very difficult and hardly depending on the variability of climate;
- the period appears to be constant on the interval of about 40 years, so it seems to have no special problems of evolution. But, analyzing the period on a larger intervals of about 50 years, we are able to confirm the hypothesis of variation of period introduced earlier by other authors (Itschenko, 1948, Zessewitsch, 1955, Ahnert, 1973).

It is easy to observe this binary, because it presents a minimum of about 1.4 magnitude deep, attained in an interval of $3^h 36^m$.

The binary VX *Lacertae* is very interesting for the jumps in the variation of its period.

2. ORBITAL PERIOD OF VX LACERTAE

Ahnert (1973) deduced the following four series of elements:

$$\text{Min I} = 2424791.488 + 1.0744391 E \quad (1)$$

$$\text{Min I} = 2432479.477 + 1.0744991 E \quad (2)$$

$$\text{Min I} = 2440750.971 + 1.0744881 E \quad (3)$$

$$\text{Min I} = 2445258.466 + 1.0744942 E \quad (4)$$

He remarks the jumps in the variation of period (all observations were obtained visually or photographically).

Binary VX *Lacertae* was introduced in the observation program at the Astronomical Observatory of Cluj-Napoca between 1952 – 1953, using the *Prin telescope* (field glass, $D = 20$ cm, $F = 300$ cm). The magnitude variation was between 10.6 – 11.9. Photographic observations were made on *Guilleminot* plates in the following time intervals: 14 July 1952 – 8 September 1953, 3 July 1961 – 31 August 1961, 27 December 1972. We have 260 photos on 43 plates obtained by several researchers: I. Armeanca – 19 photos on 7 plates, Gh. Chiş – 204 photos on 23 plates, I. Todoran – 33 photos on 7 plates and Gh. D. Chiş – 4 photos on 4 plates. The last quoted photos contain the binary system and the comparison stars.

Firstly, we determined the scale of comparison stars in the same way as other authors. The plates were measured with a *Schnell-Zeiss* microphotometer. On the last 4 plates mentioned earlier, in the neighbourhood of the variable star there are 42 standard stars. On these plates we determined, by repeated measurements, the magnitudes of the comparison stars as following:

$$f = 10.03 \pm 0.06$$

$$c = 10.54 \pm 0.07$$

$$a = 10.83 \pm 0.03$$

$$e = 10.90 \pm 0.05$$

$$h = 11.38 \pm 0.06$$

$$g = 11.44 \pm 0.13$$

$$h = 11.55 \pm 0.09$$

Using the comparison stars we determined the magnitudes of variable star. The moments of observations were corrected at Sun center. We obtained 10 observed minima. Utilizing the ephemeris given by (1) – (4), the residuals ($O-C$) were calculated (Table 1). Grouping the observations by phases, we obtained 33 normal points (Table 2, in which the first column contains the current number, the second one – the phase and the third one – the magnitude of binary). With these points we obtained the mean light curve (Figure 1).

Table 1

The observed minima and (O-C)'s obtained at Cluj Observatory

No. crt.	Min Hel 243...	O-C ₁	O-C ₂	O-C ₃	O-C ₄
1	4208.350	- 0.596	0.004	- 0.062	- 0.017
2	4210.495	- 0.600	0	- 0.066	- 0.021
3	4225.535	- 0.602	- 0.003	- 0.065	- 0.024
4	4239.477	- 0.626	- 0.029	- 0.096	- 0.050
5	4254.555	- 0.592	0.006	- 0.060	- 0.015
6	4255.628	- 0.594	0.004	- 0.062	- 0.017
7	4266.364	- 0.602	- 0.005	- 0.081	- 0.026
8	4281.422	- 0.586	0.011	- 0.056	- 0.011
9	4626.342	- 0.561	0.016	- 0.046	- 0.003
10	4629.554	- 0.612	0.005	- 0.058	- 0.015

Table 2

The normal points for VX Lacertae

No. crt.	ϕ	m	No. crt.	ϕ	m	No. crt.	ϕ	m
1	0.002	11.87	12	0.151	10.82	23	0.940	10.91
2	0.010	11.73	13	0.273	10.90	24	0.946	10.94
3	0.015	11.62	14	0.381	10.66	25	0.953	11.07
4	0.021	11.49	15	0.533	10.77	26	0.961	11.16
5	0.025	11.34	16	0.765	10.67	27	0.967	11.33
6	0.032	11.35	17	0.869	10.55	28	0.970	11.30
7	0.041	11.21	18	0.893	10.60	29	0.977	11.40
8	0.051	11.14	19	0.906	10.66	30	0.980	11.45
9	0.065	10.86	20	0.917	10.78	31	0.985	11.61
10	0.086	10.73	21	0.925	10.83	32	0.992	11.60
11	0.120	10.60	22	0.935	10.88	33	0.995	11.79

3. CONCLUDING REMARKS

The light curve obtained confirms that the binary *VX Lacertae* is an eclipsing variable star and its primary minimum is very deep (10.55 – 11.85 magnitudes). It has not constant phase and primary minimum is symetrical. The duration of the eclipse is 0.269 days. Because the aim of the observations was to put in evidence the variation of the period, there are very few observational points out of the eclipses and a secondary minimum was not found.

We determinated 10 primary minima which fit very well with those obtained by other authors. The present observations were used in a more general project of research at the Astronomical Observatory of Cluj, and we used them to set more general results for the jumps in the period variation, that could be explained through variation of mass, one component losing mass in a sudden mode. The

variable *VX Lacertae* becomes very important because in its evolution it reached the phase of the first mass loss.

More observations are necessary to give an answer about the evolution stage of this binary.

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no.	%	no.	%	no.	%	no.	%
1	0.001	11	0.001	21	0.001	31	0.001
2	0.010	12	0.010	22	0.010	32	0.010
3	0.018	13	0.018	23	0.018	33	0.018
4	0.021	14	0.021	24	0.021	34	0.021
5	0.028	15	0.028	25	0.028	35	0.028
6	0.032	16	0.032	26	0.032	36	0.032
7	0.040	17	0.040	27	0.040	37	0.040
8	0.051	18	0.051	28	0.051	38	0.051
9	0.063	19	0.063	29	0.063	39	0.063
10	0.086	20	0.086	30	0.086	40	0.086
11	0.120	21	0.120	31	0.120	41	0.120
		22	0.120	32	0.120	42	0.120

CONCLUDING REMARKS

The light curve obtained confirms that the binary *VX Lacertae* is an eclipsing variable star and its primary minimum is very deep (10.25 - 11.82 magnitudes). It has not constant phase and primary minimum is sporadic. The duration of the eclipse is 0.369 days. Because the part of the observations was to put in evidence the variation of the period, there are very few observational points out of the eclipse and a secondary minimum was not found.

We determined 10 primary minima which fit very well with those obtained by other authors. The present observations were used in a more general project of research in the Astronomical Observatory of Cluj, and we used them to set more general results for the binary in the period 1990-1995. That could be expected through variation of mass, the companion being in a similar state.

TIME STRUCTURES OF QUASI-PERIODIC PATTERNS IN SOLAR RADIO BURSTS

ADRIAN ONCICA

*Astronomical Institute of the Romanian Academy,
Str. Cușitul de Argint 5, 75212 Bucharest 28, Romania*

Abstract. We investigate a selection of (quasi) periodic structures, which are present in some solar radio bursts in search for physical processes signature. These structures include a wide range of phenomena from strictly sinusoidal oscillation up to quasi-periodic fine structure, observed in the radio, microwave and X-ray frequency range. Observational references are discussed, in order to give a key for the interpretation of observations. In this investigation, we use Joint Time-Frequency (JTF) spectrogram representations and scalogram (wavelet) representations. This was due not only to their ability to reveal the presence of dominant periods (as the usual spectrograms) but also to their ability to localize such spectral details in time.

Key words: solar physics – solar radio bursts – fine structure – time-frequency distribution.

1. INTRODUCTION

There is some time now since periodic and quasi-periodic structures have been identified in the single frequency or spectral observations of some solar radio burst. Their characteristics range from constant to rapidly variable amplitude, pulse period and frequency bandwidth. The richness of structural details is associated with the richness of their basic mechanisms. For previous attempts see Kruger (1979). The harmonic motion of the classical mechanical oscillator results from a restoring force proportional to the displacement. Any elastic body can be excited to oscillate in eigen-modes. In presence of a magnetic field the magnetic field strength, density and geometry of the hydrodynamic fluid determine the characteristic eigen-frequencies of an oscillating system. Hence, the only observation of some oscillation parameters (e.g., time period, quality factor, and modulation depth) reduces the number of free parameters. Also in complex systems such as wave-particle interactions the observable parameters (temporal and spectral characteristics) are related to relevant parameters of the resonance condition (resonance frequency, resonance bandwidth, coherence time, stability). Consequently, the establishment of a well-chosen set of parameters would enable us to discriminate between alternative pulsation models. The exploration of local coronal oscillations may provide prospective enlightening of coronal plasma processes.

'Pulsations' include a wide range of phenomena from strictly sinusoidal oscillations up to rather quasi-periodic patterns or fine structures, observed in the metric/decimetric/microwave or in the X-ray frequency range. The richness of structural details requires different approaches of theoretical modeling. It is still an open question which pulsation mechanisms are relevant for the interpretation of the observed events, and how the proposed models work in detail. The various versions of quasi-periodic models are classified in three groups according to their driver mechanisms: (1) magnetic flux tube oscillations (the emissivity of trapped particles are modulated by a standing or propagating MHD wave), (2) cyclic self-organizing systems of plasma instabilities (wave-particle, wave-wave interactions), (3) modulation of acceleration (acceleration/injection of particles into the source). For a review see Anschwanden (1987). The models form a very inhomogeneous group with respect to the degree of elaboration. Some of them explain the complex pulsation phenomenon rather completely. On the other hand, some other models only explain selected sophisticated details ignoring important steps of the puzzle such as the driving mechanism, the link of the driver to the emission, the emission mechanism, or the conversion into observable radio waves. Pulsation models can be classified by their basic driver mechanism; a further sub-classification may regard the involved emission mechanism (cf. Table 1)

Table 1

Classification of pulsation models

MHD oscillations (eigenmodes)	• Gyrosynchrotron modulation • Plasma emissivity modulation • Bounce motion of wave trains
Cyclic self-organizing systems	• Wave-wave interactions • Wave-particle interactions
Modulation of acceleration	• Periodic (fragmented) injection • Periodic magnetic reconnection

2. GENERAL MODELS

MHD waves is the first established and most elaborated model of coronal pulsations which considers MHD oscillations. Since the birth of this idea (Rosenberg, 1970) many modifications and theoretical improvements have succeeded. Theoretical aspects of MHD oscillations in different geometries of flux tubes were most thoroughly investigated by Roberts *et al.* (1984). Systematic studies of the propagation of MHD waves have been done for different geometries: (1) unbounded homogeneous medium, (2) single magnetic interface (Wentzel, 1979a, Roberts, 1981a), (3) slender magnetic slab (Roberts, 1981b;

Edwin and Roberts, 1982; Rae and Roberts, 1982), (4) cylindrical symmetry (Wentzel, 1979b; Edwin and Roberts, 1983).

According to the wave propagation characteristics, two kinds of waves may be distinguished: the body wave and the surface wave. The body wave resonates with the (homogeneous) volume as a whole; the surface wave has nonvanishing amplitude only near the well-defined magnetic surface. The solution for standing waves implies that there is no propagation of energy outside the cylinder. Hence, the oscillations of the magnetic flux tube are confined to the tube and penetrate only a short distance into their surroundings. These are the *free modes* of the oscillation (ω and κ real). Two free modes, the symmetric (sausage or pulsational) mode given by $n = 0$ and the asymmetric (kink or taut-wire) mode given by $n = 1$ were studied extensively by Edwin and Roberts (1983).

An observational test for the existence of these MHD waves in the corona may be the comparison of the time periods. Assuming a standing wave, with its knots at the footpoints of a loop with length L , the wavenumbers are $k = j\pi/L$ for integers $j = 1, 2, \dots$. Pulsation periods $\tau = 2\pi/\omega$ may be derived by inserting the phase speed $v_{ph} = \omega/k$, hence, we obtain $\tau = 2L/jv_{ph}$. The shortest time periodicities of the relevant modes compiled with estimates using typical coronal parameters (Roberts *et al.*, 1984), are: $\tau \approx 50$ s for the fast kink mode, $\tau \approx 1.5$ s for the fast sausage mode and $\tau \approx 2$ s for MHD propagating waves.

Self-organizing processes are characterized by (i) cyclic processes (closed trajectories in phase space) and (ii) connection of the considered system with the environment by other processes (energy dissipation or transfer). There is a big difference with isolated systems which tend to reach equilibrium position (converging trajectory in phase space), wherefore the evolutionary dynamics disappears. Dissipative self-organizing systems, however, communicate with the environment, in contrast to the models of MHD pulsations, which correspond to the physical picture of a classical oscillator, characterized by discrete eigen-modes. These interactions enable the system to persist in a cyclic state *far* from the equilibrium. This can be typical for: quasi-stabilized systems of non-linear plasmas governed by an oscillatory phase of wave-wave (resonant/nonresonant) or wave-particle (relaxational) interactions

The oscillating wave energies of Langmuir waves may produce radio bursts with periodic fine structure. Interpretations of fine structures invoking periodical regimes of plasma instabilities (wave-wave interactions) were proposed by Gotwols (1972) for broad-band pulsations upon an underlying type IV continuum, by Elgaroy and Sveen (1973) for wide band quasi-periodic bursts with periods of 0.1 – 0.2 s. Zaitsev (1971), solving the 'limit cycle' equation found for 'rain'-type

bursts (succession of regularly repeated short-lived broad-band bursts) periods of the order of: $\tau = 2\pi / \sqrt{\gamma \cdot \nu}$ where γ is the growth rate and ν is the collision rate.

The interaction of the unstable waves with the resonant particles may also be approximated by a Lotka-Volterra-like equation. The periodic solution predicts not only oscillatory wave energy amplitude, but also a phase-shifted oscillatory particle distribution (relaxational oscillation). The positive growth of instability dissipates energy from the resonant particles. A stationary operation of the instability will modify the particle distribution considerably. Relaxational oscillations were studied by Aschwanden and Benz (1987) who applied the electron-cyclotron maser driven by the loss-cone in coronal loops. They found for the pulse period a similar expression: $\tau = 2\pi / \sqrt{\tau_{\text{growth}} \cdot \tau_{\text{diff}}}$. The pulse amplitude was found to be highly variable and dependent on pulse period.

Modulation (fragmentation) of acceleration does not possess an internal pacemaker, but is a mechanism driven by external drive forces. The common aspects of this residual third group of pulsation models are two fold: (a) they do not fit into one of the previous two categories and (b) the resonant region where the pulsating radio emission originates is driven by external dynamics. This external driver may be represented by a: (1) modulated acceleration process (repetitive injection of particles) whose driver may be a pulsed acceleration (e.g., a quasi-periodic magnetic reconnection). The theoretical framework of such mechanisms is rather fragmentary; neither the dominant acceleration process nor its efficiency is well known for the solar environment.

Quasi-periodic fine structures during type IV are observed in a variety of pulse shapes, from emissive bursts up to clear absorption features upon a smooth continuum. Assuming plasma emission for narrow-banded decimetric type IV continua, absorption features have been interpreted in terms of injected particles into the magnetic trap (Benz and Kuijpers, 1976). The periodicity of sudden reductions is commonly not very regular. Interpretation of observations in terms of the 'sudden reduction'-model has been suggested, e.g., by Trotter *et al.* (1981) for quasi-periodic modulations of moving type IV bursts. In addition, the reported hard X-ray pulsations are currently proposed to be associated with multiple injections of particles into the flare energy region.

3. TIME FREQUENCY PROFILES ANALYSIS

The inherent dissimilarities of the previous models imposes constraints on the interpretation of observation. Nearly all models are able to match the typical observed repetitive period of order of 1s., so they are best suited to rule out a model. For systems oscillating in eigen modes the period is generally constant. In contrast,

cyclic self-organizing systems exhibit variable pulse periods. As for the third class the variability of pulse periods may be determined by the kind of driving mechanism which can belong to both first two classes. Table 2 contains an overview.

To investigate the time profiles we choose the *time frequency distribution* approach. The representation of a function in terms of its time behavior (sequence of samples) or its frequency behavior (Fourier expansion) are two extreme cases. These representations show extreme locality in one domain but no locality at all in the other. So came up the idea (Gabor, 1946) of representing signals by time-frequency shifted versions of some basic building blocks ('atoms') with good TF concentration. In the original idea was natural to choose the Gauss function $g(x) \approx \exp(-x^2)$ which happens to be both invariant under the Fourier transform and to give equality in the Heisenberg inequality. (the measure of the TF locality). Later it was shown (Janssen, 1981) that neither the representation is unique nor it can represent arbitrary signals.

Table 2

Spectral characteristics of different mechanisms

Mechanism	Emission	Model	Period	Amplitude	Bandwidth
MHD oscill.	gyromagnetic	radial	constant	damped	large
MHD oscill.	gyromagnetic	torsional	constant	constant	large
MHD oscill.	plasma	impulsive	variable	variable	narrow
Cyclic	Langmuir	wave-wave	variable	variable	narrow
Cyclic	loss-cone	wave-particle	variable	variable	narrow
Modulated	sudden	repetitive	variable	absorption	narrow
acceleration	reduction	injection			

A first (linear) class of such TF representations is illustrated by the Short Time Fourier Transform:

$$STFT_x(t, v, h) = \int_{-\infty}^{+\infty} x(u) \cdot h^*(u-t) \cdot \exp(-j2\pi \cdot v \cdot u) \cdot du \quad (1)$$

The *short time analysis window* $h(t)$ suppresses the (time) signal outside a neighborhood around $u = t$ and filter the (frequency) signal through $H^*(v)$, the Fourier transform of $h(u, t)$. Considering $|x(t)|^2$ and $|X(v)|^2$ as probability distributions their mean values (*average time* t_m and *average frequency* v_m) and standard deviations (*time spreading* T and *frequency spreading* B) can be computed through the usual formulae. The TF localization is then given by the time bandwidth product that is lower bounded ($B \times T \leq 1$). The lower bound is reached for Gaussian functions:

$$x(t) = C \cdot \exp[-\alpha(t-t_m)^2 + j2\pi\nu_m(t-t_m)] \quad (2)$$

A second (nonlinear) class of TF representations (Cohen, 1989) distribute the *energy* of the signal over the two descriptive variables (time and frequency) subject to *marginal properties* and verifying both *time and frequency covariance*. A representative of this class is the *smoothed pseudo Wigner Ville* distribution:

$$SPWV_v(t, \nu) = \int_{-\infty}^{+\infty} h(\tau) \int_{-\infty}^{+\infty} g(s-t) \cdot x(s + \frac{\tau}{2}) \cdot x^*(s - \frac{\tau}{2}) \cdot ds \cdot e^{-j2\pi\nu\tau} \cdot d\tau \quad (3)$$

Excellent representation for single component signals, this class is not suited for typical noisy, multi-component astrophysical signals (Kollath, 1997) due to the cross-terms (Jones, 1992).

Somehow belonging to the linear class of TFR is the Wavelet representation. The fundamental idea behind wavelets is to analyze according to scale (Meyer, 1993). The general form of the (continuous) wavelet transform is:

$$WT_x(t, a; \Psi) = \int_{-\infty}^{+\infty} x(s) \cdot \left[\frac{1}{\sqrt{a}} \cdot \Psi^*\left(\frac{s-t}{a}\right) \right] \cdot ds \quad (4)$$

where $\Psi(s)$ is the *mother wavelet*. It generates the daughter wavelets $\Psi_{t,a}(s)$ in the formula (4) through scaling (a) and translation (t). A time–frequency interpretation is possible through the formal identification: $\nu = \nu/a$. As we can look at gross features with a large window and at the small features with a small window we can see *both the forest and the trees*.

4. THE DATA

The radio spectrometric data used for this analysis was collected from several ftp available archives and some other published data. Astronomical Institute ETHZ-Zurich operates the PHOENIX digital spectrometer (Benz, et. al., 1991) in the 0.1–3GHz band. Of a great help in selecting the material was the Catalog of decimetric solar radio emission (Gudel and Benz, 1988). Observatory for Solar Radio Astronomy of the AIP Potsdam operates 4 sweep radio spectrographs (10 sweeps per second in the ranges 40–90 MHz; 100–170 MHz; 200–400 MHz and 400–800 MHz) at the aerial site in Trens Dorf 15 km to the south-west of Potsdam. Ondrejov Astronomical Observatory operates two digital spectrographs: RT5 (data series 'b') in the 1–2GHz band and RT4 (data series 'a') in the 2–4.5 GHz band. Some historical data from Utrecht analog spectrograph was also used (Rosenberg, 1976). A clean damped oscillations event was taken from the observations of the Coolgoora radiospectrometer (McLean and Sheridan, 1973).

The data was labeled in the format LYYMMDD. The letter L designates the observatory ('z' for Zurich, 't' for Tremsdorf, 'b' from Ondrejov series, 'u' for Utrecht, and 'c' for Coolgoora) and the next six digits the date in an obvious format. Following the classification by Isliker and Benz (1994) five classes were investigated and representative events are presented. A supplementary class was added through the c720516 example.

The TFR power spectrum (label 'SP') was computed as the square modulus of the $STFT_z$ (Eq. 1) using a Hanning window of adequate length ('Lh'). The scalogram (label 'scalo') was computed through WT_z (Eq. 4) using the Morlet wavelet as it establish a more clear distinction between the random fluctuations and the periodic features (Rioul, 1992). Most of the Cohen class TF representations were tested by the author and found unsuitable for TF representation of solar radio bursts time series. A variable threshold was applied to all spectra depending on the amount of noise in the data.

4.1. TYPE III BURSTS

Event b920716 at 07:27:00 UT. + 120 s recorded at Ondrejov. A chain of type III bursts. The most important burst at $t + 50$ s is preceded and followed by chains of smaller type III bursts. This may suggest fragmentation of the driving mechanism. (Fig. 1a, 1b)

4.2. NARROW BAND SPIKES

Event z800831 at 07:30:30 UT + 37s recorded at Zurich. Narrow band spikes at the footpoint of a type III event. (Fig. 2a, 2b)

4.3. TYPE IV BURSTS AND MODULATED CONTINUUM

Event u720306 at 12:11:34 UT + 15 s recorded at Utrecht. A broad band continuum modulated in absorption by fiber bursts and parallel absorption bands. (Fig. 3a, 3b)

4.4. PULSATIONS

Event t980427 at 09:07:27 UT + 100s recorded at Tremsdorf. Broad band pulsations. The visibility is enhanced by plotting the time derivative of the flux. (Fig. 4a, 4b)

4.5. OSCILLATIONS

The event c720516 at 03:14:30 UT + 132s recorded at Coolgoora. A narrow band extremely regulate oscillations. (Fig. 5a, 5b)

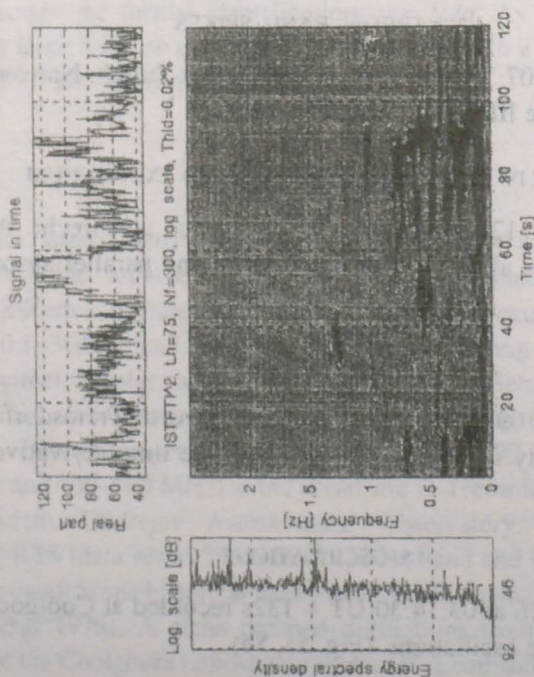


Fig. 1 a – TFR spectrum of the event b920716 scanned at 1.6 GHz. Possible quasi-stable rate around 0.6 Hz mainly in the second part of the event.

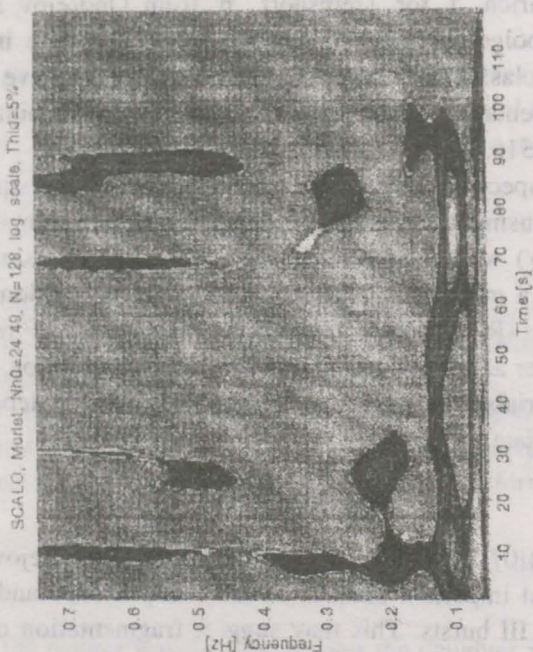


Fig. 1 b – TFR scalogram of the event b920716 scanned at 1.6 GHz. The 0.6 Hz scale from above is highly fragmented and shows a drift from 0.7 to 0.5 Hz. The drifting scale around 0.1 Hz may imply clusterization of the individual type III bursts.

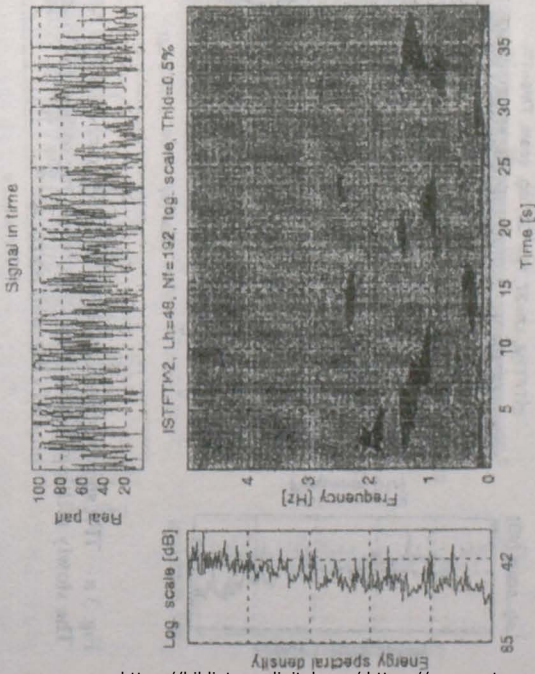


Fig. 2 a – TFR spectrogram of the event z800831 scanned at 750 MHz. A quasi-periodic structure is visible around 1.0–1.5 Hz mainly on the spectrum at left.

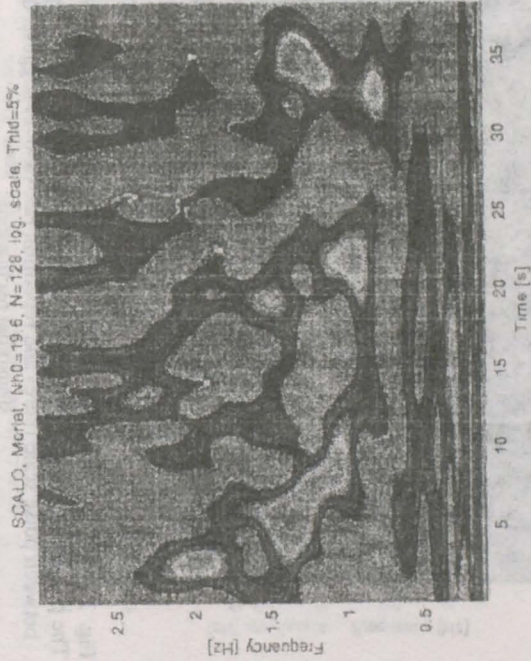


Fig. 2 b – TFR scalogram of the event z800831 scanned at 750 MHz. The quasi-periodic structure around 1.0–1.5 Hz becomes questionable and fragmented. Again a larger scale around 0.3 Hz may imply clusterization of individual spikes.

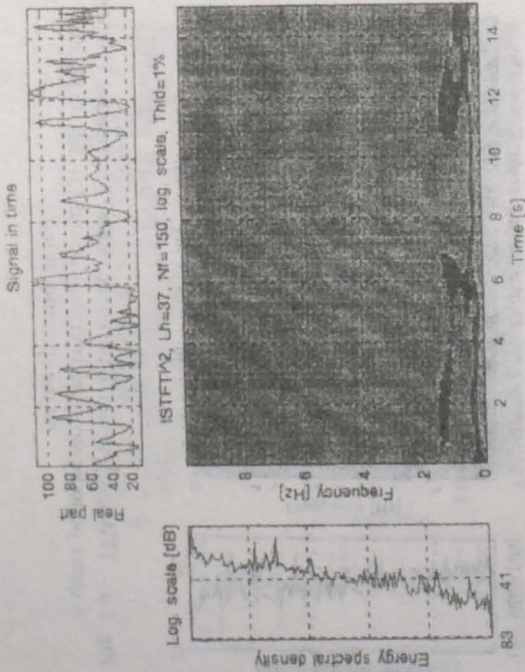


Fig. 3 a – TFR spectrogram of the event u720306 scanned at 250 MHz. The slowly drifting-periodic structure belonging to fibers is visible around 1 Hz well above noise.



Fig. 3.b. TFR scalogram of the event u720306 scanned at 250 MHz. The periodic structure around 1 Hz shows an obvious split. The equality between pulse scale (here) and repetition rate (above) suggests quasi sinusoidal oscillations.

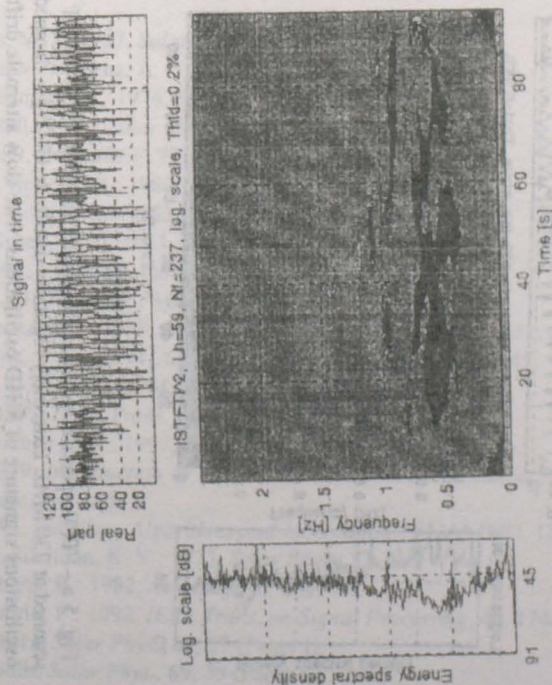


Fig. 4 a – TFR spectrogram of the event 1980427 scanned at 240 MHz. The stable-periodic structure visible around 0.6 Hz is clearly defined and well above noise (see the spectrum at left).



Fig. 4 b – TFR scalogram of the event 1980427 scanned at 240 MHz. The stable-periodic structure around 0.6 Hz is tampered by noise at the end.

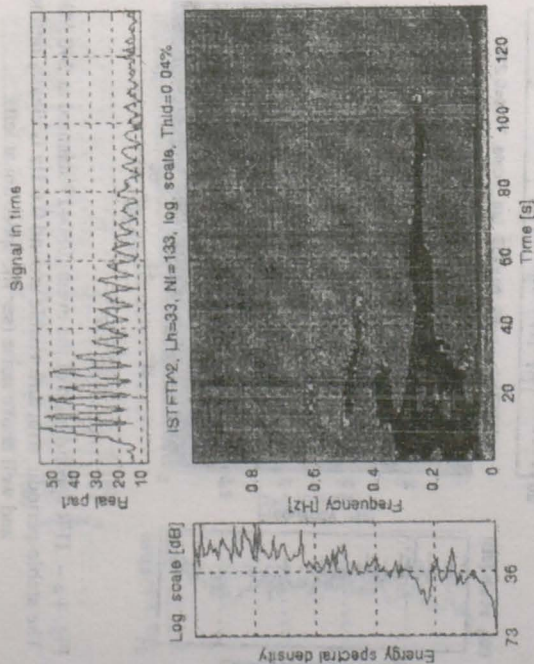


Fig. 5 a – TFR spectrogram of the event c720516 at 03:14:30 UT scanned at 230 MHz. Extremely regularly distributed 0.25 Hz damped oscillations signature of MHD oscillations. The slow alternate drift in frequency is more visible in the first harmonic around 0.5 Hz.

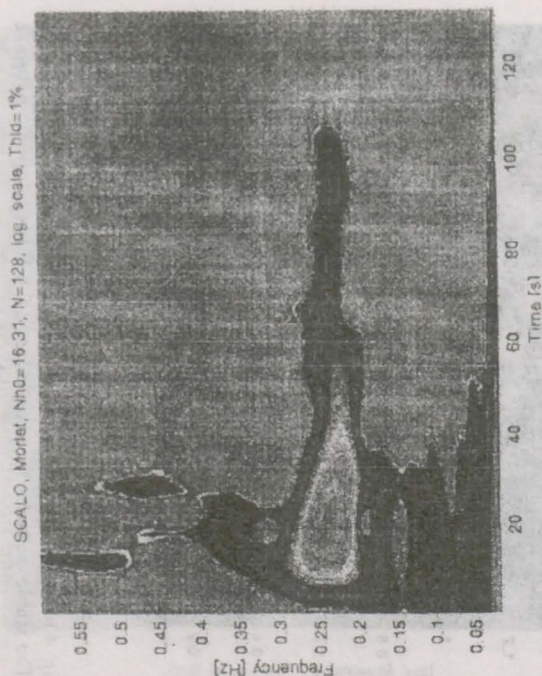


Fig. 5 b – TFR scalogram of the event c720516 at 03:14:30 UT scanned at 230 MHz. The alternate frequency drift is more clear in the scalogram representation. The perfect equality between individual pulses scale (here) and the repetition rate (above) reveal the oscillatory nature.

5. CONCLUSIONS

An observational fact is the predominance of regular MHD oscillations in the metric domain (higher corona, large loops), whereas the oscillations in the decimetric range are irregular as a rule. The bandwidth criterion for synchrotron emission is not fulfilled in the decimetric range, suggesting plasma or maser emission. In contrast, microwave observations exhibit large bandwidths in support of synchrotron emission.

We found that certain single frequency solar radio "oscillations" which have a definite narrow dominant frequency in the classical spectrogram are in fact not of constant frequency. When analyzed through both JTF and wavelet methods we found that their dominant frequency, if there is one, shows a drift or jump in time.

We investigate also some solar radio spectrograms (pulsations, oscillations, and spikes) which at the first visual inspection seems to contain periodical structures. Most of them show no definite dominant frequency, which may require additional investigation methods belonging to the chaotic time series analysis.

Whether or not such spectral and temporal behavior is relevant for the interpretation of the underlying physical processes of the observed events is a matter of further investigation.

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VASILE URECHE

"Babeş-Bolyai" University of Cluj-Napoca,
 Faculty of Mathematics and Computer Science,
 Str. M. Kogălniceanu 1, 3400 Cluj-Napoca, Romania

Abstract. The gravitational redshift in Maneff's field is analysed. The results are expressed in terms of the ratio between the Schwarzschild gravitational radius and the geometric radius of the body, both for the special relativistic approximation and for the general relativistic approximation. In both cases the difference between the Newton's field and the Maneff's field in gravitational redshift is present only in the second order term and reaches $3/8$ from the square of the above mentioned ratio. In general relativistic approximation, it is shown that an effect of black hole appears at a radius which we name Maneff gravitational radius and which is larger with the factor of $3/2$ than the Schwarzschild gravitational radius.

Key words: relativistic astrophysics, gravitational redshift, Maneff field.

1. INTRODUCTION

G. Maneff considered a post-Newtonian nonrelativistic law of gravitation, assuming that the gravitational interaction between the masses m_1 and m_2 is given by the "force function" (Maneff 1924):

$$U = \frac{G m_1 m_2}{r} \left[1 + \frac{3G(m_1 + m_2)}{2c^2 r} \right], \quad (1)$$

where r is the distance between m_1 and m_2 , G is the Newtonian gravitational constant, and c the speed of light.

Recently, (Mioc and Stoica, 1995; Diacu et al., 1995; Stoica and Mioc, 1997) the two-body problem in the Maneff gravitational field featured by (1) was studied. The complete solution of the problem was given. Also, the free-fall collapse of a homogeneous sphere in Maneff's gravitational field was analysed (Ureche, 1995).

In this paper, the problem of gravitational redshift in the Maneff field is considered.

We shall consider a massive body of mass M and a test particle of mass $m \ll M$ in the gravitational field of M . The effects of this field on m can be described by the following potential Φ , which is attached to U from (1) ($m_1 = M$, $m_2 = m$):

$$\Phi(r) = -\frac{GM}{r} - \frac{3G^2 M^2}{2c^2 r^2}, \quad (2)$$

Now, we shall analyse the gravitational redshift produced by the field described by the potential (2). The results will be compared with those obtained for the Newton gravitational field, both for the special relativistic approximation and for the general relativistic approximation.

2. SPECIAL RELATIVISTIC APPROXIMATION

2.1. NEWTON GRAVITATIONAL FIELD

Let be the Newton gravitational field of a celestial body with the mass M and the radius R . Then let be a photon with the mass m_f , the wavelength λ and the frequency ν at the surface of the body. The gravitational potential at the surface of the body is:

$$\Phi^N = -G \frac{M}{R}. \quad (3)$$

For the distant observer, the above defined quantities will be denoted with the index 0. Taking into account the conservation of the energy for the considered photon, we have:

$$m_f c^2 + m_f \Phi^N = m_{f_0} c^2 + m_{f_0} \Phi_0^N, \quad (4)$$

where c is the speed of light. But

$$m_f c^2 = h\nu, \quad \nu = \frac{c}{\lambda}, \quad (5)$$

where h is the Planck's constant.

If we consider a massive celestial body, the potential Φ_0^N can be neglected. Noting $\lambda_0 - \lambda = \Delta\lambda$ and developing (4) until the second order term, we obtain:

$$\frac{\Delta\lambda}{\lambda} = \frac{GM}{c^2 R} + \frac{G^2 M^2}{c^2 R^2}. \quad (6)$$

Putting $z_g = \frac{\Delta\lambda}{\lambda}$ and

$$R_s = \frac{2GM}{c^2} \quad (7)$$

- the Schwarzschild gravitational radius, we have:

$$(4) \quad \Phi^N = -\frac{1}{2} \frac{R_S}{R} + \frac{1}{4} \frac{R_S^2}{R^2} \quad (8)$$

Usually, in the astrophysical applications, only the first term from (6) (or (8)) is considered.

2.2. MANEFF GRAVITATIONAL FIELD

In this case, an equation analogous with (4) we can write. With $\Phi = \Phi^N$ from (2) for $r = R$ until the second order term, we have:

$$\frac{\Delta\lambda}{\lambda} = \frac{GM}{c^2 R} + \frac{5G^2}{2c^2 R^2} \quad (9)$$

or

$$z_x^M = \frac{1}{2} \frac{R_S}{R} + \frac{5}{8} \frac{R_S^2}{R^2} \quad (10)$$

that is

$$z_g^M - z_g^N = \frac{3}{8} \frac{R_S^2}{R^2} \quad (11)$$

3. GENERAL RELATIVISTIC APPROXIMATION

Let $x^0 = ct$ be the temporal coordinate and $x^1 = r$, $x^2 = \theta$, $x^3 = \varphi$ the spherical (Schwarzschild) spatial coordinates. Then, we shall associate to the potential Φ the following general relativistic metric (Landau, Lifshitz, 1973):

$$ds^2 = \left(1 + \frac{2\Phi}{c^2}\right) c^2 dt^2 - \frac{dr^2}{1 + \frac{2\Phi}{c^2}} - r^2 d\theta^2 - r^2 \sin^2 \theta d\varphi^2 \quad (12)$$

3.1. NEWTON GRAVITATIONAL FIELD

For any radial coordinate r the gravitational potential (3) will be write:

$$\Phi^N(r) = -G \frac{M}{r} \quad (13)$$

Introducing (13) in (12) we obtain the well-known Schwarzschild metric:

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{2GM}{c^2 r}} - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2. \quad (14)$$

The relation between the proper time τ and the time t of distant observer is

$$\tau = \sqrt{1 - \frac{2GM}{c^2 r}} t. \quad (15)$$

The existence condition for (15) is $r \geq R_S$, where R_S is the Schwarzschild gravitational radius from (7).

Let P be the period of an electromagnetic wave emitted by a star of mass M and radius R ($r = R$) and P_0 the period of the same wave measured by the distant observer. Then, from (15) we have:

$$P_0 = \frac{P}{\sqrt{1 - \frac{2GM}{c^2 R}}} \quad (16)$$

The relation between the corresponding wavelengths λ , λ_0 is of the same form:

$$\lambda_0 = \frac{\lambda}{\sqrt{1 - \frac{2GM}{c^2 R}}} \quad (17)$$

Then, for z_r^N we obtain

$$z_r^N = \left(1 - \frac{2GM}{c^2 R}\right)^{-1/2} - 1. \quad (18)$$

Developing this to the second order in $1/c^2$, we have:

$$z_r^N = \frac{GM}{c^2 R} + \frac{3}{2} \frac{G^2 M^2}{c^2 R^2}, \quad (19)$$

or in terms of the Schwarzschild radius:

$$z_r^N = \frac{1}{2} \frac{R_s}{R} + \frac{3}{8} \frac{R_s^2}{R^2}. \quad (20)$$

3.2. MANEFF GRAVITATIONAL FIELD

Introducing (2) in (12) we obtain the general relativistic metric:

$$ds^2 = \left(1 - \frac{2GM}{c^2 r} - \frac{3G^2 M^2}{c^4 r^4}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{2GM}{c^2 r} - \frac{3G^2 M^2}{c^4 r^4}} - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2. \quad (21)$$

Neglecting the terms of second order we have the Schwarzschild metric.

Now, the connection between the proper time τ and the time t of the distant observer is:

$$\tau = \sqrt{1 - \frac{2GM}{c^2 r} - \frac{3G^2 M^2}{c^4 r^2}} t. \quad (22)$$

The existence condition for (21) and (22) is:

$$1 - \frac{2GM}{c^2 r} - \frac{3G^2 M^2}{c^4 r^2} > 0 \quad (23)$$

that is

$$r > R_M, \quad R_M = \frac{3GM}{c^2}. \quad (24)$$

The length R_M will be named the Maneff gravitational radius, which can be compared with the Schwarzschild gravitational radius (7), namely: $R_M = (3/2) R_S$. For $R \leq R_M$ the massive body of radius R becomes a black hole. We shall name this a black hole of Maneff type.

Let P be the period of an electromagnetic wave emitted by a star of mass M and radius R ($r = R$) and P_0 the period of the same wave measured by the distant observer. Then, from (22) we have:

$$P_0 = \frac{P}{\sqrt{1 - \frac{2GM}{c^2 R} - \frac{3G^2 M^2}{c^4 R^2}}}. \quad (25)$$

The relation between the corresponding wave lengths λ, λ_0 is of the same form:

$$\lambda_0 = \frac{\lambda}{\sqrt{1 - \frac{2GM}{c^2 R} - \frac{3G^2 M^2}{c^4 R^2}}}. \quad (26)$$

Then for the Maneff gravitational redshift we obtain:

$$z_k^M = \left(1 - \frac{2GM}{c^2 R} - \frac{3G^2 M^2}{c^4 R^2} \right)^{-1/2} - 1. \quad (27)$$

Developing (27) in a power series, to the second order in $1/c^2$, we have:

$$z_R^M = \frac{GM}{c^2 R} + 3 \frac{G^2 M^2}{c^4 R^2} \quad (28)$$

or in terms of Schwarzschild radius:

$$z_R^M = \frac{1}{2} \frac{R_S}{R} + \frac{3}{4} \frac{R_S^2}{R^2} \quad (29)$$

Anew, the difference:

$$z_R^M - z_R^N = \frac{3}{8} \frac{R_S^2}{R^2} \quad (30)$$

The relative difference is the same in both the special relativistic case and in the general relativistic case, namely:

$$\frac{z_R^M - z_R^N}{(z_R^N)^2} = \frac{3}{2} \quad (31)$$

For a typical neutron star with $M = 1.4 M_\odot$ (binary pulsar 1913 + 16, Taylor and Weisberg, 1982) and $R = 15$ km, the second term in (29) can reach about 40% from the first one.

Unfortunately, the second term in (8), (10), (20) and (29) has usually a small value and will be very difficult to distinguish between the numerical results obtained with these expressions. Therefore, very accurate observations will be needful in order to discern between Newtonian gravitational field and Maneff's gravitational field.

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THE SCHWARZSCHILD - DE SITTER PROBLEM (I) : BEHAVIOUR AT COLLISION AND INFINITY

VASILE MIOC, MAGDA STAVINSCHI

*Astronomical Institute of the Romanian Academy, Astronomical Observatory Bucharest
Str. Cuţitul de Argint 5, RO-75212 Bucharest 28, Romania*

E-mail: vmioc@roastro.astro.ro, magda@roastro.astro.ro

Abstract: The Schwarzschild-de Sitter problem (i.e. the two-body problem associated to a potential of the form $A/r + B/r^3 + Dr^2$, where r = distance between particles, and A, B, D = well-established positive constants) is being qualitatively tackled. The equations of motion are written in Hamiltonian formalism, and the first integrals of energy and angular momentum are pointed out. Using McGehee's transformations, the motion equations are regularized, and the collision singularity is replaced by the collision manifold M_0 . Analogously, one introduces the infinity manifold M_∞ (for $r \rightarrow \infty$). The flows on M_0 and M_∞ and in their neighbourhood are depicted. They are identical to those corresponding to the Schwarzschild problem for M_0 , and differ drastically for M_∞ .

Key words: celestial mechanics - Schwarzschild-de Sitter problem - McGehee transformations.

1. INTRODUCTION

The Schwarzschild-de Sitter gravitational field represents a static, spherically symmetric solution of Einstein's vacuum equations with cosmological term. Written in physical units, the respective metric reads (e.g. Eddington 1925):

$$ds^2 = c^2 \left(1 - \frac{2GM}{c^2 r} - \frac{\Lambda}{3} r^2 \right) dt^2 - \left(1 - \frac{2GM}{c^2 r} - \frac{\Lambda}{3} r^2 \right)^{-1} dr^2 - r^2 d\chi^2, \quad (1)$$

with G = Newtonian gravitational constant, M = mass of the field-generating body, c = speed of light, Λ (> 0) = cosmological constant, $d\chi^2 = d\theta^2 + \sin^2 \theta d\varphi^2$, (r, θ, φ, t) = Schwarzschild coordinates. A well-known canonical formalism makes equation (1) provide

$$\frac{d^2(1/r)}{d\varphi^2} + \frac{1}{r} = \frac{GM}{L^2} + \frac{3GM}{c^2 r^2} - \frac{\Lambda c^2}{3L^2} r^3, \quad (2)$$

where L = constant angular momentum. This Binet-type equation describes the motion as governed by a force (per unit mass):

$$F(r) = F_N(r) + F_S(r) + F_D(r), \quad (3)$$

in which $F_N = -GM/r^2$ is a Newtonian-type force, $F_S = -3GML^2/(c^2r^4)$ features the local deformation of the spacetime due to the presence of the central mass, whereas $F_D = \Lambda c^2r/3$ is the de Sitter force due to the cosmological background. The force (3) may be regarded as originating in a potential of the form

$$U(q) = A/r + B/r^3 + Dr^2, \quad (4)$$

where $q \in \mathbb{R}^3$ is the radius vector of a test particle with respect to the central mass ($r = |q|$), $A = GM$, $B = GML^2/c^2$, $D = \Lambda c^2/6$.

Different authors tackled the elliptic-type motion of a test particle in such a field from a quantitative point of view. Eddington (1923) and Brumberg (1972) gave the first order variations of some Keplerian orbital elements over one anomalistic period. McVittie (1963) provided an approximate formula for the nodal (sidereal) period as a function of the orbital elements, however without expressing the change of this period. Blaga and Mioc (1992) used a perturbative method to obtain the first and second order variations of the orbital elements over one nodal period, as well as the first order variation of the nodal period itself.

In this paper we start a qualitative study of the problem. Our aim is to understand the dynamics in the neighbourhood of two limit situations: collision and escape. We have to specify that the motion will be studied within the framework of the two-body problem (in the point mass approximation) associated to the potential (4) with $A, B, D > 0$, which we shall hereafter call the *Schwarzschild-de Sitter problem*.

A mention must be made here: taking into account (2), (3) and (4), the relativistic initial problem is transposed within the framework of the classical mechanics. Therefore we can consider it from an alternative standpoint: the motion either in a Newton-type field (with perturbations featured by the terms containing B and D), or in a field described by (4) (without perturbations), with unspecified positive A, B, D . Both cases (mathematically identical) will be regarded as belonging to the realm of classical mechanics, and treated accordingly by qualitative methods.

In Section 2 we use the Hamiltonian formalism to write the equations of motion, then we point out the first integrals of energy and angular momentum.

In Section 3 we resort to the powerful tool of the McGehee-type transformations of the second kind to regularize the motion equations. Under these transformations the (collision) singularity is blown up and replaced by a manifold pasted on the phase space.

Section 4 revisits the flow on the collision manifold and the local flow near collision. The flow on the collision manifold is deprived of physical significance, but – due to the continuity of solutions with respect to the initial data – allows the understanding of near-collisional orbits. This manifold is homeomorphic to a two-

dimensional torus (imbedded in the four-dimensional full phase space), on which the flow is foliated by heteroclinic orbits joining the upper circle and the lower circle, that are entirely formed by degenerate equilibria.

These results coincide with those obtained by Mioc and Stoica (1997) or Stoica and Mioc (1997), due to the vanishing of F_D at collision. As to the local flow near collision, due to the fact that F_D is negligible at small distances from the field source, it can be extracted from the global flow of the Schwarzschild problem provided by Stoica and Mioc (1997).

In a somewhat symmetrical way, another limit situation, the escape, is approached in Section 5. The so-called infinity manifold is constructed via another set of McGehee-type transformations of the second kind. This manifold (also deprived of physical significance) proves to be homeomorphic to a two-dimensional torus, too, imbedded in the four-dimensional full phase-space. The corresponding flow consists of heteroclinic orbits joining the lower circle and the upper circle, both being formed only by degenerate fixed points.

Section 6 describes the structure of the phase space in the neighbourhood of the infinity manifold. Using the rotational symmetry of the problem, and the angular momentum integral, the dimension of the phase space is reduced from 4 to 2. The local flow near escape consists of both rectilinear and spiral orbits, which tend to/come from infinity, or come from infinity towards the central mass, reach a minimum distance then tend back to infinity.

A brief survey of the results obtained in Sections 2–6 is presented in Section 7.

2. EQUATIONS OF MOTION AND FIRST INTEGRALS

The potential (4) is central, so the motion of the test particle is confined to a plane. Consider hence the field source M to be fixed at the origin of this plane $\mathbb{R}^2$. In Hamiltonian formalism, the equations of motion which characterize the Schwarzschild-de Sitter problem read

$$\begin{aligned}\dot{q} &= \partial H(q, p) / \partial p, \\ \dot{p} &= -\partial H(q, p) / \partial q,\end{aligned}\tag{5}$$

where $q = (q_1, q_2) \in \mathbb{R}^2$ is the configuration vector of the particle, while $p = (p_1, p_2) \in \mathbb{R}^2$ is the momentum vector.

Equations (5) define a vector field on the phase space $\mathbf{Q} \times \mathbf{P}$, where $\mathbf{Q} = \mathbb{R}^2 \setminus \{(0, 0)\}$ is the configuration space, whereas $\mathbf{P} = \mathbb{R}^2$ is the momentum space.

The Hamiltonian function

$$H(q, p) = \frac{|p|^2}{2} - \frac{A}{|q|} - \frac{B}{|q|^3} - D|q|^2\tag{6}$$

has the property

$$H(q, p) = h/2 = \text{constant}, \quad (7)$$

namely it is a first integral of the motion (the energy integral; h is the energy constant).

Another first integral is that of angular momentum

$$L(q, p) = q_1 p_2 - q_2 p_1 = C, \quad (8)$$

where C is the angular momentum constant.

Explicitly, the equations of motion are

$$\begin{aligned} \dot{q} &= p, \\ \dot{p} &= \left(-\frac{A}{|q|^3} - \frac{3B}{|q|^5} + 2D \right) q, \end{aligned} \quad (9)$$

with the energy relation

$$|p|^2 - \frac{2A}{|q|} - \frac{2B}{|q|^3} - 2D|q|^2 = h. \quad (10)$$

3. MCGEEHEE'S TRANSFORMATIONS

The potential, the equations of motion, and the integral of energy have an isolated singularity at the origin: $q = (0, 0)$. It can be proved that this singularity corresponds to a collision particle-centre (see, e.g., Witner 1941, or Saari 1974). To remove it, we resort to the McGehee-type transformations of the second kind (McGehee 1974).

The first step of these transformations introduces the standard polar coordinates via the real analytic diffeomorphism

$$(0, \infty) \times (0, \infty) \times \mathbb{R}^2 \rightarrow (0, \infty) \times S^1 \times \mathbb{R} \times \mathbb{R}$$

$$(q_1, q_2, p_1, p_2) \rightarrow (r, \theta, u, v)$$

consisting of

$$\begin{aligned} r &= |q|, \\ \theta &= \arctan(q_2 / q_1), \\ u &= \dot{r} = (q_1 p_1 + q_2 p_2) / |q|, \\ v &= r \dot{\theta} = (q_1 p_2 - q_2 p_1) / |q|. \end{aligned} \quad (11)$$

The equations of motion acquire the form

$$\begin{aligned}\dot{r} &= u, \\ \dot{\theta} &= \frac{v}{r}, \\ \dot{u} &= \frac{v^2}{r} - \frac{A}{r^2} - \frac{3B}{r^4} + 2Dr, \\ \dot{v} &= -\frac{uv}{r},\end{aligned}\quad (12)$$

while the first integrals of energy and angular momentum become respectively

$$u^2 + v^2 - \frac{2A}{r} - \frac{2B}{r^3} - 2Dr^2 = h, \quad (13)$$

$$rv = C. \quad (14)$$

The singularity corresponds now to $r = 0$. Examining the energy relation, we notice that, since h is finite, the velocity must tend to infinity as r tends to zero. So, for the second step we have to scale down the polar components of the velocity.

Consider the real analytic diffeomorphism

$$(0, \infty) \times S^1 \times \mathbb{R} \times \mathbb{R} \rightarrow (0, \infty) \times S^1 \times \mathbb{R} \times \mathbb{R}$$

$$(r, \theta, u, v) \rightarrow (r, \theta, x, y)$$

consisting of

$$\begin{aligned}x &= r^{3/2}u, \\ y &= r^{3/2}v.\end{aligned}\quad (15)$$

Under these transformations, the equations of motion become

$$\begin{aligned}\dot{r} &= \frac{x}{r^{3/2}}, \\ \dot{\theta} &= \frac{y}{r^{5/2}}, \\ \dot{x} &= \frac{3x^2/2 + y^2 - 3B}{r^{5/2}} - \frac{A}{r^{1/2}} + 2Dr^{5/2}, \\ \dot{y} &= \frac{xy}{2r^{5/2}},\end{aligned}\quad (16)$$

and the integrals of energy and angular momentum read respectively

$$x^2 + y^2 = 2Dr^5 + hr^3 + 2Ar^2 + 2B, \quad (17)$$

$$y = Cr^{1/2}. \quad (18)$$

The singularity at $r = 0$ still persists. To remove it, we perform the last step of the McGehee-type transformations

$$dt = r^{5/2} ds, \quad (19)$$

which rescales the time. The equations of motion now read

$$\begin{aligned} r' &= rx, \\ \theta' &= y, \\ x' &= \frac{3}{2}x^2 + y^2 - 3B - Ar^2 + 2Dr^5, \\ y' &= \frac{1}{2}xy, \end{aligned} \quad (20)$$

where primes mean differentiation with respect to the timelike variable s , and we kept by abuse the same notations for the new functions of s .

Both the equations of motion (20) and the first integrals are now well defined for the boundary $r = 0$. This means that the phase space can be extended analytically to contain the manifold $\{(r, \theta, x, y) | r = 0\}$. This manifold is invariant under the flow because $r' = 0$ when $r = 0$. The relations (17) and (18) also extend smoothly to this boundary.

4. THE FLOW ON THE COLLISION MANIFOLD AND THE NEAR - COLLISION FLOW

Consider the sets

$$M_{col} = \{(r, \theta, x, y) | r = 0\},$$

$$E_h = \{(r, \theta, x, y) | x^2 + y^2 = 2Dr^5 + hr^3 + 2Ar^2 + 2B\},$$

and define the *collision manifold* as being their intersection $M_0 = M_{col} \cap E_h$, namely

$$M_0 = \{(r, \theta, x, y) | r = 0; \theta \in S^1; x^2 + y^2 = 2B\}. \quad (21)$$

In this way, applying the McGehee-type transformations, we have blown up the collision singularity, and pasted on the phase space, instead of it, the manifold M_0 . The flow on this invariant manifold is deprived of physical significance. Nevertheless, due

to the continuity of the solutions with respect to the initial conditions, the respective flow provides valuable informations about the behaviour of near-collisional orbits.

Since $B > 0$, M_0 is a 2-dimensional cylinder in the 3-dimensional space of the coordinates $(\theta, x, y) \in S^1 \times \mathbb{R}^2$ (see Fig. 1). But, since $\theta \in S^1$ (the segment $[0, 2\pi]$ with the end points pasted together), the M_0 cylinder may be identified with a torus, both being actually imbedded in the 4-dimensional full phase space of the coordinates (r, θ, x, y) .

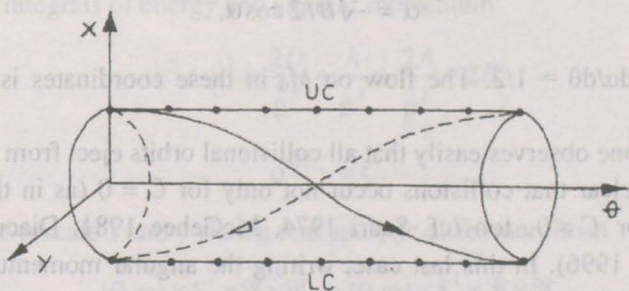


Fig. 1 – The collision manifold as a cylinder and the flow on it.

By (17) and (20), the vector field on M_0 is given by

$$\begin{aligned} \theta' &= y, \\ x' &= -\frac{1}{2}y^2, \\ y' &= \frac{1}{2}xy. \end{aligned} \quad (22)$$

and is identical with the vector field on M_0 corresponding to the Schwarzschild-type two-body problem. This is natural because, from the mathematical standpoint, making $r = 0$ in the third equation (20), which is characteristic to the Schwarzschild-de Sitter problem, is equivalent to making $r = 0$ in the same equation with $D = 0$, which corresponds to the Schwarzschild problem. In addition, the collision manifold is featured by the same expression for both problems.

The flow on M_0 (for the Schwarzschild problem) was described by Mioc and Stavinschi (1997), Mioc and Stoica (1997), or Stoica and Mioc (1997). We shall revisit it here.

Equations (22) admit the fixed points $(\theta, x, y) = (\theta_0, \pm\sqrt{2B}, 0)$, with arbitrary $\theta_0 \in S^1$. In other words, there are two circles of degenerate equilibria on the torus, at $y = 0$. Since, by (22), $x' < 0$ for $y \neq 0$, all other orbits of the flow on

M_0 are heteroclinic and move from the *upper circle* (UC): $(\theta \in S^1, x = \sqrt{2B}, y = 0)$ to the *lower circle* (LC): $(\theta \in S^1, x = -\sqrt{2B}, y = 0)$.

To deduce the slope of these trajectories, put $x = \sqrt{2B} \sin \alpha$, $y = -\sqrt{2B} \cos \alpha$. This leads to

$$\theta' = -\sqrt{2B} \cos \alpha,$$

$$\alpha' = -\sqrt{B/2} \cos \alpha,$$

which means $d\alpha/d\theta = 1/2$. The flow on M_0 in these coordinates is illustrated in Fig. 1.

By (18) one observes easily that all collisional orbits eject from UC or tend to LC. It is also clear that collisions occur not only for $C = 0$ (as in the Newtonian model), but for $C \neq 0$, too (cf. Saari 1974, McGehee 1981; Diacu et al. 1995, Delgado et al. 1996). In this last case, writing the angular momentum integral in polar coordinates, one sees that $\dot{\theta} \rightarrow \infty$ for $r \rightarrow 0$, which means that the particle spirals infinitely many times around the centre before collision / after ejection. This is the so-called *black hole effect* (Diacu et al. 1995).

An important remark is to be made here: M_0 does not depend on h , therefore every level of energy shares this boundary.

The local flow near collision is easy to obtain. Recall that the cosmological constant Λ has a very small value (of the order of 10^{-56} cm^{-2} ; see e.g. Brumberg 1972). Taking into account equations (2) and (3), this means that F_D is negligible at small distances from the field source. This is also obvious from (4): in the Schwarzschild-de Sitter field the particle behaves near collision just like in the Schwarzschild field.

But Stoica and Mioc (1997) depicted the global structure of the phase space corresponding to Schwarzschild-type problems ($A, B \in \mathbb{R}, D = 0$). Consequently, the near-collision flow of the Schwarzschild-de Sitter problem can be deduced from the global flow of the Schwarzschild problem for $A > 0, B > 0$ (Stoica and Mioc 1997, Figs. 2 and 3).

5. INFINITY MANIFOLD AND THE FLOW ON IT

By some symmetry of the treatment, let us see what happens with the escape orbits when $r \rightarrow \infty$. With this end in view, put $\rho = 1/r$; the vector field (20) acquires the form

$$\rho' = -\rho x,$$

$$\theta' = y,$$

$$x' = \frac{3}{2}x^2 + y^2 - 3B - \frac{A}{\rho^2} + \frac{2D}{\rho^5}, \quad (23)$$

$$y' = \frac{1}{2}xy,$$

with the first integrals of energy and angular momentum:

$$x^2 + y^2 = \frac{2D}{\rho^5} + \frac{h}{\rho^3} + \frac{2A}{\rho^2} + 2B, \quad (24)$$

$$\rho^{1/2}y = C. \quad (25)$$

Now we rescale x and y via the real analytic diffeomorphism

$$(0, \infty) \times S^1 \times \mathbb{R} \times \mathbb{R} \rightarrow (0, \infty) \times S^1 \times \mathbb{R} \times \mathbb{R}$$

$$(\rho, \theta, x, y) \rightarrow (\rho, \theta, \xi, \eta),$$

consisting of the McGehee-type transformation

$$\xi = \rho^{5/2}x, \quad (26)$$

$$\eta = \rho^{5/2}y.$$

Under this change of variables the equations of motion read

$$\rho' = \frac{\xi}{\rho^{3/2}},$$

$$\theta' = \frac{\eta}{\rho^{5/2}},$$

$$\xi' = \frac{-\xi^2 + \eta^2 + 2D}{\rho^{5/2}} - A\rho^{1/2} - 3B\rho^{5/2},$$

$$\eta' = -\frac{2\xi\eta}{\rho^{5/2}},$$

whereas the first integrals (24) and (25) become respectively

$$\xi^2 + \eta^2 = 2B\rho^5 + 2A\rho^3 + h\rho^2 + 2D, \quad (28)$$

$$\eta = C\rho^2. \quad (29)$$

To remove the singularity $\rho = 0$, we perform the last step of these McGehee-type transformations by rescaling the timelike variable s via

$$ds = \rho^{5/2} d\tau. \tag{30}$$

Under this transformation, the equations of motion (27) acquire the regular form

$$\begin{aligned} d\rho/d\tau &= -\rho\xi, \\ d\theta/d\tau &= \eta, \\ d\xi/d\tau &= -\xi^2 + \eta^2 + 2D - A\rho^3 - 3B\rho^5, \\ d\eta/d\tau &= -2\xi\eta, \end{aligned} \tag{31}$$

where we preserved by abuse the same notations for the new functions of τ .

Let us denote

$$\begin{aligned} M_{\text{inf}} &= \{(\rho, \theta, \xi, \eta) \mid \rho = 0\}, \\ \tilde{E}_h &= \{(\rho, \theta, \xi, \eta) \mid \xi^2 + \eta^2 = 2B\rho^5 + 2A\rho^3 + h\rho^2 + 2D\}, \end{aligned}$$

and define the *infinity manifold* as being their intersection, $M_\infty = M_{\text{inf}} \cap \tilde{E}_h$, namely

$$M_\infty = \{(\rho, \theta, \xi, \eta) \mid \rho = 0; \theta \in S^1; \xi^2 + \eta^2 = 2D\}. \tag{32}$$

In the 3-dimensional space of the coordinates $(\theta, \xi, \eta) \in S^1 \times \mathbb{R}^2$ the manifold (32) is homeomorphic to a 2-dimensional cylinder, actually imbedded in the 4-dimensional full phase space (see Fig. 2). Because $\theta = 0$ and $\theta = 2\pi$ can

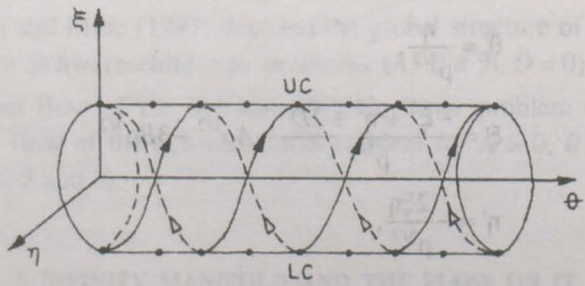


Fig. 2 – The infinity manifold as a cylinder and the flow on it.

be identified, whereas $D > 0$, M_∞ is homeomorphic to a torus.

The vector field on the M_∞ torus is

$$\begin{aligned}d\theta/d\tau &= \eta, \\d\xi/d\tau &= 2\eta^2, \\d\eta/d\tau &= -2\xi\eta,\end{aligned}\tag{33}$$

where the energy relation (28) for $\rho = 0$ was used to remove ξ and D from the right-hand side of the second equation.

As in the case of the collision manifold, there are two circles of degenerate equilibria on the M_∞ torus: the *upper circle* (UC): $(\theta \in S^1, \xi = \sqrt{2D}, \eta = 0)$ and the *lower circle* (LC): $(\theta \in S^1, \xi = -\sqrt{2D}, \eta = 0)$. Since, by (33), $d\xi/d\tau > 0$ for $\eta \neq 0$, all other trajectories of the flow on M_∞ are heteroclinic and move from LC to UC. Their slope can be easily deduced by introducing the angle α through the relations $\xi = \sqrt{2D} \sin \alpha$, $\eta = -\sqrt{2D} \cos \alpha$, which leads to

$$\begin{aligned}d\theta/d\tau &= -\sqrt{2D} \cos \alpha, \\d\alpha/d\tau &= \sqrt{8D} \cos \alpha,\end{aligned}\tag{34}$$

meaning $d\alpha/d\theta = -2$. The flow on M_∞ in these coordinates is illustrated in Fig. 2.

As in the case of the collision manifold, we have to remark that M_∞ does not depend on the constant of energy h . Consequently, every energy level shares this boundary.

Another important remark concerns the way of obtaining the equations of motion (31) and the first integrals (28) and (29).

They could be found by using the variable changes

$$\begin{aligned}\xi &= \dot{r}/r, \\ \eta &= \dot{\theta},\end{aligned}\tag{35}$$

without resorting to the chain of McGehee-type transformations. However, the way we have chosen seemed us more elegant.

As a final remark, observe by (19) and (30) that the flow on M_∞ has the physical time t as time scale.

6. NEAR - INFINITY FLOW

In this section we shall describe the phase space structure in the neighbourhood of the infinity manifold. Consider ρ to be sufficiently small to neglect ρ^n , $n > 2$, in equations (28) and (31). Under these conditions, the latter ones become

$$\begin{aligned}
 d\rho/d\tau &= -\rho\xi, \\
 d\theta/d\tau &= \eta, \\
 d\xi/d\tau &= -\xi^2 + 2D, \\
 d\eta/d\tau &= -2\xi\eta.
 \end{aligned}
 \tag{36}$$

Observe now that θ does not appear explicitly in either motion equations or energy integral. So, we can reduce the 4-dimensional full phase space to dimension 3 by factorizing the flow to S^1 . In this way we get the reduced phase space of the coordinates (ρ, ξ, η) . Of course, a solution in the reduced phase space must be regarded $\times S^1$ in the full phase space (hence as a manifold of solutions).

We can further reduce the dimension of the phase space by resorting to the integral of angular momentum (29), which relates η to ρ . So we obtain the phase plane of the coordinates (ρ, ξ) . It is obvious that what we said above about the orbits in the reduced phase space is to be applied to the solutions in the phase plane, too.

In the phase plane (ρ, ξ) the vector field (36) reads

$$\begin{aligned}
 d\rho/d\tau &= -\rho\xi, \\
 d\xi/d\tau &= -\xi^2 + 2D,
 \end{aligned}
 \tag{37}$$

whereas M_∞ , which is a torus in full phase space and is restricted to the circle $\{(\rho, \xi, \eta) | \rho=0; \xi^2 + \eta^2 = 2D\}$ in the reduced phase space, is further restricted in the phase plane (ρ, ξ) to two points: M ($\rho=0; \xi = \sqrt{2D}$) and N ($\rho=0, \xi = -\sqrt{2D}$).

In our approximation, the integral of energy becomes

$$\xi^2 = h\rho^2 + 2D.
 \tag{38}$$

Plotting the function $\xi = \xi(\rho)$, we can depict the near-infinity flow of the Schwarzschild-de Sitter problem (Fig. 3).

The points M and N are equilibria (see Section 7 below). In the physical space they represent degenerate "orbits" which remain forever at infinity.

For $h > 0$ we have the curves starting from N or tending to M . In the physical plane they represent radial ($C = 0$) or spiral ($C \neq 0$) motion coming from infinity, or tending to infinity, respectively.

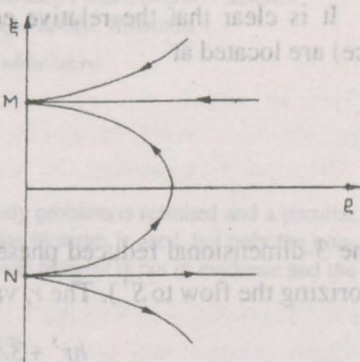
For $h = 0$ we have the two halflines, one starting from N , the other tending to M . Their physical interpretation is wholly similar to that corresponding to the positive energy case.

As to the heteroclinic trajectories which start from N and tend to M , they correspond to $h < 0$. They physically represent motions in which the particle comes radially or spiralling from infinity, reaches a minimum distance to the centre

$$\rho_{\max}(=r_{\min}) = -2D/h, \quad (39)$$

then tends back to infinity.

Fig. 3 – The near-infinity flow on the (ρ, ξ) -plane.



7. CONCLUDING REMARKS

We obtained in Sections 4 and 5 the flow on the collision manifold M_0 , and that on the infinity manifold M_∞ . Of course, these flows have no physical significance; however their description allows the understanding of the behaviour of the physical motion in two special situations: near collision, respectively very far from the centre.

Taking into account the McGehee-type transformations (19) and (30), one sees that the M_0 and M_∞ manifolds constitute equilibria for the phase orbits.

The time scales for the flows on M_0 and M_∞ are considerably different; however, the time scale for the flow on M_∞ coincides with the physical time t .

The flow on M_0 and the near-collisional flow for the Schwarzschild-de Sitter model coincide with those corresponding to the Schwarzschild problem for $A > 0$, $B > 0$.

As regards the flow on M_∞ and the near-infinity flow, the Schwarzschild-de Sitter problem differs drastically from the Schwarzschild problem (with $A > 0$, $B > 0$). The infinity manifold has a similar structure within the framework of both problems. But, on the one hand, the slope of the heteroclinic orbits differs in the two cases. On the other hand (and this is more important), the infinity manifold does not depend on the energy level h in the Schwarzschild-de Sitter problem, but only on D . This means that every energy level can share the boundary represented by the infinity.

To make a step further, (although this exceeds the framework of the present study) we shall mention another difference between the Schwarzschild-de Sitter problem and the Schwarzschild one. The natural first step in understanding the global flow is searching for equilibria. Consider hence the system (20) in which, by virtue of the energy relation (17), the third equation is replaced by

$$x' = \frac{5}{2}x^2 + 2y^2 - 5B - 3Ar^2 - hr^3. \quad (40)$$

It is clear that the relative equilibria for $r \neq 0$ (circular orbits in physical space) are located at

$$\begin{aligned} r &= r_e, \\ x &= 0, \\ y &= C\sqrt{r_e} \end{aligned} \quad (41)$$

in the 3-dimensional reduced phase space of the coordinates (r, x, y) (obtained by factorizing the flow to S^1). The r_e values are the real positive roots of the equation

$$hr^3 + 3Ar^2 - 2C^2r + 5B = 0. \quad (42)$$

The interplay among the field characteristic quantities A and B , the level of energy h and the angular momentum C makes possible the existence of three, two, one, or zero relative equilibria (for the Schwarzschild-de Sitter problem). As to the Schwarzschild problem with $A > 0$, $B > 0$, its global flow admits at most two relative equilibria (see Stoica and Mioc 1997)

Problems as the location of the relative equilibria, their nature, the manifolds associated to them, the existence of orbits connecting the equilibria on M_0 and M_∞ , and so forth will be treated elsewhere.

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THE ELLIPTIC RESTRICTED THREE - BODY PROBLEM. CONSTANT EQUIPOTENTIAL SURFACES

RODICA ROMAN

„Babeş-Bolyai” University, Faculty of Mathematics and Computer Science

Str. M. Kogălniceanu 1, 3400 Cluj-Napoca, Romania

E.mail: rroman@math.ubbcluj.ro

Abstract. The study of the elliptic restricted three-body problem is resumed and a peculiar case is considered. For the radius-vector an expansion in series is used, but only the non-periodic contribution is taken into account. A *Jacobian integral* is put in evidence and the positions of the Lagrangian points are determined.

Key words: celestial mechanics - three body problem.

1. INTRODUCTION

In a previous paper, Todoran (1993) has analysed the effects of the orbital eccentricity on the shape of the equipotential surfaces in close binary systems. So it was put in evidence their pulsational character. Nevertheless, there are some problems where we need a constant Roche limit, e.g. the use of equipotential surfaces for the study of the stellar interior, or some statistical investigations. An analysis of such constant equipotential surfaces is the aim of the present note.

2. BASIC EQUATIONS OF THE PROBLEM

Consider a non-uniform rotating barycentric coordinate system (x, y, z) , assumed in rotation with the binary system, x -axis being always directed towards the mass-centres of the two finite bodies. Here the true anomaly, v , is considered as an “angular” function reckoned from the periastron, while xy -plane coincides with the orbital plane. In such conditions, the differential equations for the motion of an infinitesimal body may be written in the form [see Todoran, 1993; Eqs. (5) and (6)].

$$\begin{aligned} \frac{d^2 x}{dt^2} - 2 \left(\frac{dv}{dt} \right) \left(\frac{dy}{dt} \right) &= \left(\frac{dv}{dt} \right)^2 x - G \frac{m_1}{r_1^3} (x + R_1) - G \frac{m_2}{r_2^3} (x - R_2) + y \frac{d^2 v}{dt^2}, \\ \frac{d^2 y}{dt^2} + 2 \left(\frac{dv}{dt} \right) \left(\frac{dx}{dt} \right) &= \left(\frac{dv}{dt} \right)^2 y - G \frac{m_1}{r_1^3} y - G \frac{m_2}{r_2^3} y - x \frac{d^2 v}{dt^2}, \end{aligned} \quad (1)$$

$$\frac{d^2 z}{dt^2} = -G \frac{m_1}{r_1^3} z - G \frac{m_2}{r_2^3} z,$$

where

$$r_1^2 = (x + R_1)^2 + y^2 + z^2, \quad r_2^2 = (x - R_2)^2 + y^2 + z^2. \quad (2)$$

Here m_1 and m_2 are stellar masses, R_1 and R_2 stand for the distances between the common mass centre, M , and the two finite bodies, S_1 and S_2 , respectively.

The separation between the two stars $R (= R_1 + R_2)$ is given by

$$R = \frac{A(1 - e^2)}{1 + e \cos v}, \quad (3)$$

where A = semi-major axis of the relative orbit, e = orbital eccentricity.

From gravitational point of view, the two component stars S_1 and S_2 are assumed as being mass-points.

In equations (1) the second terms on the left-hand side, multiplied by dy/dt and, respectively, by dx/dt , represent the effects of Coriolis force - Coriolis accelerations.

The first terms on the right-hand side, multiplied by x and respectively y , represent centrifugal effects. The next two terms represent gravitational effects, while the forth terms represent the acceleration normal to the radius vector due to non-uniform rotation of the coordinate system.

Now, let us have in view the fact that in a binary system, the distances between the common mass centre and the two finite bodies are inversely proportional to the corresponding masses. That is

$$\frac{R_2}{R_1} = \frac{m_1}{m_2},$$

whence we have at once

$$R_1 = \frac{m_2}{m_1 + m_2} R, \quad R_2 = \frac{m_1}{m_1 + m_2} R \quad (4)$$

In addition, from the areas law, it follows that

$$\frac{dv}{dt} = \frac{C}{R^2}; \quad C = \sqrt{G(m_1 + m_2)A(1 - e^2)} \quad (5)$$

whence we may write

$$\frac{dv}{dt} = \sqrt{\frac{G(m_1 + m_2)}{A^3(1 - e^2)^3}} (1 + e \cos v)^2.$$

3. AN ELEMENTARY EXPANSION IN SERIES FOR THE RADIUS-VECTOR

As it is easy to see, from equation (3) we obtain the following expansion in series:

$$R = A(1 - e^2) \left[1 + (-1)^n \sum_{n=1}^{\infty} e^n \cos^n \nu \right],$$

or

$$R = A(1 - e^2) \left[1 + \sum_{k=1}^{\infty} e^{2k} \cos^{2k} \nu - \sum_{k=1}^{\infty} e^{2k-1} \cos^{2k-1} \nu \right] \quad (6)$$

Moreover, if we have in mind the corresponding formulas for the connection between $\cos^n \nu$ and $\cos n\nu$, equation (6) leads to

$$R = A(1 - e^2) \left[1 + \sum_{j=1}^{\infty} C_{2j}^j \left(\frac{e}{2} \right)^{2j} + \text{periodic terms} \right],$$

where periodic terms are multiplied by $\cos \nu$, $\cos 2\nu$ and so on. Here C_{2j}^j denote algebraical combinations.

Now, if we are neglecting the periodic terms, the average radius-vector will be given by

$$R = AF_1(e), \quad (7)$$

where, for brevity, we have used:

$$F_1(e) = (1 - e^2) \left[1 + \sum_{j=1}^{\infty} C_{2j}^j \left(\frac{e}{2} \right)^{2j} \right]. \quad (8)$$

Equation (7) shows that, in such conditions, the radius-vector R and the orbital motion are considered as being constants.

Consequently, equations (4) and (5) become:

$$R_1 = \frac{m_2}{m_1 + m_2} AF_1(e), \quad R_2 = \frac{m_1}{m_1 + m_2} AF_1(e) \quad (9)$$

and

$$\frac{dv}{dt} = \frac{\sqrt{G(m_1 + m_2)}}{A^{3/2}} F_2(e) \quad (10)$$

where we have used the substitution

$$F_2(e) = \frac{(1 + e \cos v)^2}{(1 + e^2)^{3/2}}$$

Here we shall make the substitution

$$1 + 2e \cos v + e^2 \cos^2 v = 1 + \frac{e^2}{2} + 2e \cos v + \frac{e^2 \cos 2v}{2}$$

where the terms containing $\cos v$ and $\cos 2v$ will be ignored, so that:

$$F_2(e) = \frac{2 + e^2}{2(1 - e^2)^{3/2}} \quad (11)$$

It is well known that for close binary systems, generally, we have $e < 0.5$ and equation (8) may be written in the form

$$F_1(e) = (1 - e^2) \left(1 + \frac{e^2}{2} + \frac{3}{8}e^2 + \dots \right). \quad (12)$$

4. EQUATIONS OF THE CONSTANT EQUIPOTENTIAL SURFACES

Consider now the following system of units: the semi-major axis ($A = 1$) as unit of length, the sum of masses ($m_1 + m_2 = 1$) as unit of mass, and the reciprocal of the mean angular velocity as unit of time. Therefore, we have $P = 2\pi$ for the orbital period and $G = 1$.

In such conditions we may write:

$$m_1 = \frac{1}{1+q}; \quad m_2 = \frac{q}{1+q}; \quad q = \frac{m_2}{m_1}$$

and equations (9) and (10) become:

$$R_1 = \frac{q}{1+q} F_1; \quad R_2 = \frac{1}{1+q} F_1; \quad \frac{dv}{dt} = F_2 = \text{const.} \quad (13)$$

Therefore, equations (1) will be written in the form:

$$\begin{aligned} \frac{d^2x}{dt^2} - 2 \frac{dy}{dt} F_2 &= x F_2^2 - \frac{1}{1+q} \frac{x + \frac{q}{1+q} F_1}{r_1^3} - \frac{q}{1+q} \frac{x - \frac{1}{1+q} F_1}{r_2^3} = \frac{\partial U}{\partial x} \\ \frac{d^2y}{dt^2} + 2 \frac{dx}{dt} F_2 &= y F_2^2 - \frac{1}{1+q} \left(\frac{y}{r_1^3} \right) - \frac{q}{1+q} \left(\frac{y}{r_2^3} \right) = \frac{\partial U}{\partial y} \end{aligned} \quad (14)$$

$$\frac{d^2 z}{dt^2} = -\frac{1}{1+q} \left(\frac{z}{r_1^3} \right) - \frac{q}{1+q} \left(\frac{z}{r_2^3} \right) = \frac{\partial U}{\partial z},$$

with

$$r_1^2 = \left(x + \frac{q}{1+q} F_1 \right)^2 + y^2 + z^2; \quad r_2^2 = \left(x - \frac{1}{1+q} F_1 \right)^2 + y^2 + z^2 \quad (15)$$

In equations (14) the corresponding potential function is defined by:

$$U = \frac{1}{1+q} \left(\frac{1}{r_1} \right) + \frac{q}{1+q} \left(\frac{1}{r_2} \right) + \frac{1}{2} F_2^2 (x^2 + y^2) \quad (15a)$$

5. THE LOCATION OF THE LAGRANGIAN POINTS

We must have in mind the fact that, for Lagrangian points the vector field (14) is zero (see, also Kopal, 1989, p.13). So (14) lead to

$$xF_2^2 - \frac{1}{1+q} \frac{x + \frac{q}{1+q} F_1}{r_1^3} - \frac{q}{1+q} \frac{x - \frac{1}{1+q} F_1}{r_2^3} = 0 \quad (16)$$

$$y \left(F_2^2 - \frac{1}{1+q} \frac{1}{r_1^3} - \frac{q}{1+q} \frac{1}{r_2^3} \right) = 0 \quad (17)$$

$$z \left(\frac{1}{1+q} \frac{1}{r_1^3} + \frac{q}{1+q} \frac{1}{r_2^3} \right) = 0 \quad (18)$$

Now, for $x \neq 0$, $y = z = 0$, from equation (16) we obtain the abscissas of the three colinear Lagrangian points, while for $x \neq 0$, $y \neq 0$ and $z = 0$, from equation (16) we have the corresponding coordinates for the two triangular points.

a) *The inner Lagrangian point L_1 .* Consider that M represents the position of the common mass-centre. In such a case, for the inner Lagrangean point $L_1 \in (M, S_2)$, that is, for $q < 1$, we have to put:

$$x = x_{L_1}; \quad r_1 = x_{L_1} + R_1; \quad r_2 = R_2 - x_{L_1}; \quad y = z = 0,$$

and equation (16) becomes:

$$x_{L_1} F_2^2 - \frac{1+q}{[x_{L_1}(1+q) + qF_1]^2} + \frac{q(1+q)}{[x_{L_1}(1+q) - F_1]^2} = 0 \quad (19)$$

Moreover, from equation (15a) we may write:

$$C_1 = \frac{1}{2} x_{L_1}^2 F_2^2 + \frac{1}{|x_{L_1}(1+q) + qF_1|} + \frac{q}{|x_{L_1}(1+q) - F_1|}. \quad (20)$$

Then, from equation (15a) for each value of the abscissa, x , we may calculate the corresponding ordinate, y . In such a way, the intersection between the Roche limit and $z = 0$ plane may be obtained.

b) *The triangular solution.* For $y \neq 0$ and $z = 0$, from equation (17) it follows that

$$F_2^2 - \frac{1}{1+q} \left(\frac{1}{r_1^3} \right) - \frac{q}{1+q} \left(\frac{1}{r_2^3} \right) = 0 \quad (21)$$

while equation (16) may be written in the form:

$$x \left[F_2^2 - \frac{1}{1+q} \left(\frac{1}{r_1^3} \right) - \frac{q}{1+q} \left(\frac{1}{r_2^3} \right) \right] - \frac{qF_1}{(1+q)^2} \left(\frac{1}{r_1^3} - \frac{1}{r_2^3} \right) = 0 \quad (22)$$

whence we have at once:

$$r_1 = r_2 = r.$$

Thus, from equation (21) we may write:

$$F_2^2 - \frac{1}{r^3} = 0,$$

or

$$r = r_1 = r_2 = F_2^{2/3}. \quad (23)$$

Moreover, in the present case, the triangular solution is referring to such a situation when the three bodies are placed at the corners of an isosceles triangle

$$S_1 P = S_2 P = (1 - e^2) \left(\frac{2}{2 + e^2} \right)^{2/3},$$

and

$$S_1 S_2 = F_1 = (1 - e^2) \left[1 + \sum_{j=1}^{\infty} C_{2,j}^j \left(\frac{e}{2} \right)^{2j} \right]$$

Now, from equation (15) it follows that:

$$\left(x + \frac{q}{1+q} F_1\right)^2 = \left(x - \frac{1}{1+q} F_1\right)^2$$

whence we have:

$$x_{4,5} = \left(\frac{1}{2}\right) \left(\frac{1-q}{1+q}\right) F_1. \quad (24)$$

Let us consider once again equations (15) and (22) whence we may write :

$$y^2 = r^2 - \left(x + \frac{q}{1+q} F_1\right)^2 = (F_2)^{-4/3} - \frac{1}{4} F_1^2$$

Therefore, for the ordinates of the two triangular points we have:

$$y_{4,5} = \pm \sqrt{F_2^{-4/3} - \frac{1}{4} F_1^2}, \quad (25)$$

where:

$$F_2^{-4/3} - \frac{1}{4} F_1^2 > 0,$$

for $e \leq 1$.

Remark: It is easy to see that for $e = 0$, the three bodies (S_1, S_2, P') will be placed at the corners of an equilateral triangle:

$$S_1 S_2 = S_1 P' = S_2 P' = 1.$$

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NATIONAL SEMINAR

"THE TOTAL SOLAR ECLIPSE OF AUGUST 11, 1999 INTERDISCIPLINARY APPROACH"

BUCHAREST, OCTOBER 15, 1998

A really exceptional astronomical event will focus the attention of the whole world on Romania: **the total solar eclipse of August 11, 1999**. Considered from many standpoints to be the eclipse of the century, its maximum will be in our country; this is a unique chance for many domains of the Romanian scientific research.

The Romanian astronomers are preparing a series of specific programmes in cooperation with specialists from abroad. But the importance of the phenomenon requires a national cooperation, too, with all research institutes, centres, or groups that are interested in the scientific capitalization of such an event.

With this end in view, on October 15, 1998 the National Seminar "The Total Solar Eclipse of August 11, 1999 - Interdisciplinary Approach" was held in Bucharest at World Trade Center. The Seminar was organized by the Romanian Academy and the Ministry of Research and Technology through the agency of the Astronomical Institute, the Romanian National Astronomical Committee, the International Association "Eclipsa '99", and the Romanian Space Agency.

Its main goal was to facilitate the exchange of ideas concerning the scientific exploitation of the results of eclipse observation.

The Seminar was intended to:

- present the specific research programmes in view of the scientific capitalization of the event;
- identify the possibilities and ways of collaboration, to merge all efforts in order to obtain a maximum scientific benefit for all implied institutions;
- discuss the possibilities of achieving the proposed projects with maximum efficiency.

There were presented 25 programmes proposed by the Astronomical Institute of the Romanian Academy, the Institute of Space Sciences, the National Institute for Physics of Materials, the Institute of Geodynamics "Sabba Ștefănescu", the National Institute of Meteorology and Hydrology, the Geologic Institute, the Institute for Optoelectronics, S.A., the National Institute of Biological Sciences, the National Institute for Glass - Ploiești Branch, the Faculty of Mathematics and Computer Science of the "Babeș-Bolyai" University, Cluj-Napoca, the Faculty of Physics of the Bucharest University, the Faculty of Electronics and Communications of the "Politehnica" University, and the Romanian representation of "Silicon Graphics".

The National Seminar also benefitted of an international participation; the stage of preparations and the main programmes for the eclipse in France and Bulgaria were presented by Zadiğ Mouradian, director of programs at Observatoire de Paris-Meudon, and Vladimir Dermendzhiev, deputy director of the Institute of Astronomy of the Bulgarian Academy.

The fact that other institutes announced in the last moment their intention to participate, as well as the necessity to present the proposed programmes in an advanced stage, imposes to resume the National Seminar in the spring of '99, some months before the event.

MAGDA STAVINSCHI

THE FIRST ROMANIAN ASTRONOMER : THE MONK DIONYSIUS EXIGUUS

Overlooking some preoccupations of empirical observation of the celestial vault from other centuries, the first quoted name of a Romanian astronomer is that of the Father Superior Silvan of Putna. To him a *Paschalia*, drawn up around 1493, has been attributed.

We would like to point out that, judging by the same merits of paschaliography, the title of the first Romanian astronomer should better be attributed to the monk Dionysius Exiguus, dead in Italy at about 555.

Across the meridians of Europe, this ancient Romanian scholar is known as the initiator of the Christian Era, which led to the abolishment of the Martyrs Era, named after the cruel Roman emperor Diocletian († 316).

Dionysius Exiguus concluded this universally important result after long efforts to establish the paschal cycle.

For this goal he gathered numerous data of ecclesiastical history, theology, canonical tradition, astronomy (lunar phases and eclipses, the date of the Jewish Easter, etc.).

The scientific conclusions of the Dacian-Roman monk from the pontifical court set forth as the most sure solution. They were obtained following a centuries old confrontation between the Western and the Eastern Christianity. Due to Dionysius Exiguus' stand, and following the official request of the Roman chancellor's office, the West adopted again the paschaliographical tradition of the Ecumenical Synod of Niceea (325). This forum requested the Patriarchy of Alexandria to calculate the date of the Easter celebration and to communicate it to the other ecumenical christian centers as well.

Being forced to prolong the paschal cycle of Saint Chiril the Great († 444), Dionysius Exiguus had to specify also the date of the Saviour's birth, which he fixed in the year 754 from the foundation of Rome. For this goal he made use only of the means in the field of the ecclesiastical comput, extracted from the Gospel. As he did his best to match the years of the Christian Era with those of the 19 years cycle, it becomes evident that the computation of the Tomis-born scholar actually begins with the announcement of the Lord's birth made by the Saint Archangel Gabriel in 753 B.C.

Dionysius followed the same trend of paschal elements also when he took into consideration the date of the Saviour's crucification. He thus established that the Lord revived on 25 March, the year 31.

For the establishment of the Embodiement date he also took into consideration the deductions concerning Irod's death. Irod died 37 years after receiving the decree of the Roman Senate by which he was appointed ruler. Jesus Christ was over four years old at the death of this King and of the infants of Bethlehem killed at his order.

The specification of the bloody tyrant's end is made also from the astronomical point of view. Today it is known that from the three eclipses of the epoch, the last one occurred on the 12th to the 13th of March 750; after a while there followed the death of the sad memory monarch.

Chapter 2 of Saint Lucas gives more precise informations for the date when Jesus Christ came to life. Today it is known that the Emperor Octavian the Augustus ordered three censuses in his empire. The second of these three censuses took place in 749, the probable year of the Saviour's birth.

Quirinius was then Emperor's legate for Judeea, due to his rich experience, as one who ten years back, when the first census was carried out, enjoyed the status of Governor of Syria.

As for the year 15 of Tiberius' reign, when the Lord was baptized at the age of 30 (Lucas 3, 23). Dionysius found an explanation in keeping with the events of the Roman history. The above mentioned year has to be counted from the date when Tiberius became an associate to Augustus' reign. In this case, the date of the latter one's death indicates the baptism age of the Saviour, and then, through other deductions, the year of his birth.

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NOTICE TO AUTHORS

ROMANIAN ASTRONOMICAL JOURNAL is a publication which appears twice a year and is open to original contributions in Astronomy and related disciplines. The contributions – in English or French – can be accepted only if they were neither published before nor destined to any other publication.

Manuscripts should be submitted in duplicate; they must be typewritten on white A4-sized paper, onesided and doublespaced (enclosing abstract, references, footnotes and figure captions). The first page should contain: the article's title (brief and informative), author's name and affiliation, followed by an Abstract in English and Key words. The text should be clear and concise (it is recommended not to exceed 10 pages). *The Abstract* will present clearly the principal conclusions of the work, in no more than 10–15 lines.

Chapter and Paragraphs. Papers, except short notes, should be divided into Chapters, numbered by Arabic numerals. Chapters may be divided into Paragraphs denoted by the number of the Chapter and the number of the Paragraph; each Chapter and each Paragraph should have a short descriptive title (e.g. "3.2. Results").

Formulae have to be numbered consecutively in Arabic numerals, too, but included in parenthesis on the right-hand side of the manuscript; all formulae should be written in legible form. The author should underline, in the text and in formulae, with lead pencil, all characters which he wants to be italics; the bold appearance will be indicated by double underline.

Tables should be numbered consecutively in Arabic numerals; each should be typed on a separate sheet.

Figures and Illustrations should be submitted separately in such a form as to permit reproduction without retouching. Any lettering should be large enough to be legible after the figure has been reduced in size for printing. Captions should be given on a separate sheet and labelled to show which illustration each is accompanying. All the figures should be numbered consecutively in Arabic numerals and referred to in the text as e. g. Fig. 2 or Figs. 2–5. Photographs should be given only if essential and should be enlarged enough to permit clear reproduction.

The Places of tables and figures within the text have to be marked with lead pencil on the left margin of the manuscript.

References are indicated in the text by the author's name and year of publication. They should be listed in alphabetic and chronologic order at the end of the paper, as follows: name and the initials of the author(s), the year of publication, suitable abbreviation of the journal (or title of the book and editing house), its volume and page.

The content of the papers will be introduced on floppy disk (3.5"), in well-known editor, preferably Word v. 6. The collections and emphasis will be made respecting (as far as possible) the prototype of the journal, 11/13 font for the text proper, 12/14 for the paper titles and 9/11 for annexes (tables, references, explanation, footnotes, etc.).

Please pay attention to these recommendations; it will contribute to a faster publishing of the manuscript.

ROMANIAN ASTRONOMICAL JOURNAL

Vol. 8, No. 2, 1998

CONTENTS

<i>IN MEMORIAM</i> - Alexandru Dinescu (1929 – 1998)	61
D. M. ȘURAN, Nedelia Antonia POPESCU, The Model of Two Points Correlation Function for the Determination of the Large Scale Structure of the Universe (II)	63
Iordana AȘTEFĂNOAEI, G. MAFTEI, The Primordial Gravitational Waves with Cylindrical Symmetries	79
A. HORVATH, O. GHEREGA, L. FARKAȘ, Period Changes of BW Vulpeculae: the Influence of Motion in a Binary System.....	89
G. D. CHIȘ, About the System VX Lacertae	101
A. ONCICA, The Structures of Quasi-Periodic Patterns in Solar Radio Bursts	105
V. URECHE, Gravitational Redshift in Maneff's Field	119
V. MIOC, Magda STAVINSCHI, The Schwarzschild-de Sitter Problem (I): Behaviour at Collision and Infinity.....	125
Rodica ROMAN, The Elliptic Restricted Three-Body Problem. Constant Equipotential Surfaces	139

MISCELLANEA

National Seminar "The Total Solar Eclipse of August 11, 1999 – Interdisciplinary Approach", Bucharest, October 15, 1998 (Magda Stavinschi).....	147
The First Romanian Astronomer: The Monk Dionysius Exiguus (G. Drăgulin)	148

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Lei 12 500