

Pp 2

ROMANIAN ACADEMY

*ROMANIAN
ASTRONOMICAL
JOURNAL*

*Vol. 15, No. 2
2005*



EDITURA ACADEMIEI ROMÂNE

ROMANIAN ACADEMY
ROMANIAN ASTRONOMICAL JOURNAL

EDITORIAL BOARD

Honorary Editor in Chief: Arpad PAL

Editor in Chief: Vasile MIOC

Secretary: Petre POPESCU

Members:

José Luis BALLESTER (Palma de Mallorca, SPAIN), Maria Magdalena CÂRȘMARU, Florin DIACU (Victoria, B.C., CANADA), Alexandru DUMITRESCU, Eugeniu GREBENICOV (Moscow, RUSSIA), Georgeta MARIȘ, Ieronim MIHĂILĂ, Vasile POP, Helen ROVITHIS-LIVANIOU (Athens, GREECE), Jean SOUCHAY (Paris, FRANCE), Magda STAVINSCHI, Vasile URECHE, Gheorghe VASS

Editorial assistant: Dan-Florin DUMITRESCU

Computer operator: Sofia MORAR

The ROMANIAN ASTRONOMICAL JOURNAL appears twice a year. Orders from abroad (issues or subscriptions) should be sent to:

EDITURA ACADEMIEI ROMÂNE, Calea 13 Septembrie 13, sector 5, 050711, Bucharest, Romania, Phone: +(4021) 318 81 46, +(4021) 318 81 06, Fax: +(4021) 318 24 44, E-mail: edacad@ear.ro, web: www.ear.ro

RODIPET S.A., Piața Presei Libere 1, P.O. Box 33–57, Bucharest, Romania, Phone: +(4021) 318 70 00, +(4021) 318 70 01, Fax: +(4021) 318 70 02, +(4021) 318 70 03 E-mail: rodipet@rodipet.ro

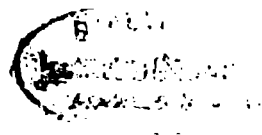
ORION PRESS IMPEX 2000, Șos. Viilor nr. 101, bl. 1, sc. 4, ap. 98, parter, sector 5 (P.O. Box 77-19, sector 3), Bucharest, Romania, Phone: +(4021) 301 87 86, Fax: +(4021) 335 02 96, E-mail: office@orionpress.ro

The manuscripts, the books and journals proposed in exchange and the mail should be sent to the Editorial Board.

Editorial Board's Address:

ROMANIAN ASTRONOMICAL JOURNAL
Astronomical Institute of the Romanian Academy
Str. Cujitul de Argint 5
040557 Bucharest 28
Phone: +(4021) 335 68 92, +(4021) 335 80 10
Phone/Fax: +(4021) 337 33 89
E-mail: roaj@aira.astro.ro
<http://www.astro.ro/~roaj/>
Romania

© 2005, EDITURA ACADEMIEI ROMÂNE
Calea 13 Septembrie 13, sector 5, București, tel. +(4021) 318 81 46
E-mail: edacad@ear.ro
Internet: <http://www.ear.ro/>



ROMANIAN ASTRONOMICAL JOURNAL

Vol. 15, No. 2, 2005

CONTENTS

Ioannis Iraklis HARANAS, Pressure Calculation of a Constant-Density Star in the Dynamic Theory of Gravity	95
Ibrahim Mohamed SELIM, Inall Adham HASSAN, Hamed Abd-Alhamed ISMAIL, Magdy Mahros ALKATEEB, Virialized Hickson Compact Groups of Galaxies	107
Peter ROVITHIS, Alexandru DUMITRESCU, Helen ROVITHIS-LIVANIOU, Gabriela OPRESCU, Marian Doru ȘURAN, The Eclipsing Binary AW Uma: Observations at Athens and Bucharest	113
Nabil Shoukry AWADALLA, Monir Ahmed HAMDY, Magdy Abdel-Malek HANNA, Abdel-Fady Beshara MORCOS, Study of the Orbital Period Variation of the Eclipsing Binary System BV Dra	129
Marilena MIERLA, Rainer SCHWENN, Luca TERIACA, Guillermo STENBORG, Borut PODLIPNIK, Dynamics of the Solar Corona Using LASCO-C1 Spectral Data	137
Dumitru PRICOPI, Vasile MIOC, Hénon-Heiles' Two-Body Problem. II. Main Features of the Global Flow	153
Ciprian CHIRUȚĂ, Tiberiu OPROIU, Note on the Photogravitational Restricted Three-Body Problem	165
Ovidiu VĂDUVESCU, Observing Near Earth Asteroids with a Small Telescope. II. The Program at York University Observatory	171
Minodora LIPCANU, Astronomical Refraction Calculation for the Earth's Ellipsoidal Atmosphere at Great Zenith Distances	183
<i>MISCELLANEA</i>	
The International Scientific Session "Scientific Programs and Astronomy Education in SEE and Ukraine", Bucharest, Romania, 16–17 September 2005 (Vasile MIOC)	193
The Third Meeting of the Sub-Regional European Astronomical Committee (SREAC) within the Framework of the UNESCO-ROSTE Funded Project "Enhancing Astronomical Research and Observation in SEE and Ukraine", Bucharest, Romania, 18 September 2005 (Maeda Stavinschi)	194

PRESSURE CALCULATION OF A CONSTANT-DENSITY STAR IN THE DYNAMIC THEORY OF GRAVITY

IOANNIS IRAKLIS HARANAS

*Department of Physics and Astronomy
York University, Toronto, Ontario, Canada
E-mail: ioannis@yorku.ca*

Abstract. In a new theory called Dynamic Theory of Gravity, the gravitational potential is derived from gauge relations and has a different form than the classical Newtonian potential. In the same theory an analytical expression for the pressure is derived from the equation of the hydrodynamic equilibrium, which is solved for a star of constant density and the results are compared with those of Newtonian gravity. Changes in the central pressure and radius are calculated, too, and finally a redshift calculation is performed, so that the dynamic gravity effects, if any, might be shown to be of some detectable magnitude.

Key words: dynamic theory of gravity – gauge fields – Weyl's quantum principle.

1. INTRODUCTION

There is a new theory called the Dynamic Theory of Gravity (DTG). It is derived from classical thermodynamics and requires that Einstein's postulate of the constancy of the speed of light holds (Williams 1976). Given the validity of the postulate, Einstein's theory of special relativity follows right away (Williams 1977). The dynamic theory of gravity through Weyl's quantum principle also leads to a nonsingular electrostatic potential of the form:

$$V(r) = -\frac{K_0}{r} e^{-\frac{\lambda}{r}}, \quad (1)$$

where K_0 is a constant and λ is a constant defined by the theory. The DTG describes physical phenomena in terms of five dimensions: space, time and mass. (Williams 1980). By conservation of the fifth dimension, we obtain equations identical to Einstein's field equations and describe the gravitational field. These field equations are similar to those of general relativity and are given below:

$$K_0 T^{\alpha\beta} = G^{\alpha\beta} = R^{\alpha\beta} - \frac{g^{\alpha\beta}}{2} R. \quad (2)$$

Now $T^{\alpha\beta}$ is the surface energy-momentum tensor, which may be found within the space tensor and is given by:

$$T^{\alpha\beta} = T_{sp}^{\alpha\beta} - \frac{1}{c^2} \left[F_4^\alpha F^{4\beta} - \frac{h^{\alpha\beta}}{2} F^{4\nu} F_{4\nu} \right] \quad (3)$$

and $T_{sp}^{\mu\nu}$ is the space energy-momentum tensor for matter under the influence of the gauge fields and is given by (Williams 1997):

$$T_{sp}^{ij} = \gamma u^i u^j + \frac{1}{c^2} \left[F_k^i F^{kj} + \frac{1}{4} a^{ij} F^{kl} F_{kl} \right], \quad (4)$$

which further can be written in terms of the surface metric as follows (Williams 1997):

$$T_{sp}^{\alpha\beta} = \gamma u^\alpha u^\beta + \frac{1}{c^2} \left[F_k^\alpha F^{k\beta} + F_4^\alpha F^{4\beta} + \frac{1}{4} (g^{\alpha\beta} - h^{\alpha\beta}) (F^{\mu\nu} F_{\mu\nu} + F^{4\nu} F_{4\nu}) \right], \quad (5)$$

since

$$u^4 = \frac{dy^4}{dt} \Rightarrow \frac{\partial y^4}{\partial t} + \overline{\nabla} \cdot (y^4 \overline{u}) = 0 \quad (6)$$

is the statement required by the conservation of the fifth dimension, and the surface indices $\nu, \alpha, \beta = \overline{0, 3}$ and space indices $i, j, k, l = \overline{0, 4}$, and

$$g_{\alpha\beta} = a_{ij} y_\alpha^i y_\beta^j = a_{\alpha\beta} + h_{\alpha\beta} = a_{\alpha\beta} + 2a_{\alpha 4} y_\beta^4 + a_{44} y_\alpha^4 y_\beta^4,$$

where the surface field tensor is given by:

$$\begin{aligned} F_{\alpha\beta} &= F_{ij} y_\alpha^i y_\beta^j, \\ y_\alpha^i &= \frac{\partial y^i}{\partial x^\alpha} = \delta_\alpha^i, \quad i = \overline{0, 3}, \\ y_\alpha^4 &= \frac{\partial y^4}{\partial x^\alpha}, \end{aligned} \quad (7)$$

and

$$F_{ij} = \begin{bmatrix} 0 & E_1 & E_2 & E_3 & V_0 \\ -E_1 & 0 & B_3 & -B_2 & V_1 \\ -E_2 & -B_3 & 0 & B_1 & V_2 \\ -E_3 & B_2 & -B_1 & 0 & V_3 \\ -V_0 & -V_1 & -V_2 & -V_3 & 0 \end{bmatrix}. \quad (8)$$

It was shown by Weyl that the gauge fields may be derived from the gauge potentials and the components of the 5D field tensor F_{ij} given by the 5×5 matrix given in (8) (Williams 1997).

Now the determination of the fifth dimension may be seen, for the only physically real property that could give Einstein's equations is the gravitating mass or its equivalent, mass (Williams 2001). Finally the dynamic theory of gravity further argues that the gravitational field is a gauge field linked to the electromagnetic field in a 5D manifold of space-time and mass, but, when conservation of mass is imposed, it may be described by the geometry of the 4D hypersurface of space-time, embedded into the 5D manifold by the conservation of mass. The 5D field tensor can only have one nonzero component V_0 , which must be related to the gravitational field, and the fifth gauge potential must be related to the gravitational potential.

The theory makes its predictions for redshifts by working in the 5-dimensional geometry of space, time, and mass, and determines the unit of action in the atomic states in a way that can be calculated with the help of quantum Poisson brackets when covariant differentiation is used (Williams 1993):

$$[x^\mu, p^\nu] \Phi = i\hbar g^{\nu q} \{ \delta_{\mu q} + \left| \Gamma_{s,q}^\mu \right| x^s \} \Phi. \quad (9)$$

In (9) the vector curvature is contained in the Christoffel symbols of the second kind and the gauge function Φ is a multiplicative factor in the metric tensor $g^{\nu q}$, where the indices take the values $\nu, q = \overline{0,4}$. In the commutator, x^μ and p^ν are the space and momentum variables respectively, and finally $\delta_{\mu q}$ is the Kronecker delta. In DTG the momentum ascribed as a variable canonically conjugated to the mass is the rate at which mass may be converted into energy. The canonical momentum is defined as follows:

$$p_4 = m v_4, \quad (10)$$

where the velocity in the fifth dimension is given by:

$$v_4 = \frac{\dot{\gamma}}{\alpha_0}, \quad (11)$$

dot signifies time derivative, γ has units of mass density (kg/m^3), and α_0 is a density gradient with units of kg/m^4 . In the absence of curvature, (8) becomes

$$[x^\mu, p^\nu]\Phi = i\hbar g^{\nu\mu}\Phi. \quad (12)$$

2. THE GRAVITATIONAL POTENTIAL OF DYNAMIC GRAVITY

As it turns out in the DTG the gravitational potential takes the form:

$$V(r) = -\frac{GMm}{r} \exp\left[-\frac{\lambda}{r}\right], \quad (13)$$

where M is the mass of the main body, m is the mass of the test particle, r is the distance from the center of main body to the center of the test particle body, and λ is a distance factor, which may be different for each particle, defined as (Williams 1993)

$$\lambda = \frac{GM}{c^2}. \quad (14)$$

We can easily note that equation (3) is a nonsingular equation, almost of the same form as Yukawa equation. Yukawa equation goes as r rather than $1/r$ in the exponent. For distances much greater than λ , it has the familiar $1/r$ form. When $\lambda = r$ the potential has its maximum value. At $r = 0$ the potential becomes zero due to the overriding effect of the exponential.

3. EQUATIONS OF HYDROSTATIC EQUILIBRIUM

The structure of any stable star, where nonrelativistic effects are not included in hydrostatic equilibrium, obeys the following differential equations:

$$\begin{aligned} \frac{dP(r)}{dr} &= -g(r)\rho(r) = -\frac{GM(r)}{r^2}\rho(r), \\ \frac{dM(r)}{dr} &= 4\pi r^2\rho(r), \end{aligned} \quad (15)$$

where the notation is the usual one. If relativistic effects are taken into account, then the first equation in (15) becomes the so called *Oppenheimer-Volkoff* differential equation of stellar structure below:

$$\frac{dP(r)}{dr} = \frac{Gm(r)\rho(r)\left(1 - \frac{p(r)}{\rho(r)c^2}\right)\left(1 + \frac{4\pi r^3 p(r)}{m(r)c^2}\right)}{r^2\left(1 - \frac{2Gm(r)}{rc^2}\right)}, \quad (16)$$

where the relativistic correction terms are obvious.

4. STELLAR STRUCTURE IN DTG

To study any possible effects in DTG, let us first find, using equation (3), the force due to such a potential as given by the DTG:

$$F(r) = -\frac{\partial V(r)}{\partial r} = -\frac{\partial}{\partial r} \left[-\frac{GMm}{r} \exp\left(-\frac{\lambda}{r}\right) \right]. \quad (17)$$

Then equation (3) finally gives the following force function:

$$F(r) = \frac{GMm}{r^3} (\lambda - r) \exp\left(-\frac{\lambda}{r}\right), \quad (18)$$

from which we find that the acceleration of gravity per unit mass takes the form:

$$g(r) = \frac{GM(r)}{r^3} (\lambda - r) \exp\left(-\frac{\lambda}{r}\right). \quad (19)$$

If we substitute (19) into the first equation of hydrostatic equilibrium, we obtain

$$\frac{dP(r)}{dr} = -g(r)\rho(r) = -\frac{GM(r)}{r^3} (\lambda - r) \rho(r) \exp\left(-\frac{\lambda}{r}\right). \quad (20)$$

As the next step, we have to express the mass $M(r)$ as function of distance r . First we assume a constant density function $\rho(r) = \rho_0$. Then the second equation for hydrodynamic equilibrium gives that:

$$M(r) = \frac{4\pi}{3} \rho_0 r^3 \quad \text{for } 0 \leq r \leq R, \quad (21)$$

which finally makes equation (20) turn to

$$\frac{dP(r)}{dr} = -\frac{4\pi G}{3} \rho_0^2 (\lambda - r) \exp\left(-\frac{\lambda}{r}\right). \quad (22)$$

Integrating now from $(0, r)$ and changing the variable back to r , we obtain

$$P(r) - P(0) = -\frac{4\pi G}{6} \rho_0^2 \exp\left(-\frac{\lambda}{r}\right) \left[r(r - 3\lambda) + 3\lambda^2 \exp\left(\frac{\lambda}{r}\right) \Gamma\left(0, \frac{\lambda}{r}\right) \right], \quad (23)$$

where $\Gamma(0, \lambda/r)$ is the Incomplete Gamma function of argument $(0, \lambda/r)$. This expression can be further simplified to

$$P(r) - P(0) = -\frac{2\pi G}{3} \rho_0^2 r^2 \left[\left(1 - \frac{3\lambda}{r}\right) \exp\left(-\frac{\lambda}{r}\right) + 3\left(\frac{\lambda}{r}\right)^2 \Gamma\left(0, \frac{GM_{\text{star}}}{rc^2}\right) \right]. \quad (24)$$

If we now apply the boundary condition that $P(r = R) = 0$, we obtain the central pressure $P(0)$ to be equal to

$$P(0) = \frac{2\pi G}{3} \rho_0^2 R^2 \left[\left(1 - \frac{3\lambda}{R}\right) \exp\left(-\frac{\lambda}{R}\right) + 3\left(\frac{\lambda}{R}\right)^2 \Gamma\left(0, \frac{\lambda}{R}\right) \right], \quad (25)$$

which can be further simplified as follows:

$$P(0) \approx \frac{2\pi G}{3} \rho_0^2 R^2 \left[1 - \frac{4\lambda}{R} + 40.441 \frac{\lambda^2}{R^2} \right], \quad (25a)$$

where $\Gamma(0, \lambda/R) = \Gamma(0, 2.117 \times 10^{-6}) = 12.4883$ for a star of radius $R = 7R_{\text{Sun}}$. The central pressure effect due to dynamic gravity is zero when $R = 10.1112 \lambda = 14\,986$ km. That would probably mean that dynamic gravity effects on the central pressure die out within a sphere of radius $R = 14\,986$ km from the center of the star having a mass equal to M_{Sun} . Finally the total pressure at any point r can be written as follows:

$$P(r) = \frac{2\pi G}{3} \rho_0^2 \left[R^2 \left(1 - \frac{3\lambda}{R}\right) \exp\left(-\frac{\lambda}{R}\right) + 3\left(\frac{\lambda}{R}\right)^2 \Gamma\left(0, \frac{GM_{\text{star}}}{Rc^2}\right) - r^2 \left(1 - \frac{3\lambda}{r}\right) \exp\left(-\frac{\lambda}{r}\right) + 3\left(\frac{\lambda}{r}\right)^2 \Gamma\left(0, \frac{GM_{\text{star}}}{rc^2}\right) \right]. \quad (26)$$

In the case where the radius of the star R is much greater than λ , then equation (17) can be simplified as follows, after expanding the exponentials to first order:

$$P(r) = \frac{2\pi G}{3} \rho_0^2 (R^2 - r^2) \left\{ 1 - \frac{4\lambda}{(R+r)} + 3\left(\frac{\lambda}{r}\right)^2 \left[\Gamma\left(0, \frac{\lambda}{r}\right) + \left(\frac{r}{R}\right)^2 \Gamma\left(0, \frac{\lambda}{R}\right) \right] \right\}. \quad (27)$$

Taking into account the fact that $\lambda = GM/c^2$, equation (14) can be further written as follows:

$$P(r) = \frac{2\pi G \rho_0^2}{3} (R^2 - r^2) \times \left[1 - \frac{4GM}{c^2(R+r)} + 3 \left(\frac{GM}{c^2 r} \right)^2 \Gamma \left(0, \frac{\lambda}{r} \right) + 3 \left(\frac{GM}{c^2 R} \right)^2 \Gamma \left(0, \frac{\lambda}{R} \right) \right]. \quad (28)$$

If we look at equation (28), we see that the first term in the right-hand side goes as $(R^2 - r^2)$ and is similar to the density profile of a Newtonian gravity star of constant density (Phillips 1999). There is now a difference in the pressure profile due to the DTG and its new gravitational potential. This difference can be shown as two extra-correction terms in equation (28).

5. CHANGE IN CENTRAL PRESSURE

The change in central pressure $P(0)$ between dynamic and Newtonian gravity star becomes

$$\frac{\Delta P_c}{P_{cN}} = \frac{P_{cD} - P_{cN}}{P_{cN}} = -\frac{4\lambda}{R} + \frac{121.323\lambda^2}{R^2}. \quad (29)$$

Therefore we now further have (Stuart et al. 1983):

$$P'_{cD} = P_{cN} + \Delta P_c = [3GM^2/(8\pi R^4)](1 - 4\lambda/R + 121.323\lambda^2/R^2). \quad (30)$$

Apart from the solar-type constant-density stars (Table 1), two more stars were used so that the dynamic gravity effects could be calculated on the central pressure. These ones were neutron stars (Stuart et al. 1983), and we must say these stars can not be really thought as constant-density stars to really fit a simple constant-density calculation. A different relativistic treatment has to be performed in order to study these effects.

Table 1

Dynamic gravity effects on the central pressure

Star radius (R_{Sun})	Star mass (R_{Sun})	λ (km)	Change (square bracket in (30))
1.00	1.000	1.4822	0.9999
3.00	0.180	0.2668	0.9999
8.80	0.720	1.0672	2.7842

We can now see that the value for the dynamic correction of the central pressure takes a value of 0.9999 for the two first stars, and 2.7842 times for the star of radius equal to $8.8 R_{\text{Sun}}$. Next an expression for the pressure change at any r is also derived:

$$\Delta P(r)/P(r) = -4\lambda/(R+\lambda) + 3(\lambda/r)^2 \Gamma(0, \lambda/r) + 3(\lambda/R)^2 \Gamma(0, \lambda/R), \quad (31)$$

which can be further written if the numerical factors taken into account:

$$\Delta P(r)/P(r) = 3(\lambda/r)^2 \Gamma(0, \lambda/r) - 8.468 \times 10^{-6}. \quad (32)$$

Plots of $\Delta P(r)/P(r)$ vs. r are shown for the radial ranges of zero to thirty kilometers from the center of a sunlike star, and where hundred thousand points have been plotted for this and all the graphs below.

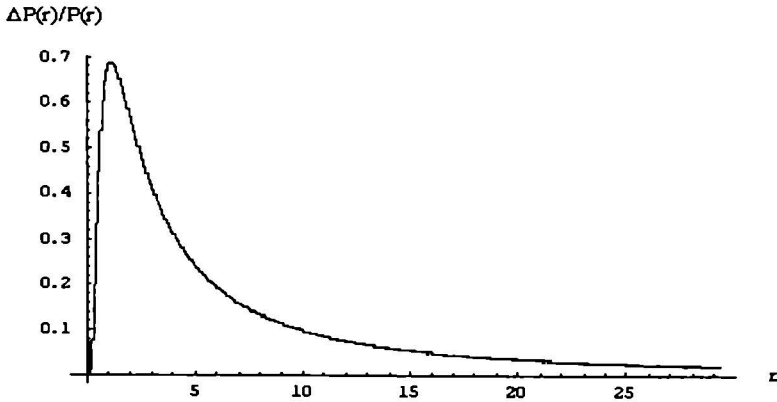


Fig. 1 – $\Delta P(r)/P(r)$ for $0 \text{ km} \leq r \leq 30 \text{ km}$.

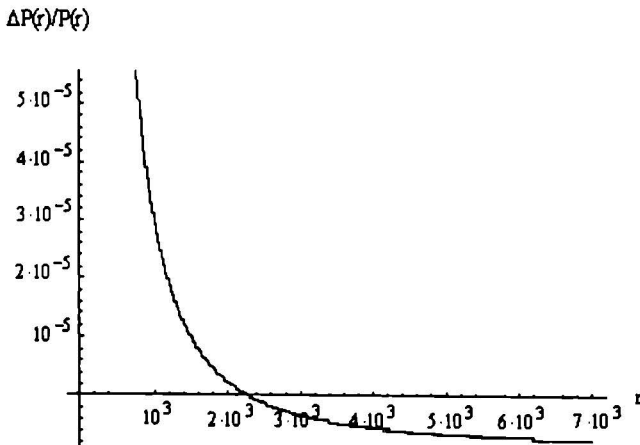


Fig. 2 – $\Delta P(r)/P(r)$ for $30 \text{ km} \leq r \leq 7000 \text{ km}$.

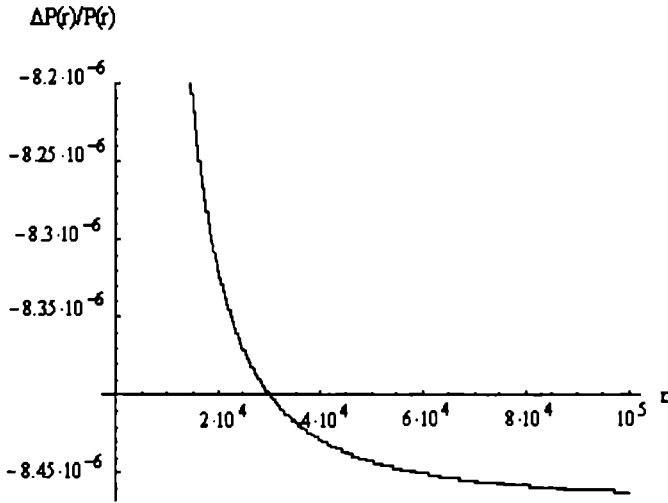


Fig. 3 – $\Delta P(r)/P(r)$ for $7000 \text{ km} \leq r \leq 100\,000 \text{ km}$.

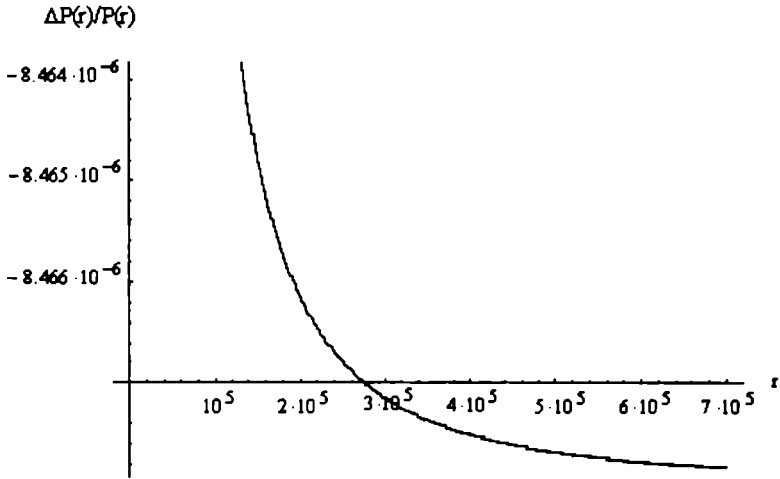


Fig. 4 – $\Delta P(r)/P(r)$ for $100\,000 \text{ km} \leq r \leq 700\,000 \text{ km}$.

6. CHANGE IN RADIUS

A change in the radius of the star, if any, due to the change in the central pressure becomes:

$$\frac{\Delta R}{R_N} = \frac{R_D - R_N}{R_N} = 2.624 \times 10^{-6} \quad (33)$$

which makes the radius of the star to increase by:

$$R_D = R_N(1 + 2.624 \times 10^{-6}) = 1.000002624 R_N. \quad (34)$$

7. REDSHIFT IN DTG

The redshift in DTG is given by the formula

$$z = \frac{\Delta\lambda}{\lambda} = \exp \left\{ \left(-\frac{G}{c^2} \right) \left[\frac{M_r \exp \left(-\frac{\lambda_r}{R_r} \right)}{R_r} - \frac{M_e \exp \left(-\frac{\lambda_e}{R_e} \right)}{R_e} \right] \right\} - 1, \quad (35)$$

where M_r is the mass of the body where the redshifted photons are received, M_e the mass of the body from where the redshifted photons are emitted, R_r and R_e the radii of the corresponding bodies, and λ_r and λ_e the values of dynamic theory parameter at the receiving and emitting bodies. Using the really small increase in the radius of the star $R_e = R_D = 1.200\,000\,262\,4 R_N$ we find that:

$$z = \frac{\Delta\lambda}{\lambda} = \exp(2.117 \times 10^{-6}) - 1 = 2.117 \times 10^{-6}, \quad (36)$$

where the mass and radius of the receiving body is taken to be the mass and the radius of the Earth. The dynamic gravity redshift contribution due to the minute change in the star's radius is calculated above and it is exactly identical to that of a star where the radius remains unchanged under Newtonian gravity and equal to $R = R_{\text{Sun}}$. It is also of the same numerical value as the redshift created by Newtonian gravity. Taking or not the really small change of the radius into account in dynamic gravity, we see that the corresponding redshift is very small and of the order of approximately two parts per million. For such a star it will be a really difficult task for an observer to differentiate with certainty if he is really observing dynamic or Newtonian gravity effects, not to mention that a redshift of two parts per million is a really small change to be observed for Newtonian or dynamic gravity. For such a star dynamic gravity effects do not produce a drastic change in the redshift, which can be easily identified from those of which are caused by Newtonian gravity.

8. CONCLUSIONS

In the DTG a very simple calculation has been performed using the nonrelativistic equation of hydrodynamic equilibrium for a star of constant density, and

thus an analytical expression for the pressure at any point r has been obtained. The pressure $P(r)$ is found to be given in terms of the incomplete gamma function of argument $(0, r/\lambda)$, where λ is the dynamic gravity parameter.

Next, by applying boundary conditions, an expression of the central pressure was also obtained. Both expressions resemble those of a constant-density star in Newtonian gravity but they involve extra-correction terms. The central pressure of such a star appears to be slightly smaller than a constant-density Newtonian star. If that is the case, we anticipated some increase in the radius of the star, which was estimated to be of the order of two parts per million, small enough to produce any noticeable redshift. For our simple model, it appears to be no difference in magnitude between Newtonian and dynamic gravity redshift, and especially when the photons are emitted from such a $1 M_{\text{Sun}}$ star and observed on the Earth. The small increase in radius does not effect the redshift, which is identically the same as the Newtonian or that of dynamic gravity with $R_{\text{star}} = 1 R_{\text{Sun}}$.

An expression for the changes in pressure $\Delta P(r)/P(r)$ at any r has also been derived and plotted for four different regions of r , namely (in km): 0–30; 30–7 000; 7 000–100 000; 100 000–700 000. All graphs have been plotted for one hundred thousand points. From the behaviour of the function we can see that close to the center there is a positive change in pressure due to dynamic gravity, which has a maximum value of approximately 0.7 and at approximately 2 km from the center. That would agree with the maximum value of the dynamic gravity potential at $\lambda = r = 1.482$ km for a solar-type star. Next, at approximately 220 km, the change in pressure drops to positive 10^{-5} , finally becoming zero at approximately 2 200 km. Lastly, in the range of 30 000 km and 280 000 km from the center it takes the values -8.350×10^{-6} and -8.466×10^{-6} , respectively. Our expression derived for the central pressure predicts a correction of 0.999, for $M = 1 M_{\text{Sun}}$ and $R = 1 R_{\text{Sun}}$ star. It seems that throughout the star and at different radial distances the corrections to the pressure due to dynamic gravity changes sign from positive to negative. To conclude, we admit that, in spite of the fact that this is not a very realistic kind of star, it could give us an idea of how the dynamic gravity might affect the stellar structure.

REFERENCES

- Phillips, A. C.: 1999, *The Physics of Stars*, John Wiley and Sons, New York.
 Stuart L. Shapiro, S. A., Teukolsky: 1983, *Black Holes, White Dwarfs, and Neutron Stars*, John Wiley and Sons, New York.
 Williams, P. E.: 1976, Thesis, U. S. Naval Postgraduate School.
 Williams, P. E.: 1977, Research Report EW-77-4, U.S. Naval Academy.

- Williams, P. E.: 1980, Los Alamos Scientific Laboratory Report LA-8370-MS.
- Williams, P. E.: 1993, *The Dynamic Theory: A New View of Space-Time – Matter*, <http://www.nmt.edu>.
- Williams, P. E.: 1997, Proc. Symp. on *Causality and Locality in Modern Physics and Astronomy: Open Questions and Possible Solutions*, Kluwer Academic Publishers, Dordrecht.
- Williams, P. E.: 2001, *Entropy*, **3**, 76.

Received on 14 December 2004

VIRIALIZED HICKSON COMPACT GROUPS OF GALAXIES

IBRAHIM MOHAMED SELIM¹, INALL ADHAM HASSAN²,
HAMED ABD-ALHAMED ISMAIL¹, MAGDY MAHROS ALKATEEB¹

¹ *National Research Institute of Astronomy and Geophysics, Helwan, Cairo, Egypt*
E-mail: i_selim@yahoo.com

² *Astronomy Department, Faculty of Sciences, Al-Azhar University, Cairo, Egypt*

Abstract. We investigate the dynamical stability of Hickson compact groups of galaxies through the virial theorem. The mass-luminosity ratio values of galaxies can be used to obtain the luminous mass in compact groups of galaxies. The average luminous mass of a member of the group is approximately $1.3 \times 10^{11} M_{\odot}$, whereas the average total luminous mass of the group is approximately $4.6 \times 10^{11} M_{\odot}$. The virial theorem has been used to estimate the virial mass. The average virial mass of the group of galaxies is approximately $6.3 \times 10^{13} M_{\odot}$. This is attributed to the presence of a large amount of dark matter: more than 90% in this group. The virial to luminous masses ratio is due to the instability of groups. We conclude that the Hickson compact groups of galaxies are unstable.

Key words: galaxies – groups of galaxies – luminosity – masses.

1. INTRODUCTION

The distribution of galaxies is clumpy, in the form of clusters and groups. The dynamical evolution of gravitationally bound groups of galaxies should account for some of the objects seen in the universe. There is little consensus on the nature of compact groups of galaxies. They are relatively small, dense, isolated systems, containing typically three or more members, with very low median projected intergalactic separation. A numerical simulation of the merging of compact groups shows that the process may result in the formation of bright elliptical galaxies. If the compact groups are interpreted as transit phases in the dynamical evolution of looser groups, the merger remnants they leave behind exemplify galaxy formation at the present epoch. The compact group formation could act on the majority of galaxies in the immediate surrounding of the group: as a suggestion, large potential wells of dark matter form first. All galaxies in the area of these wells would fall in producing a compact group before finally merging. The extent of such potential wells would have to

be considerably larger than the extent of the present Hickson compact groups. X-ray observation is most suitable to investigate the potential well by revealing the extension of the gas surrounding the group. Therefore they are extremely useful in elucidating the nature and dynamical state of these objects and should allow discriminating between the various compact group formation scenarios given by Diaferio et al. (1994). The virial mass necessary to bind a group was larger than the luminous mass. The problem, which has still not found a definite answer, is: the dynamical instability of large galaxy aggregates has always been demonstrated by assuming that if they were stable they should obey the virial theorem. Early studies of stability of the galaxy groups show that the virial mass necessary to bind a group was larger than the visible mass by a factor of 50 (Materne and Tammann 1974). The aim of the present paper is to show that the case for stability of some Hickson compact groups of galaxies as defined by Hickson et al. (1992). We shall use the virial theorem to check whether these groups are stable or not.

2. VISIBLE MASS OF GROUPS

In the present study we reinvestigate the data of some Hickson compact groups of galaxies, which have been compiled from the papers of Hickson et al. (1992), Prandoni et al. (1994) and Barton et al. (1996). In general all member galaxies are of known radial velocities (Hickson et al. 1992). It is now clear that many galaxies emit a large fraction of their radiation at other wavelengths, especially radio and infrared. The luminosity of our sample has been calculated from the absolute magnitude relation: $L/L_{\odot} = 10^{0.4(m_{\odot} - m_g)}$. Here L is the galaxy luminosity, $L_{\odot}$ is the Sun's luminosity, m_g is the magnitude of galaxies, and $m_{\odot}$ is the magnitude of the Sun. For the magnitudes of galaxies we used the results of Mendes de Oliveira and Hickson (1991). The visible mass of individual galaxies in Hickson compact groups has been obtained from the adopted mass-to-light ratio. The mass-to-luminosity ratio in Hickson compact groups is somewhat lower than for relatively isolated galaxies. We assumed the mass-to-light ratio to be 9–13 for spiral, 10 for elliptical and 3 for irregular galaxies (Sersic 1982).

The estimated average luminous mass of galaxies members is $\sim 1.3 \times 10^{11} M_{\odot}$. The total luminous mass of the groups is reached by summation of the individual masses of the galaxy members is $\sim 4.6 \times 10^{11} M_{\odot}$.

3. STABILITY OF GROUPS

The gravitational bound system is in equilibrium state if its total energy is zero. So, one half of the time-averaged potential energy equals the kinetic energy, according

to the virial theorem. Using the virial theorem, we shall determine the virial mass of Hickson compact groups of galaxies. To approach the problem, we have to use the equation of the virial theorem:

$$2T = U ,$$

where the notation is the usual one. This equation applies to a wide variety of systems, from an ideal gas to a cluster of galaxies (Carroll and Ostlie 1996).

The kinetic energy of a relaxed group of galaxies is given by

$$T = \frac{3}{2} \sum_i m_i v_i^2 ,$$

where T is the kinetic energy of the group, m_i and v_i are the mass and radial velocity of the i -th group member with respect to the center of the group (Selim 1999).

The potential energy of the group is given by

$$-U = \frac{2G}{\pi} \sum_i \sum_j \frac{m_i m_j}{r_{ij}} ,$$

where m_i and m_j are the masses of the i -th and the j -th group members, respectively, G is the Newtonian gravitational constant, and r_{ij} is the 3D real distance between the center of the galaxy m_i and the center of the galaxy m_j .

The total virial mass of the group is approximated by Materne and Tammann (1974) as

$$m_{\text{vt}} = -2 \frac{T}{U} m_{\text{L}} ,$$

where m_{vt} is the total virial mass of the group, whereas m_{L} is the total luminance mass of the group.

We study the stability of Hickson compact groups of galaxies by applying the virial theorem and define therefore the S ratio ($S = M_{\text{virial}} / M_{\text{visible}}$) of the virial to the visible masses of the group. If the virial theorem holds, we have $S \approx 1$. For stability it would be sufficient, however, if $0 < S < 2$ (Materne and Tammann 1974).

The calculations of visible and virial masses of Hickson compact groups of galaxies in units of solar mass and the S ratio are listed in Table 1. The virial theorem is due to the fact that mass-to-luminosity ratios used are global values and are subject to some uncertainty. An alternative explanation that Hickson compact groups of galaxies are bound supports the hypothesis of the presence of large amounts of dark matter in these groups. This is supported by the fact that the average mass of a member, as deduced using the mass-to-luminosity ratio, is approximately $1.3 \times 10^{11} M_{\odot}$, and the average total luminous mass of a group is approximately $4.6 \times 10^{11} M_{\odot}$. The average

virial mass (average dynamical mass of group) ranges up to $6.3 \times 10^{13} M_{\odot}$. This can be attributed to the presence of large amounts of dark matter in Hickson compact groups of galaxies. This makes nearly a value of more than 90% of the whole of mass of groups be dark matter. Hickson et al. (1992) found that the average virial mass is $5.5 \times 10^{13} M_{\odot}$ and suggested the presence of more than 85% of dark matter; so the systems of groups are bounded by dark matter not by luminous matter.

The *S* ratio for Hickson compact groups of galaxies has been calculated with the mean value 99.7. The variation of *S* values could be two times lower or two times higher than the adopted values. In addition to the effect of random errors in *S* there are several sources of systematic errors, which could affect the results. Probably the mass of galaxies or the mass of certain type of galaxies is a matter of doubt. This leads to an overestimate of the *S* values.

Table 1 presents the virial-to-luminosity mass ratios for Hickson compact groups. Column 1 is the groups number, column 2 is the luminous mass, column 3 is the virial mass, and column 4 is the *S* ratio.

Table 1

Virial-to-luminosity mass ratios for Hickson compact groups

Groups number	$M_L (10^{11} M_{\odot})$	$M_V (10^{13} M_{\odot})$	<i>S</i>
1	4.353	6.5	149.3223
4	6.063	7.91	130.4635
5	6.57	1.81	27.54947
6	5.148	7.07	137.3349
7	5.259	6.59	125.3090
8	15.71	5.29	33.67282
12	5.91	2.04	34.51777
13	7.942	3.66	46.08411
15	5.858	2.39	40.79891
16	4.975	1.49	29.94975
19	1.155	1.3	112.5541
20	2.649	1.21	45.67761
21	9.045	13	143.7258
23	1.823	1.85	101.4811
24	3.384	8.18	241.7258
25	3.415	1.7	49.78038
26	1.7277	4.5	260.4619
30	5.034	6.93	137.6639
32	10.89	4.49	41.23049
35	10.406	3.1	29.79051

Table 1 (continued)

37	6.66	5.87	88.13814
39	4.472	1.39	31.08229
40	5.888	1.83	31.08016
43	3.658	2.64	72.17059
44	1.838	1.19	64.74429
45	1.0105	2.9	286.9866
46	1.359	3.23	237.6748
47	4.16	1.39	33.41346
49	1.2497	1.96	156.8376
56	3.766	1.72	45.6718
57	12.881	4.66	36.17732
58	9.353	3.29	35.17588
59	1.244	2.5	200.9646
60	12.01	4.7	39.13405
62	2.581	1.04	40.29446
65	26.5	81.03	305.7736
66	9.369	5.54	59.13118
67	10.476	2.24	21.38221
68	5.143	3.4	66.10927
69	3.814	8.7	228.107
74	9.83	28.65	291.4547
75	7.993	12.2	152.6336
76	9.453	14.5	153.3905
79	1.7984	1.04	57.82918
80	2.366	1.09	46.06932
82	8.919	5.28	59.19946
83	4.444	5.06	113.8614
84	12.384	22.25	179.6673
87	5.212	1.28	24.55871
88	9.031	2.88	31.89016
89	4.506	2.14	47.49223
90	3.546	1.171	33.02312
91	9.442	17.57	186.0835
93	7.151	12.56	175.6398
94	13.059	4.25	32.54461
95	6.52	2.14	32.82209
96	7.0676	5.2	73.57519
97	4.86	1.89	38.88889
99	8.091	3.41	42.14559
100	2.099	4.42	210.5765

4. CONCLUSION

The mass to luminosity relation for each morphological type is used to derive the galaxies luminous masses, since the luminosity of the members is previously estimated. The results show that the average luminous mass of the members of the Hickson compact groups is approximately $1.3 \times 10^{11} M_{\odot}$ and the estimated average total luminous mass of the groups is approximately $4.6 \times 10^{11} M_{\odot}$. The average dynamical group masses range up to $6.3 \times 10^{13} M_{\odot}$. This is attributed to the presence of large amount of dark matter in Hickson compact groups. Hickson et al. (1992) concluded that average dynamical groups masses range up to $5.5 \times 10^{13} M_{\odot}$ and he found that more than 85% of the mass in compact groups could be nonluminous mass (dark matter). These results have been used to test the stability of Hickson compact groups through the virial theorem. In all groups the S ratio is more than 2, so the system of groups is bound by dark matter, not by luminous matter. This means that these groups are unstable for the luminous mass; the instability could lead to the collapse of these systems and increase the galaxies merging.

REFERENCES

- Barton, E., Geller, M., Ramella, M., Marzke, R. O., Nicolaci da Costa, L.: 1996, *Astron. J.*, **112**, 871.
 Carroll, B. W., Ostlie, D. A.: 1996, *An Introduction to Modern Astrophysics*, Addison-Wesley, Reading, Mass.
 Diaferio, A., Geller, M. J., Ramella, M.: 1994, *Astron. J.*, **107**, 868.
 Hickson, P., Kindl, E., Huchra, J. P.: 1988, *Astrophys. J.*, **331**, 64.
 Hickson, P., Mendes de Oliveira, C., Huchra, J. P., Palumbo, G. G.: 1992, *Astrophys. J.*, **399**, 353.
 Materne, A., Tammann, G. A.: 1974, *Astron. Astrophys.*, **37**, 383.
 Mendes de Oliveira, C., Hickson, P.: 1991, *Astrophys. J.*, **380**, 30.
 Prandoni, I., Iovino, A., MacGillivray, H. T.: 1994, *Astron. J.*, **107**, 1235.
 Selim, I. M.: 1999, Thesis, Cairo University.
 Sersic, J. L.: 1982, *Extragalactic Astronomy*, D. Reidel, Dordrecht.

Received on 1 July 2005

THE ECLIPSING BINARY AW UMA: OBSERVATIONS AT ATHENS AND BUCHAREST

PETER ROVITHIS¹, ALEXANDRU DUMITRESCU², HELEN ROVITHIS-LIVANIOU¹,
GABRIELA OPRESCU², MARIAN DORU ŞURAN²

¹ *Institute of Astronomy and Astrophysics
National Observatory of Athens, P.O. Box 20048, Athens 118 10, Greece
E-mail: rovithis@astro.noa.gr*

² *Astronomical Institute of the Romanian Academy
Str. Cuştitul de Argint 5, 040557 Bucharest, Romania
E-mail: alex@aira.astro.ro, suran@aira.astro.ro*

³ *Section of Astrophysics-Astronomy and Mechanics
Department. of Physics, Athens University
Panepistimiopolis, Zografos 157 84, Athens, Greece
E-mail: elivan@cc.uoa.gr*

Abstract. BV and UBV photoelectric observations of the contact binary AW UMa – carried out at the Kryonerion Astronomical Station of the National Observatory of Athens, Greece, and at the Bucharest Observatory, Romania, respectively – are given. The derived times of minimum light are given, too, and some of the corresponding light curves are presented. A general discussion concerning this very interesting eclipsing binary is made.

Key words: eclipsing binaries – light curves – AW UMa.

1. INTRODUCTION

The eclipsing binary AW UMa (HD 99967, BD +30° 2163) is a short-period ($P = 0.4387$ days) bright variable star (6.84–7.13 in V light), discovered by Paczynski (1964). In the IDS catalogue a 9.0 magnitude companion at 67" is referred (Eggen 1967).

AW UMa shows very shallow eclipses, and is considered one of the systems with the smallest known mass-ratio ($q \approx 0.075 - 0.085$) among contact binaries. For these reasons it has attracted the interest of many investigators. Among others, Istomin et al. (1980), Woodward et al. (1980), Hrivnak (1982), Bakos et al. (1991), Demirçan et al. (1992), Pribulla et al. (1999) have observed AW UMa.

Spectroscopic studies – performed by Mochnacki and Doughty (1972), McLean (1981), Rensing et al. (1985), Rucinski (1992), Pribulla et al. (1999) – point out the fact that the two components of the system correspond to F0 and F2 spectral types, respectively.

Polarimetric observations of the binary AW UMa were performed by Oschepkov (1974) and Pirola (1975), while Rucinski et al. (1984) analyzed the IUE observations of the system.

2. OBSERVATIONS

The BV and UBV photoelectric observations of AW UMa have been carried out with the 48-inch Cassegrain reflector at the Kryonerion Astronomical Station of the National Observatory of Athens, and with the 50-cm Cassegrain telescope of the Bucharest Observatory. The observations were performed using the classical photometers of the Stations, and the filters used in all cases are in close accordance to the standard UBV system. Correction and reduction of the observations have been made in the usual way (e.g., Hardie 1962; Heuden and Kaitchuk 1987).

The observations at the Kryonerion Station correspond to 9 observational nights spread from 1982 to 1996, and those obtained at the Bucharest Observatory to 10 nights (9 in 1994 and 1 in 1995). The later are given in Tables 1, 2 and 3 as individual observational points in the three filters used.

Moreover, Figs. 1 and 2 present the B and V light curves obtained at the Kryonerion Station during 1986, while Figs. 3, 4 and 5 those from Bucharest Observatory, in the three observational bands UBV, respectively. The phases in all cases have been calculated using Kholopov's (1985) ephemeris formula:

$$\text{HJD}_{\min 1} = 2444664.7993 + 0.4387299 E.$$

Finally, three times of minimum light were derived from our observations made at the Bucharest Observatory, which are given in Table 4.

Table 1

Observations of AW UMa in U

Hel. JD 2440000.+	Phase	ΔU	Hel. JD 2440000.+	Phase	ΔU
1	2	3	1	2	3
9425.812726	0.949	0.190	.856996	0.050	0.294
.816199	0.957	0.211	.862031	0.062	0.290
.820018	0.966	0.200	.872100	0.085	0.364
.835295	0.001	0.212	.875572	0.093	0.376
.838941	0.009	0.182	.882517	0.109	0.378
.842240	0.017	0.189	.888072	0.121	0.403
.845538	0.024	0.173	.891544	0.129	0.440
.848837	0.032	0.223	.897968	0.144	0.470
.853351	0.042	0.251	9426.823643	0.254	0.447

Table 1 (continued)

1	2	3	1	2	3
.827463	0.262	0.420	.850198	0.783	0.407
.830935	0.270	0.442	.853323	0.790	0.458
.833886	0.277	0.421	.857489	0.800	0.413
.837358	0.285	0.412	.860267	0.806	0.409
.840310	0.292	0.425	.862871	0.812	0.374
.843782	0.300	0.385	.865996	0.819	0.376
.847080	0.307	0.381	.870509	0.829	0.359
.850379	0.315	0.359	.873634	0.836	0.333
.856802	0.329	0.356	.876412	0.843	0.379
.860448	0.338	0.354	.879884	0.851	0.330
.863920	0.345	0.351	.882661	0.857	0.324
.866698	0.352	0.341	.885266	0.863	0.295
.870344	0.360	0.323	9443.778628	0.899	0.191
.873469	0.367	0.326	.782447	0.908	0.197
.876941	0.375	0.330	.785225	0.914	0.173
9434.808554	0.454	0.225	.788871	0.923	0.165
.812315	0.462	0.252	.792690	0.931	0.161
.815613	0.470	0.235	.796336	0.940	0.178
.819780	0.479	0.259	.800849	0.950	0.161
.823599	0.488	0.250	.803801	0.957	0.194
.826724	0.495	0.244	.817862	0.989	0.177
.830428	0.504	0.267	.820467	0.995	0.183
.833205	0.510	0.241	.823418	0.001	0.141
.836099	0.516	0.216	.826543	0.008	0.154
.840265	0.526	0.229	.829147	0.014	0.184
.843159	0.533	0.258	.834876	0.027	0.218
.846862	0.541	0.256	.851541	0.065	0.342
.849467	0.547	0.212	.855534	0.075	0.308
.811831	0.696	0.385	.858312	0.081	0.324
.815477	0.704	0.427	.862131	0.090	0.333
.818775	0.711	0.430	.864735	0.096	0.343
.821553	0.718	0.446	.869770	0.107	0.424
.824331	0.724	0.449	.874110	0.117	0.409
.826761	0.730	0.449	.878797	0.128	0.448
.831101	0.740	0.391	.881922	0.135	0.446
.834400	0.747	0.457	9450.797894	0.898	0.177
.837351	0.754	0.448	.802061	0.908	0.189
.840649	0.761	0.416	.805880	0.917	0.222
.847420	0.777	0.459	.809178	0.924	0.103

Table 1 (continued)

1	2	3	1	2	3
.816122	0.940	0.118	.867507	0.733	0.462
.819942	0.949	0.162	.870805	0.740	0.436
.824108	0.958	0.123	.873583	0.747	0.465
.829837	0.971	0.158	.877228	0.755	0.473
.832441	0.977	0.097	.879485	0.760	0.466
.837128	0.988	0.151	9457.788456	0.832	0.346
.839211	0.992	0.158	.791407	0.839	0.351
9456.757617	0.482	0.204	.795053	0.847	0.336
.760569	0.489	0.187	.798004	0.854	0.307
.763346	0.495	0.204	.801129	0.861	0.317
.766471	0.503	0.206	.804254	0.868	0.315
.769596	0.510	0.204	.806858	0.874	0.294
.772547	0.516	0.206	.811198	0.884	0.259
.776366	0.525	0.210	.813802	0.890	0.234
.778970	0.531	0.214	.816753	0.896	0.195
.782095	0.538	0.190	.819704	0.903	0.202
.785220	0.545	0.218	.823003	0.911	0.200
.787824	0.551	0.246	.826127	0.918	0.217
.790949	0.558	0.247	.829599	0.926	0.198
.794074	0.565	0.259	.834287	0.936	0.205
.797546	0.573	0.311	.837411	0.944	0.200
.800844	0.581	0.268	.840710	0.951	0.200
.803448	0.587	0.311	.844008	0.959	0.185
.805358	0.591	0.326	.847133	0.966	0.161
.809871	0.601	0.343	.850258	0.973	0.181
.812822	0.608	0.351	.853556	0.980	0.188
.815947	0.615	0.375	.856681	0.987	0.199
.818898	0.622	0.382	.859806	0.995	0.204
.821676	0.628	0.386	.862584	0.001	0.193
.824454	0.635	0.387	.865882	0.008	0.204
.828794	0.645	0.399	.868833	0.015	0.190
.831571	0.651	0.396	.873520	0.026	0.223
.834522	0.658	0.402	.876298	0.032	0.222
.838515	0.667	0.430	.878555	0.037	0.210
.852403	0.698	0.435	.883936	0.050	0.258
.854834	0.704	0.459	.886714	0.056	0.240
.857611	0.710	0.462	.889318	0.062	0.275
.861778	0.720	0.441	9467.786737	0.621	0.377
.864555	0.726	0.450	.790209	0.629	0.384

Table 1 (continued)

1	2	3	1	2	3
.796805	0.644	0.407	.905905	0.198	0.471
.800625	0.653	0.439	.907814	0.203	0.476
.804097	0.661	0.429	.913891	0.217	0.460
.807742	0.669	0.417	.915801	0.221	0.476
.811561	0.678	0.396	.917537	0.225	0.492
.814860	0.685	0.447	.922398	0.236	0.453
.818505	0.694	0.419	.924134	0.240	0.445
.839163	0.741	0.452	.925870	0.244	0.456
.841767	0.747	0.449	.930210	0.254	0.436
.844719	0.753	0.451	.931946	0.258	0.434
.848191	0.761	0.440	.933335	0.261	0.442
.850795	0.767	0.410	.938370	0.272	0.438
.853919	0.774	0.407	.939933	0.276	0.443
.856697	0.781	0.394	.942189	0.281	0.422
.859648	0.787	0.400	.946356	0.291	0.372
.862599	0.794	0.407	.948092	0.295	0.366
.865377	0.800	0.386	.949828	0.299	0.369
9776.879516	0.138	0.435	.953648	0.307	0.377
.880905	0.141	0.450	.955037	0.310	0.380
.882641	0.145	0.436	.956773	0.314	0.364
.888023	0.158	0.448	.961634	0.326	0.384
.889585	0.161	0.454	.963717	0.330	0.371
.892710	0.168	0.454	.965453	0.334	0.352
.897050	0.178	0.472	.968926	0.342	0.334
.898787	0.182	0.487	.970488	0.346	0.331
.900523	0.186	0.480	.972051	0.349	0.343
.904169	0.195	0.463			

Table 2

Observations of AW Uma in B

Hel. JD 2440000.+ 1	Phase 2	ΔB 3	Hel. JD 2440000.+ 1	Phase 2	ΔB 3
9425.812553	0.949	0.260	.848490	0.031	0.272
.816025	0.957	0.256	.853003	0.041	0.296
.819844	0.966	0.248	.856823	0.050	0.338
.835122	0.000	0.284	.861684	0.061	0.351
.838768	0.009	0.266	.871927	0.084	0.411
.842066	0.016	0.273	.875225	0.092	0.407
.845191	0.023	0.234	.882170	0.108	0.431

Table 2 (continued)

1	2	3	1	2	3
.887725	0.120	0.516	.830754	0.739	0.499
.891197	0.128	0.514	.834053	0.746	0.498
.897273	0.142	0.515	.837177	0.753	0.516
9426.823296	0.253	0.495	.840302	0.761	0.475
.827115	0.262	0.473	.847246	0.776	0.498
.830588	0.269	0.485	.850024	0.783	0.481
.833713	0.277	0.473	.852975	0.789	0.512
.837185	0.285	0.448	.857142	0.799	0.490
.840136	0.291	0.474	.860093	0.806	0.461
.843608	0.299	0.447	.862697	0.812	0.446
.846907	0.307	0.443	.865822	0.819	0.442
.850032	0.314	0.418	.870162	0.829	0.428
.856629	0.329	0.414	.873287	0.836	0.406
.860101	0.337	0.413	.876065	0.842	0.433
.863747	0.345	0.397	.879537	0.850	0.380
.866524	0.351	0.383	.882488	0.857	0.372
.870170	0.360	0.359	.885092	0.863	0.344
.873295	0.367	0.354	.888217	0.870	0.347
.876767	0.375	0.355	9443.778281	0.898	0.284
34.808669	0.454	0.284	.785051	0.914	0.228
.812489	0.463	0.297	.788697	0.922	0.246
.815961	0.471	0.320	.792343	0.931	0.254
.819954	0.480	0.301	.795988	0.939	0.242
.823773	0.488	0.286	.800502	0.949	0.229
.826898	0.496	0.283	.803627	0.956	0.235
.830659	0.504	0.290	.817515	0.988	0.233
.833321	0.510	0.310	.820119	0.994	0.272
.836273	0.517	0.273	.785051	0.914	0.228
.840439	0.526	0.295	.788697	0.922	0.246
.843564	0.533	0.306	.792343	0.931	0.254
.847036	0.541	0.295	.795988	0.939	0.242
.849640	0.547	0.295	.800502	0.949	0.229
9442.808533	0.688	0.485	.803627	0.956	0.235
.811484	0.695	0.463	.817515	0.988	0.233
.815130	0.703	0.475	.820119	0.994	0.272
.818255	0.710	0.497	.785051	0.914	0.228
.821379	0.717	0.503	.788697	0.922	0.246
.823984	0.723	0.483	.823244	0.001	0.195
.826414	0.729	0.491	.826369	0.008	0.218

Table 2 (continued)

1	2	3	1	2	3
.828973	0.014	0.243	.812649	0.608	0.404
.834702	0.027	0.281	.815600	0.615	0.415
.851194	0.065	0.373	.818725	0.622	0.428
.855187	0.074	0.337	.821502	0.628	0.445
.857965	0.080	0.366	.824280	0.634	0.444
.861784	0.089	0.379	.828446	0.644	0.453
.864562	0.095	0.406	.831224	0.650	0.466
.869423	0.106	0.437	.834175	0.657	0.464
.873936	0.116	0.435	.838342	0.666	0.466
.878450	0.127	0.446	.840772	0.672	0.480
.881575	0.134	0.466	.846674	0.685	0.482
9450.797721	0.898	0.241	.849452	0.692	0.496
.801887	0.907	0.252	.852230	0.698	0.508
.805706	0.916	0.294	.854660	0.704	0.520
.808831	0.923	0.203	.857438	0.710	0.522
.815949	0.939	0.192	.861431	0.719	0.504
.819768	0.948	0.217	.864382	0.726	0.543
.823761	0.957	0.176	.867333	0.732	0.555
.829663	0.971	0.207	.870631	0.740	0.526
.832094	0.976	0.178	.873235	0.746	0.496
.836955	0.987	0.207	.876881	0.754	0.525
.839038	0.992	0.202	.879311	0.760	0.514
9456.757270	0.482	0.264	9457.788283	0.832	0.420
.760395	0.489	0.275	.791060	0.838	0.424
.763173	0.495	0.291	.794359	0.845	0.383
.766297	0.502	0.265	.797831	0.853	0.370
.769075	0.508	0.249	.800608	0.860	0.358
.772373	0.516	0.288	.803733	0.867	0.362
.776019	0.524	0.269	.806684	0.873	0.354
.778797	0.531	0.277	.811024	0.883	0.318
.781922	0.538	0.267	.813628	0.889	0.288
.785046	0.545	0.278	.816579	0.896	0.268
.787650	0.551	0.306	.819531	0.903	0.260
.790775	0.558	0.328	.822655	0.910	0.256
.793900	0.565	0.332	.825954	0.917	0.266
.797198	0.573	0.364	.829252	0.925	0.259
.800497	0.580	0.353	.833939	0.936	0.246
.803274	0.586	0.368	.837238	0.943	0.232
.805705	0.592	0.392	.840536	0.951	0.236
.809524	0.601	0.389	.843661	0.958	0.251

Table 2 (continued)

1	2	3	1	2	3
.846959	0.965	0.228	.850621	0.767	0.471
.849911	0.972	0.218	.853746	0.774	0.491
.853383	0.980	0.243	.856350	0.780	0.475
.856508	0.987	0.252	.859301	0.786	0.498
.859632	0.994	0.264	.862252	0.793	0.467
.862410	0.001	0.260	.865203	0.800	0.454
.865708	0.008	0.254	9776.879168	0.138	0.433
.868660	0.015	0.274	.880731	0.141	0.474
.873173	0.025	0.284	.882293	0.145	0.474
.876124	0.032	0.277	.887675	0.157	0.505
.878381	0.037	0.287	.892537	0.168	0.509
.881332	0.044	0.292	.896703	0.178	0.506
.883763	0.049	0.287	.898613	0.182	0.506
.886367	0.055	0.325	.900175	0.185	0.503
.889144	0.061	0.358	.903995	0.194	0.508
9467.786563	0.621	0.425	.905557	0.198	0.519
.796632	0.644	0.474	.907467	0.202	0.512
.800277	0.652	0.485	.913717	0.216	0.538
.803923	0.660	0.491	.915453	0.220	0.505
.807568	0.669	0.486	.917189	0.224	0.529
.811214	0.677	0.462	.921877	0.235	0.528
.814512	0.684	0.468	.923960	0.240	0.505
.818158	0.693	0.495	.925696	0.244	0.505
.821630	0.701	0.510	.929863	0.253	0.499
.825449	0.709	0.504	.931599	0.257	0.506
.834303	0.730	0.511	.933162	0.261	0.493
.838816	0.740	0.497	.938196	0.272	0.476
.841594	0.746	0.518	.939585	0.275	0.481
.844545	0.753	0.505	.941842	0.280	0.500
.847843	0.760	0.490	.945835	0.289	0.473

Table 3

Observations of AW UMa in V

Hel. JD 2440000.+	Phase	ΔV	Hel. JD 2440000.+	Phase	ΔV
1	2	3	1	2	3
9425.812206	0.948	0.117	.838420	0.008	0.134
.815678	0.956	0.123	.841719	0.016	0.130
.819497	0.965	0.140	.845017	0.023	0.128
.834948	0.000	0.135	.848316	0.031	0.148

Table 3 (continued)

1	2	3	1	2	3
.852830	0.041	0.177	.826240	0.728	0.387
.856476	0.049	0.198	.830580	0.738	0.354
.861510	0.061	0.216	.833879	0.746	0.394
.871579	0.084	0.245	.836830	0.753	0.391
.875052	0.092	0.268	.840129	0.760	0.350
.881822	0.107	0.280	.847073	0.776	0.311
.887378	0.120	0.303	.849677	0.782	0.315
.890850	0.128	0.371	.852628	0.789	0.359
.896926	0.141	0.362	.856968	0.799	0.364
9426.822949	0.252	0.370	.859746	0.805	0.338
.826942	0.261	0.322	.865648	0.818	0.311
.830414	0.269	0.343	.869988	0.828	0.293
.833365	0.276	0.329	.873113	0.835	0.291
.836837	0.284	0.322	.875891	0.842	0.289
.839962	0.291	0.325	.879363	0.850	0.251
.843435	0.299	0.302	.882314	0.856	0.233
.846559	0.306	0.307	.884745	0.862	0.213
.849858	0.313	0.271	.887870	0.869	0.222
.856455	0.328	0.274	9443.778107	0.898	0.175
.859927	0.336	0.275	.782100	0.907	0.147
.863573	0.345	0.246	.784878	0.913	0.122
.866351	0.351	0.253	.788350	0.921	0.134
.869996	0.359	0.221	.792169	0.930	0.136
.873121	0.366	0.221	.795815	0.938	0.132
.876594	0.374	0.200	.800329	0.949	0.123
9434.808843	0.454	0.190	.803453	0.956	0.129
.812662	0.463	0.192	.817342	0.987	0.114
.816134	0.471	0.174	.819946	0.993	0.145
.827071	0.496	0.170	.823071	0.001	0.088
.840613	0.527	0.174	.826022	0.007	0.106
.843738	0.534	0.189	.828799	0.014	0.126
.847210	0.542	0.194	.834355	0.026	0.156
.849814	0.548	0.187	.851021	0.064	0.228
9442.808359	0.688	0.368	.855014	0.073	0.235
.811310	0.694	0.383	.857791	0.080	0.260
.814956	0.703	0.378	.861610	0.088	0.280
.818081	0.710	0.389	.864215	0.094	0.262
.821206	0.717	0.395	.869075	0.105	0.293
.823810	0.723	0.374	.873589	0.116	0.306

Table 3 (continued)

1	2	3	1	2	3
.878103	0.126	0.316	.834002	0.656	0.327
.881401	0.134	0.331	.838168	0.666	0.343
9450.797373	0.897	0.137	.840598	0.671	0.356
.801713	0.907	0.129	.843723	0.679	0.361
.805533	0.916	0.133	.846501	0.685	0.346
.808658	0.923	0.096	.848931	0.690	0.371
.815775	0.939	0.073	.852056	0.698	0.373
.819594	0.948	0.112	.854486	0.703	0.372
.823587	0.957	0.084	.857090	0.709	0.363
.829490	0.970	0.095	.861257	0.719	0.375
.831920	0.976	0.088	.864208	0.725	0.403
.836607	0.987	0.114	.867159	0.732	0.391
.838864	0.992	0.107	.873062	0.745	0.380
9456.756923	0.481	0.155	.876707	0.754	0.387
.760221	0.488	0.142	.879138	0.759	0.363
.762999	0.495	0.142	9457.788109	0.831	0.286
.765950	0.501	0.138	.790887	0.837	0.277
.768901	0.508	0.151	.794185	0.845	0.271
.772026	0.515	0.148	.797657	0.853	0.260
.775845	0.524	0.143	.800435	0.859	0.240
.778623	0.530	0.152	.803559	0.866	0.226
.781574	0.537	0.141	.806511	0.873	0.223
.784873	0.544	0.165	.810677	0.883	0.200
.787477	0.550	0.177	.813455	0.889	0.171
.790428	0.557	0.188	.816406	0.896	0.165
.793726	0.565	0.205	.819357	0.902	0.159
.796851	0.572	0.220	.822482	0.909	0.137
.800323	0.580	0.240	.825780	0.917	0.149
.803101	0.586	0.248	.829079	0.925	0.125
.805531	0.592	0.262	.833766	0.935	0.132
.809350	0.600	0.281	.836891	0.942	0.111
.812475	0.607	0.267	.840363	0.950	0.121
.815426	0.614	0.293	.843487	0.957	0.117
.818551	0.621	0.298	.846786	0.965	0.114
.821329	0.628	0.300	.849737	0.972	0.132
.823933	0.633	0.314	.853036	0.979	0.118
.828273	0.643	0.335	.856334	0.987	0.123
.831050	0.650	0.323	.859459	0.994	0.119

Table 3 (continued)

1	2	3	1	2	3
.862236	0.000	0.119	.881946	0.144	0.333
.865535	0.008	0.135	.887328	0.156	0.351
.868486	0.014	0.147	.889064	0.160	0.343
.873000	0.025	0.141	.892363	0.168	0.368
.875951	0.031	0.149	.896530	0.177	0.386
.878208	0.037	0.159	.898266	0.181	0.380
.881159	0.043	0.165	.899828	0.185	0.368
.883589	0.049	0.182	.903648	0.193	0.362
.886193	0.055	0.191	.905384	0.197	0.394
.888971	0.061	0.215	.907294	0.202	0.392
9467.786389	0.620	0.304	.913370	0.215	0.399
.789688	0.628	0.304	.915106	0.219	0.386
.796285	0.643	0.361	.917016	0.224	0.386
.799930	0.651	0.347	.921356	0.234	0.375
.803576	0.659	0.363	.923787	0.239	0.360
.807395	0.668	0.371	.925523	0.243	0.348
.810867	0.676	0.361	.929689	0.253	0.348
.814339	0.684	0.364	.931426	0.257	0.342
.817984	0.692	0.365	.932872	0.260	0.341
.821456	0.700	0.385	.938023	0.272	0.328
.825102	0.709	0.386	.939412	0.275	0.350
.834129	0.729	0.370	.941321	0.279	0.341
.838643	0.739	0.378	.945314	0.288	0.332
.841247	0.745	0.374	.947687	0.294	0.327
.844371	0.752	0.388	.949308	0.297	0.326
.847670	0.760	0.372	.953127	0.306	0.327
.850274	0.766	0.360	.954516	0.309	0.299
.853572	0.773	0.352	.956252	0.313	0.288
.844371	0.752	0.388	.961113	0.324	0.294
.856176	0.779	0.328	.962849	0.328	0.299
.859127	0.786	0.369	.964933	0.333	0.292
.862078	0.793	0.333	.968405	0.341	0.277
.865030	0.800	0.322	.969967	0.345	0.269
9776.878995	0.137	0.324	.971530	0.348	0.267
.880557	0.141	0.358			

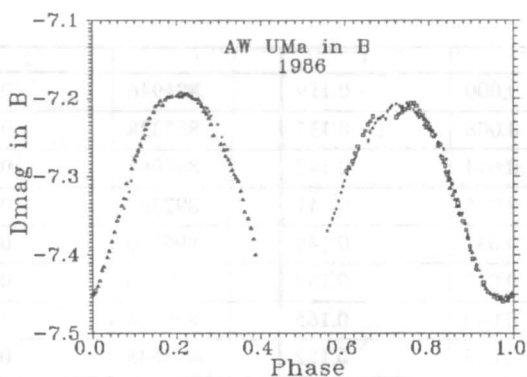


Fig. 1 – The B light curve of AW UMa obtained at Kryonerion during 1986.

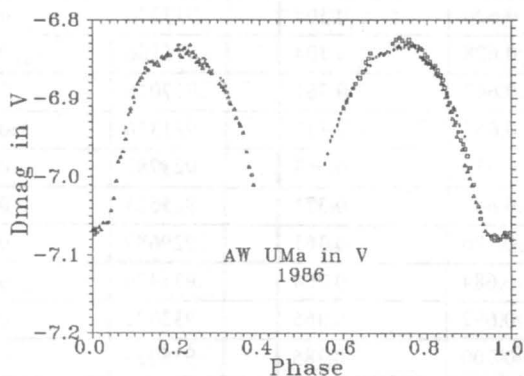


Fig. 2 – The V light curve of AW UMa obtained at Kryonerion during 1986.

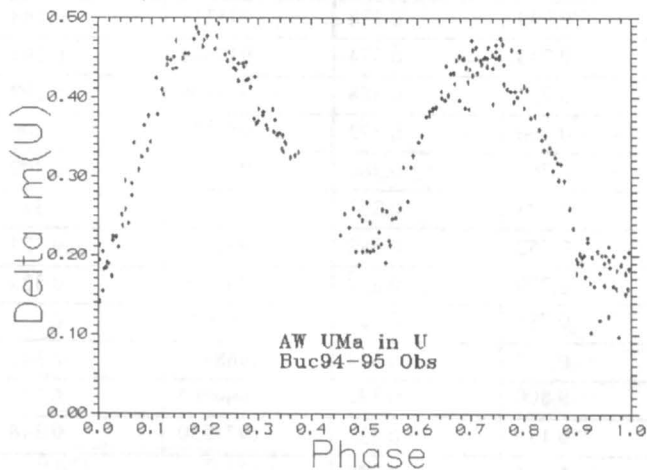


Fig. 3 – The U light curve of AW UMa obtained in Bucharest during 1994–1995.

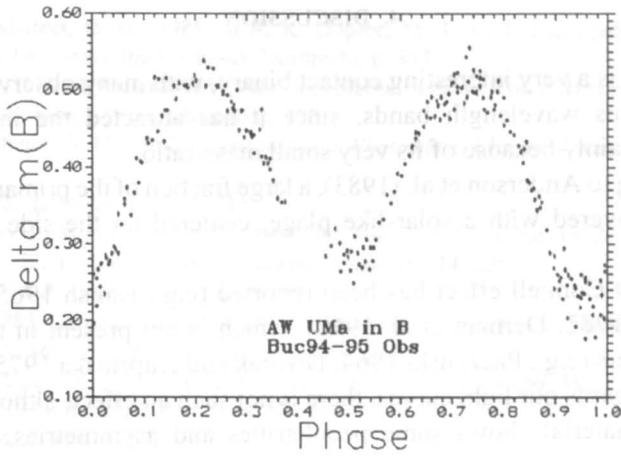


Fig. 4 – The B light curve of AW UMa obtained in Bucharest during 1994–1995.

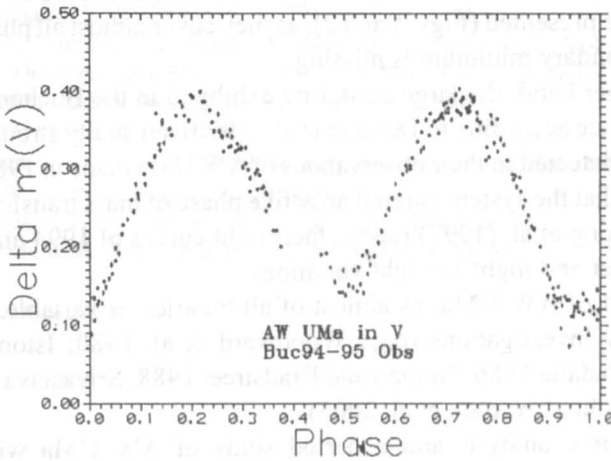


Fig. 5 – The V light curve of AW UMa obtained in Bucharest during 1994–1995.

Table 4

Minima times of AW UMa

Hel. JD 2440000.+	Filter
9426.3406±0.0032	V
.3412±0.0036	B
.3410±0.0030	U
9444.3213±0.0042	V
.3207±0.0037	B
.3217±0.0042	U
9458.3491±0.0005	V
.3474±0.0010	B
.3480±0.0007	U

4. DISCUSSION

AW UMa is a very interesting contact binary, with many observations and light curves in various wavelength bands, since it has attracted the interest of many investigators; mainly because of its very small mass-ratio.

According to Anderson et al. (1983), a large fraction of the primary component of AW UMa is covered with a solar-like plage, centered on the side away from the secondary.

A small O'Connell effect has been reported (e.g., Kalish 1965; Istomin et al. 1980; Hrivnak 1982; Derman et al. 1990), which is not present in the light curves obtained by others (e.g., Paczynski 1964; Dworak and Kuprinska 1975; Woodward et al. 1980). As regards our light curves, there is not such an effect, although the present observational material shows some peculiarities and asymmetries. This could be explained as being the result of the long time spanned among the observations. For this reason, from the obtained at the Kryonerion Station light curves of AW UMa, only these of 1986 are presented (Figs. 1 and 2), as they cover almost all phases and only the part around secondary minimum is missing.

On the other hand, the large scattering exhibited in the Bucharest observations might be real, since according to Derman et al. (1990) unusually strong light and color variations were detected in their observations of AW UMa made in 1989 and 1990, and they concluded that the system entered an active phase of mass transfer in recent years. Moreover, as Jeong et al. (1997) report, their light curves of 1994 and some of 1995 show large scatter and night-to-night variations.

The period of AW UMa, as almost of all binaries, is variable. It has been the subject of many investigations (e.g., Woodward et al. 1980; Istomin et al. 1981; Srivastava and Padalia 1986; Guinan and Bradstreet 1988; Srivastava 1989, Demirçan et al. 1992; Rovithis-Livaniou et al. 2005).

A light curve analysis and a period study of AW UMa will be presented elsewhere.

Acknowledgments. This work was partially financially supported by the Athens University (under Grant No. 70/4/3305). Moreover, P.R and H. R-L would like to thank very much the staff at the Kryonerion Station of the National Observatory of Athens, and especially Mr. George Dimou.

REFERENCES

- Anderson, L., Stanford, D., Leinger, D.: 1983, *Astrophys. J.*, **270**, 200.
Bakos, G. A., Horak, T. B., Tremko J.: 1991, *Bull. Astron. Inst. Czechosl.*, **42**, 331.
Demirçan, O., Derman, E., Müyesseroglu, Z.: 1992, *Astron. Astrophys.*, **263**, 165.
Derman, E., Demirçan, O., Müyesseroglu, Z.: 1990, *Inf. Bull. Var. Stars*, No. 3540.
Dworak, T. Z., Kuprinska, M.: 1975, *Acta Astron.*, **25**, 417.
Eggen, O.: 1967, *Mem. Roy. Astron. Soc.*, **70**, 111.

- Guinan, E. F., Bradstreet, D. H., 1988, in A. K. Dupree, M. T. V. T. Lago (eds), *Formation and Evolution of Low Mass Stars*, Kluwer, Dordrecht, p. 345.
- Hardie, R. H.: 1962, in W. A. Hiltner (ed.), *Stars and Stellar Systems*, Vol. 2, *Astronomical Techniques*, University of Chicago Press, Chicago, p. 178.
- Heuden, A. A., Kaitchuk, R. H.: 1987, in *Astronomical Photometry*, Chapter 4, van Nostrand Reinhold Co., p. 80.
- Hrivnak, B. J.: 1982, *Astrophys. J.*, **260**, 744.
- Istomin, L. F., Orlov, L. M., Kulagin, V. V.: 1980, *Inf. Bull. Var. Stars*, No. 1802.
- Jeong, J. D., Lee, Y.-S., Yim, J. R.: 1997, *J. Astron. Space Sci.*, **14**, 225.
- Kalish, M. S.: 1965, *Publ. Astron. Soc. Pacific*, **77**, 36.
- Kholopov, P. N.: 1985, *General Catalogue of Variable Stars*, 4th Ed., Vol. 2, Nauka, Moscow.
- McLean, B. J.: 1981, *Mon. Not. Roy. Astron. Soc.*, **195**, 931.
- Mochmacki, S. W., Doughty, N. A.: 1972, *Mon. Not. Roy. Astron. Soc.*, **156**, 51.
- Oschepkov, V. A.: 1974, *Inf. Bull. Var. Stars*, No. 884.
- Paczynski, B.: 1964, *Astron. J.*, **69**, 124.
- Pirola, V.: 1975, *Inf. Bull. Var. Stars*, No. 1060.
- Pribulla, T., Chochol, D., Rovithis-Livaniou H., Rovithis, P.: 1999, *Astron. Astrophys.*, **345**, 137.
- Rensing, M. J., Mochmacki, S. W., Bolton, C. T.: 1985, *Astron. J.*, **90**, 767.
- Rovithis-Livaniou, H., Tsantilas, S., Kalimeris, A., Rovithis, P., Djurašević, G., Voloshina I.: 2005, in C. Sterken (ed.), *The Light-Time Effect in Astrophysics*, Proc. ASP Conf. Series, Vol. 335, Brussels, July 19–22, 2004, Astron. Soc. Pacific, San Francisco, p. 251.
- Rucinski, S. M., 1992: *Astron. J.*, **104**, 1968.
- Rucinski, S. M., Brunt, C. C., Pringle, J. E.: 1984, *Mon. Not. Roy. Astron. Soc.*, **208**, 309.
- Srivastava, R. K.: 1989, *Astrophys. Space Sci.*, **154**, 179.
- Srivastava, R. K., Padalia, T. D., 1986, *Astrophys. Space Sci.*, **120**, 121.
- Woodward, E. J., Koch, R. H., Eisenhardt, P. R.: 1980, *Astron. J.*, **85**, 50.

Received on 1 June 2005

STUDY OF THE ORBITAL PERIOD VARIATION OF THE ECLIPSING BINARY SYSTEM BV DRA

NABIL SHOUKRY AWADALLA, MONIR AHMED HAMDY,
MAGDY ABDEL-MALEK HANNA, ABDEL-FADY BESHARA MORCOS

National Research Institute of Astronomy and Geophysics, Helwan, Cairo, Egypt
E-mail: magdyam@yahoo.com

Abstract. The (O–C) diagram of BV Draconis has been analyzed. The depth variability of the light curves' minima for the system has been studied. The depth variation may be explained as magnetic cycles effect. A relation between the magnetic cycling and the period variation has been proposed.

Key words: astrophysics – eclipsing binaries – period variation – magnetic activity.

1. INTRODUCTION

BV Dra (ADS 9537 A, mag, 7.88, spectral type F8V+F9V) is a contact binary of W UMa type. It is one of the two members of the visual binary system ADS 9537. The other member is BW Dra (ADS 9537 B). The separation angle between the two members is given by Aitken (1932) to be $16''.1$. Batten and Hardie (1965) estimated the orbital period of the visual binary and found it to be of the order of 10^4 years. They discovered the binary nature of it as a W UMa stars. They also determined the orbital period and primary and secondary depths for the two eclipsing binaries.

The system has been observed by Batten (1967), Wood (1970), Yamasaki (1979), Rovithis and Rovithis-Livanou (1981, 1982), Geyer et al. (1982), Rucinski and Kaluzny (1982), Hamdy et al. (1985), Gorda (1986), Dapergolas et al. (1989, 1990, 1991) and Lee et al. (1999). Batten and Lu (1986) found the radial velocity curve and the spectral classification for BV Dra. The light curves of the system show symmetry/asymmetry variations from time to time. Some of these light curves (in B and V) have been analyzed using different methods, such as Schneller's (1949, 1959) method, syntheses program of Rucinski (1973, 1974, 1976a,b), Kopal's Frequency Domain method (Kopal 1979), and Wilson and Devinney (1971, 1973) model. The absolute parameters from these analyses are different due to the depth changes of the light curves.

Lee et al. (1999) interpreted that the asymmetry in the light curves may be produced due to the existence of two spots: a hot one on the secondary and a cool one on the primary.

Hardie and Hall (1990) found an orbital period variation for BV Dra. They proposed a magnetic cycle, of about 19 years length, as an explanation for such a variation. On the other hand, Kaluzny and Rucinski (1986) did not come to a conclusion because their (O–C) curve had shorter baseline in time.

From the above presentation, we can report that the study of the W UMa binary BV Dra is not complete and many points are still unconfirmed, especially the magnetic cycle, the orbital period change, the depth variation between the minima, in addition to the relation between the orbital period variation, the depth changes and magnetic cycling.

2. PERIOD VARIATION

To study the variation in the orbital period and the possible connection with the asymmetry of the light curves of the contact binary BV Dra, we compiled and critically examined all the available published time of minima. The residual values of the (O–C) diagram have been computed using the light elements, which were given by Kreiner et al. (2001) as

$$\text{JD Hel (Min)} = 2445739.1151 + 0^d.350066568E \quad (1)$$

and listed in a table (available electronically). All the residuals are presented graphically versus cycles and Heliocentric Julian Day (HJD) in Fig. 1.

A brief inspection of the (O–C) curve (Fig.1), leads to the following: (1) The scattered points shown in the (O–C) curve are due to the uncertainty in the determination of the times of minima. This uncertainty comes from the asymmetry and changes of the BV Dra light curves from cycle to cycle. (2) Both primary and secondary times of minima follow the same trend. (3) There are three major peaks or jumps around HJD 2438882.9, 2444752.4 and 2451312.1 besides two major depth decreases around HJD 2440362.8 and 2447398.4. These jumps and depths are superimposed on a short cycling increase-decrease changes.

Any mean light elements derived by a linear best fit for all the residual points are not suitable to estimate the minima time in future. It is more convenient to apply a linear least square fit to the sections $E_i - E_{i+1}$, $i = \overline{1, 4}$, of the (O–C) variation to find the exact intersection of the linear fits (see Fig. 1). The light elements from the last section (E_4 to E_5) may be used for estimating the minimum times for the next few years (see Table 1). The extreme points, the intervals and the best fitting data (light elements) with standard deviation (SD), correlation coefficient (r) and the residual sum of squares are summarized in Table 1. The value of the time interval ΔE and its corresponding change in the period ΔP in each section according to the best fit to the (O–C) curve are also listed in Table 1.

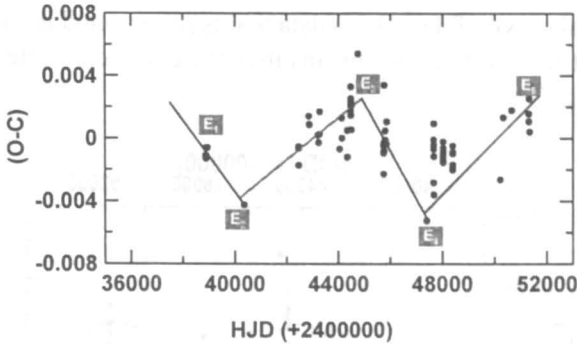


Fig. 1 – The (O–C) versus HJD diagram for BV Draconis. The four straight lines represent the linear best fits of the four sections ($E_i - E_{i+1}$, $i = 1, \dots, 4$).

Table 1

Linear fit sections, intervals, and rate of change of the period and mass of BV Dra

E (HJD)	$E_1 - E_2$	$E_2 - E_3$	$E_3 - E_4$	$E_4 - E_5$
Interval (JD:2400000+)	38882.9–40362.8	40362.8–44752.5	44752.5–47398.4	47398.4–51312.1
Epoch (HJD)	2445739.2021	2445739.0571	2445739.2941	2445739.1121
Period (day)	0.350064302	0.350067917	0.350062659	0.350066842
SD	0.00038	0.0013	0.0013	0.0015
r	0.976	0.732	0.883	0.613
Residual sum of squares	4.38045×10^{-7}	4.17027×10^{-5}	2.56691×10^{-5}	4.65491×10^{-5}
ΔE (day)	1479.9	4389.7	2645.9	3913.7
ΔP (day)	-2.266×10^{-6}	1.349×10^{-6}	-3.909×10^{-6}	0.967×10^{-6}
$\Delta P/P$	-6.473×10^{-6}	3.854×10^{-6}	-0.112×10^{-6}	2.762×10^{-6}
$\Delta P/\Delta E$ (d/cycle)	-5.360×10^{-10}	1.076×10^{-10}	-5.172×10^{-10}	0.865×10^{-10}
$\Delta M_1 (M_\odot)$	-1.429×10^{-6}	8.508×10^{-7}	-2.473×10^{-8}	6.098×10^{-7}
$dM_1/dt (M_\odot/\text{yr})$	-3.527×10^{-7}	7.079×10^{-8}	-3.414×10^{-9}	5.691×10^{-8}

It is noticed from the (O–C) curve (Fig. 1) that the period variation of BV Dra may have a cyclic behaviour. The cycling or the extremely maxima and minima behaviour can be represented as the function $P(E)$. In what follows, we are going to determine the function $P(E)$ in two different ways, to prove any existence of a cycling or periodicity feature:

(1) We try to represent the (O–C) data by a polynomial in E . The best sinusoidal fit with $SD = 0.0015$ and $r = 0.65$ yields the following equation:

$$P(E) = -0.0003 + 0.0017 \cos(0.0003E + 1.4587). \quad (2)$$

The sinusoidal curve of the (O–C) data that is represented in Fig. 2. It shows an oscillation of about two cycles. This means that one complete cycle may be equal to 19.45 ± 1.010 years.

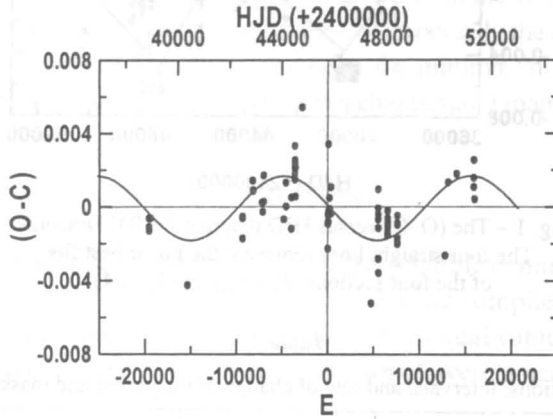


Fig. 2 – The sinusoidal fit for the (O–C) residuals.

(2) The Data Compensated Discrete Fourier Transform (DCDFT) technique is used to obtain the power spectrum of a time series of $P(E)$. The periodicity or cycling is determined by using the function (Ferraz-Mello 1981):

$$F(\omega) = \sum_{j=1}^n f(t) \exp(2\pi i \omega t_j), \quad (3)$$

which leads to a significant highest peak, corresponding to a true periodicity.

The spectra obtained using the DCDFT technique for (O–C) data as function of time are shown in Fig. 3, where the main spectral component (marked by $\omega_0 = 0.05$) corresponds to a periodicity of 20 years.

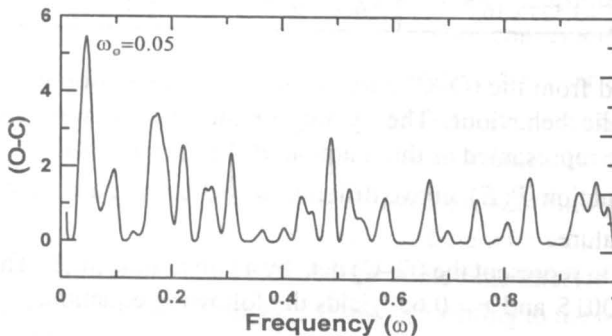


Fig. 3 – The spectrum obtained from the (O–C) residuals.

3. THE DEPTH VARIATIONS

Most of the B and V light curves for the system BV Dra, which are available and published in the literature, have been collected and examined. The primary and secondary depth difference ($\Delta D = d_{\text{pri}} - d_{\text{sec}}$) has been calculated. The magnitude differences in B and V and the corresponding dates of the observations are listed in Table 2.

Table 2 leads to a conclusion that there is a wavelike character. This may be modulated by a periodic action of some physical mechanism.

The DCDFT technique is used to find the power spectrum of a time series $\Delta D_{B,V}(t)$. The spectra obtained for $\Delta D_{B,V}(t)$ are shown in Fig. 4, where the mean spectral component (marked by $\omega_{B,V}$) corresponds to the periodicity $\Delta D_B = 7.9$ years, whereas $\Delta D_V = 7.6$ years.

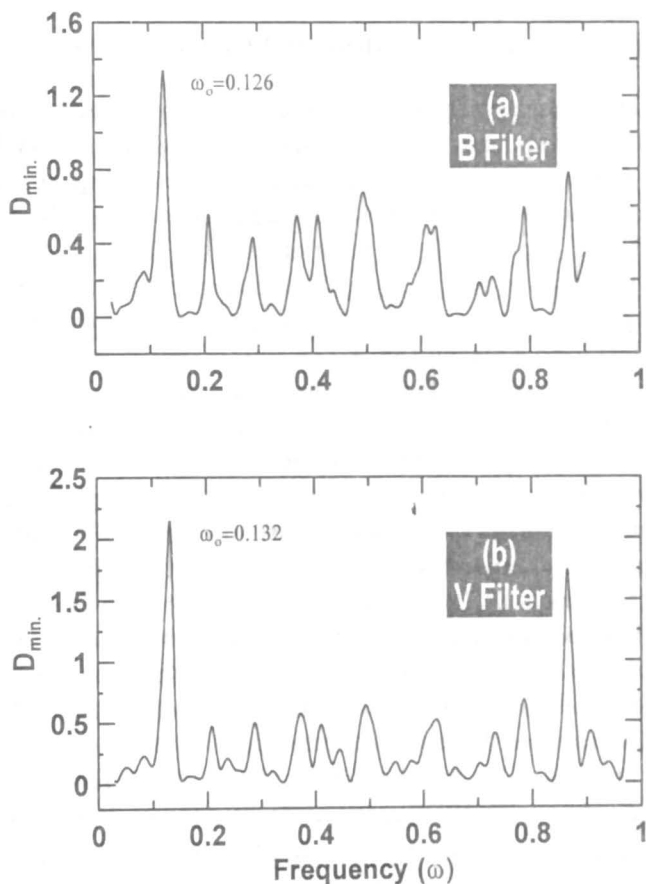


Fig. 4 – The spectrum of the differences between the depths of the two minima for B and V filters with cycling $1/0.126 = 7.9$ years and $1/0.132 = 7.6$ years, respectively.

Table 2

The depth variation for BV Dra light curves in B and V colours

Date	ΔD_B	ΔD_V	Reference
1965.38	0.00	–	Batten and Hardie (1965)
1975.12	0.03	0.03	Rucinski and Kaluzny (1982)
1977.38	0.02	0.02	Yamasaki (1979)
1981.38	0.04	0.02	Rovithis and Rovithis-Livaniou (1982)
1984.29	0.00	0.00	Gorda and Yu (1986)
1984.42	0.00	0.00	Hamdy et al. (1985)
1989.38	0.05	0.02	Dapergoles et al. (1989)
1990.38	0.00	0.00	Dapergoles et al. (1990)
1991.38	0.07	0.07	Dapergoles et al. (1991)
1999.42	0.04	0.04	Lee et al. (1999)

4. ORBITAL PERIOD – DEPTH VARIATION

It is of interest to search for the possible connection between the depth difference variations and the period changes of the eclipsing W UMa binary BV Dra.

From the above-mentioned analysis, the orbital period (Figs. 1 and 2) of BV Dra shows an increasing-decreasing behaviour, in addition to a sinusoidal oscillation. On the other hand, the depth difference variations (see Table 2 and Fig. 4) show symmetries/asymmetries in periodicity of the light curves. So this phenomenon may be due to different physical mechanisms such as: (1) magnetic activity cycles and (2) mass exchange (Rucinski and Kaluzny 1982).

The changes between the depths of the primary and the secondary may refer to some dark spots on one of the components. Since BV Dra has late spectral type, hotter primary less massive component F8V and secondary more massive component F9V (Batten and Lu 1986), so it could be considered as a chromospheric active star. This could cause a magnetic activity cycle with a mean period of 7.75 years. This magnetic activity cycle cause symmetry/asymmetry of the light curves of the system. The asymmetry light curves in BVR of the system have been observed and analyzed by Lee et al. (1999). They concluded that the asymmetry light curves might be interpreted as produced by the existence of two spots, hot spot on the secondary and cool on the primary. So, the possible existence of solar-like cycle (magnetic cycle hypothesis) with 7.75 years for BV Dra may be strongly expected.

From the analysis of the (O–C) residuals (Fig. 1) the difference between the two points E_2 and E_4 is equal to 19.27 years, while the difference between the two (peaks) E_3 and E_5 is about 17.96 years. These jumps may be expected as a result of rapid mass exchange between the two components, but the short interval of the jumps (several years) cannot support this explanation. That is to say, the mass exchange is not the only dominant mechanism. However, if the decrease or increase of the orbital

period is due to a conservative exchange, then we can estimate the mass transfer rate, by the well-known equation

$$\frac{\Delta P}{P} = \left(3 \frac{M_1}{M_2} - 1 \right) \frac{\Delta M_1}{M_1}, \quad (4)$$

where ΔM_1 is the rate of mass transfer from one component to the other.

The values of these rates, according to the spectroscopic mass ratio $q_{sp} = 0.402$ (Batten and Lu 1986) and M_1 (primary) = 0.40 and M_2 (secondary) = 1.01 (Lee et al. 1999), are listed in Table 1 and found to be of order of magnitude $10^{-8} (M_{\odot}/\text{yr})$.

On the other hand, Hardie and Hall (1990) reported that the late spectral W-type W UMa system might be expected to have magnetic cycles (but not strictly periodic). They calculated, from the (O–C) residuals of the binary system BV Dra, the time between one period decrease and the next decrease and suggested a full magnetic cycle length of about 19 years.

As it is mentioned in the previous section, a 19.45 years cycling has been found from the sinusoidal fitting (equation (2)) of (O–C) residuals. On the other hand, the power spectrum of the (O–C) curve presented by Fig. 3 gives a 20 years cycling. The cycle of about 19.45 years is in agreement with Hardie and Hall (1990). However, any explanation for the period variation via only a magnetic activity cycling may not be convenient because the cycle length of the asymmetry light curves for BV Dra is not the same as that of the (O–C) variation, but the magnetic activity may cause a fluctuation of the (O–C) residuals.

5. CONCLUSION

The W UMa system BV Dra may be considered as a chromospheric active solar-like star. Appearing and disappearing of spots activity may cause the asymmetry/symmetry light curves of the system. The depth changes may cause the orbital period fluctuation. The correlation between the magnetic activity cycling and the orbital period variation may be found if it is studied quantitatively.

The thermal relaxation oscillation effect after the rapid mass exchange or loss between the contact components can not be ignored and it may not be the main cause for the period variation and fluctuation of BV Dra system.

We stress that systematic monitoring based on the analysis of complete seasonal light curves is necessary, in order to assess the phase relationship between the period change and the activity cycle, because short-term fluctuations induced by appearance and disappearance of huge star spots, may severely affect the magnitude determination.

REFERENCES

- Aitken, R. G.: 1932, *New General Catalogue of Double Stars*, Carnegie Institute, Washington.
- Batten, A. H.: 1967, *Publ. DAO*, **13**, 119.
- Batten, A. H., Hardie, R. H.: 1965, *Astron. J.*, **70**, 666.
- Batten, A. H., Lu, W.: 1986, *Publ. Astron. Soc. Pacific*, **98**, 92.
- Dapergoles, A., Kontizas, E., Kontizas, M.: 1989, *Inf. Bull. Variable Stars* No. 3377.
- Dapergoles, A., Kontizas, E., Kontizas, M.: 1990, *Inf. Bull. Variable Stars* No. 3538.
- Dapergoles, A., Kontizas, E., Kontizas, M.: 1991, *Inf. Bull. Variable Stars* No. 3714.
- Ferraz-Mello, S.: 1981, *Astron. J.*, **86**, 619.
- Geyer, E. H., Hoffmann, M., Karimie, M. T.: 1982, *Astron. Astrophys. Suppl.*, **48**, 85.
- Gorda, S. Yu.: 1986, *Inf. Bull. Variable Stars* No.2 906.
- Hamdy, M. A., Awadalla, N. S., Morcos, A. B.: 1985, *Inf. Bull. Variable Stars* No. 2682.
- Hardie, R. H., Hall, D. S.: 1990, *J. Astrophys. Astron.*, **11**, 265.
- Kaluzny, J., Rucinski, S. M.: 1986, *Astron. J.*, **92**, 666.
- Kopal, Z.: 1979, *Language of the Stars*, D. Reidel Publ. Co., Dordrecht, Boston.
- Kreiner, J. M., Kim C. H., Nha, I. S.: 2001, *An Atlas of (O-C) Diagrams of Eclipsing Binary Stars*, Wydawnictwo Naukowe Akademii Pedagogicznej, Cracow.
- Lee, J. W., Han, W., Kim, C. H.: 1999, *J. Astron. Space Sci.*, **16**, 277.
- Rovithis, P., Rovithis-Livaniou, H.: 1981, *Inf. Bull. Variable Stars* No. 1994.
- Rovithis, P., Rovithis-Livaniou, H.: 1982, *Astrophys. Space Sci.*, **82**, 229.
- Rucinski, S. M.: 1973, *Acta Astron.*, **23**, 79.
- Rucinski, S. M.: 1974, *Acta Astron.*, **24**, 119.
- Rucinski, S. M.: 1976a, *Publ. Astron. Soc. Pacific*, **88**, 244.
- Rucinski, S. M.: 1976b, *Publ. Astron. Soc. Pacific*, **88**, 777.
- Rucinski, S. M., Kaluzny, J.: 1982, *Astrophys. Space Sci.*, **88**, 433.
- Schneller, H.: 1949, *Veröff. Sternw. Sonneberg*, **1**, 360.
- Schneller, H.: 1959, *Publ. Astrophys. Obs. Potsdam*, **30**, 1.
- Wilson, R. E., Devinney, E. J.: 1971, *Astrophys. J.*, **166**, 605.
- Wilson, R. E., Devinney, E. J.: 1973, *Astrophys. J.*, **182**, 539.
- Wood, D. B., 1970, *Bull. Amer. Astron. Soc.*, **2**, 357.
- Yamasaki, A.: 1979, *Astrophys. Space Sci.*, **60**, 173.

Received on 9 August 2005

DYNAMICS OF THE SOLAR CORONA USING LASCO-C1 SPECTRAL DATA

MARILENA MIERLA¹, RAINER SCHWENN², LUCA TERIACA²,
GUILLERMO STENBORG³, BORUT PODLIPNIK²

¹ *Astronomical Institute of the Romanian Academy
Str. Cuștitul de Argint 5, 040557 Bucharest, Romania
E-mail: marilena@aira.astro.ro*

² *Max Planck Institute for Solar System Research
Max-Planck-Str. 2, 37191 Kallenburg-Lindau, Germany
E-mail: schwenn@mps.mpg.de, teriaca@mps.mpg.de, podlipnick@mps.mpg.de*

³ *NASA Goddard Space Flight Center, Greenbelt, MD, 20771, USA
E-mail: stenborg@kreutz.nascom.nasa.gov*

Abstract. The LASCO-C1 telescope on board of Solar and Heliospheric Observatory was designed to perform spectral analysis of coronal structures. The tunable Fabry-Perot interferometer allows obtaining images at different wavelengths. Results from spectral scans of the coronal green emission line during 28–30 March, 1998 interval are presented. From the line profiles physical quantities like temperatures (from line widths), and flow velocities (from Doppler shifts) along the line of sight are got. Line of sight speeds in the range -7 to 7 km/s were derived and effective temperatures much higher than the typical coronal values (around 2 MK) were obtained. Three coronal mass ejections could be identified by the large scale line shifts in the speed maps. In the radiance maps no changes were observed during the coronal material passage. In the case of the coronal ejection observed on 28 March (14:04 UT) an increase of the effective temperature was detected while the ejection material was passing.

Key words: Sun – green corona – dynamics – coronal mass ejections.

1. INTRODUCTION

Spectroscopic observations of the solar corona in emission lines can provide important information about the physical processes taking place there. The emission profile from forbidden transition of Fe XIV (5 303 Å) contains information on the physical parameters such as temperature, mass motion, turbulence. Coronal emission lines at visible wavelengths have been observed using Fabry-Perot (FP) techniques or spectrographs during total solar eclipses and with coronagraphs at high altitudes

(e.g., Singh 1985; Ichimoto et al. 1995; Hara and Ichimoto 1999). This kind of observations is constrained by scattering effects in the terrestrial atmosphere and rarely goes up to $1.2 R_{\odot}$.

The LASCO (Large Angle Spectrometric Coronagraph) (Brueckner et al. 1995) C1 instrument, with its field of view (FOV) from 1.1 to $3 R_{\odot}$ on board the SOHO (Solar Heliospheric Observatory) spacecraft has provided unprecedented observations of these spectral lines since launch in December 1995 until June 1998. In the C1 design, a Fabry-Perot interferometer is used as a narrow passband, tunable filter. It allows generating a series of images at slightly different wavelengths such that line spectra at any point of the image plane can be assembled.

The study of the obtained line profiles can be invaluable in:

- determining the line widths and, hence, ion temperatures over the entire extent of the corona;
- providing unambiguous evidence of mass flow in the corona from the Doppler shift of the line.

The studies carried out in this work are done on sets of Fe XIV scan data, taken with the C1 interferometer on 28–30 March 1998, i.e. during the rising phase of solar cycle 23. In particular three coronal mass ejections (CMEs) could be observed in this period and a detailed analysis of them is performed. Even if CMEs are characterized by high velocities, their signatures could be seen in our spectral data by large scale line shifts.

The paper is organized as follows:

In Section 2 a detailed description of the instrument is presented. In Section 3 a summary of the data reduction is shown. The interpretation of the emission line parameters is presented in Section 4. A case study of three CMEs is also performed. In Section 5 the main results are summarized.

2. THE INSTRUMENT

The LASCO instrument consists of three individual coronagraphs with nested fields of view. The three telescopes comprising LASCO are:

LASCO – C1, with coverage from 1.1 to $3.0 R_{\odot}$;

LASCO – C2, extending from 2.0 to $6.0 R_{\odot}$, and

LASCO – C3, which spans the outer corona from about 3.7 to $32 R_{\odot}$.

The C1 instrument is a newly developed mirror version of the classic Lyot internally occulted coronagraph, while the C2 and C3 are externally occulted instruments. C1 is equipped with a Fabry-Perot interferometer that allows spatially resolved high-resolution coronal spectroscopy in selected spectral ranges.

The FP is an interference (comb-type passband) filter, which passes many interference orders. It works in conjunction with a broader blocking filter, with a passband sufficiently narrow to eliminate all but one selected order passed by the FP, but broad enough to allow tuning of the FP over nearly a full free spectral range (the wavelength range between successive orders). During series of exposures, the FP passband is stepped across a part of the free spectral range to cover a desired spectral line, with the scanning activated by piezoelectric drivers. The final image is formed on a 1024×1024 pixel CCD camera with a spatial resolution of $11''$ (for more details see Mierla 2005).

The Fabry-Perot has an intrinsic full width at half maximum (FWHM) of approximately $0.7 \text{ \AA}$, and a free spectral range of $35 \text{ \AA}$. Therefore the finesse (the ratio of the free spectral range to the spectral passband) is about 50. Cavity length and wedge defect (departure from parallelism) of the interferometer plates are controlled from a microprocessor that uses capacitance micrometers for position sensing. Three piezoelectric transducers determine the cavity length. This combination allows automatic adjustment of wavelength and finesse. The coronal spectrum is referenced to the solar disk Fraunhofer spectrum using three optical control channels, which are located on the perimeter of the interferometer plates. The coatings of the interferometer plates limit the useful wavelength range to approximately $1300 \text{ \AA}$. The coating was selected to cover the range from Fe XIV at $5303 \text{ \AA}$ to H-alpha at $6562 \text{ \AA}$.

As we mentioned above, sets of individual monochromatic images can be obtained by scanning over a range of wavelengths around each observed coronal emission line. This is illustrated in Fig. 1. The top panel shows the Airy profile of the FP used in C1 for the $5303 \text{ \AA}$ line. The three peaks represented by continuous line show the transmission profile of the FP without any blocking filters. The dashed curve shows the blocking filter profile. The resultant filter transmission is given by the product of these two profiles and it is shown by the middle peak.

The middle panel shows an enlarged view of the FP transmission peak at the line center (within the two vertical dashed lines in the top figure). The horizontal line indicates the FWHM of the filter ($0.7 \text{ \AA}$). The external profile is an assumed Lorentzian profile that is similar with the Fe XIV line to be diagnosed.

To determine the line profile, the FP pass band is tuned in steps (of $0.3 \text{ \AA}$) from the blue side of the emission profile (represented by negative values) to the red side (positive values) as sketched in the lower panel of Fig. 1. The reproduced profile, for each pixel in an image, is a convolution of the filter profile and the line profile.

In order to reproduce the line profile accurately it is necessary to have a narrow band filter (high spectral resolution) and fine tuning steps. The tuning range of the FP is $\pm 7.6 \text{ \AA}$. In terms of Doppler shifts, this range will correspond to velocities of $\pm 400 \text{ km/s}$. The dashed curve in Fig. 1 shows the line for 500 km/s , i.e. beyond the measurement range.

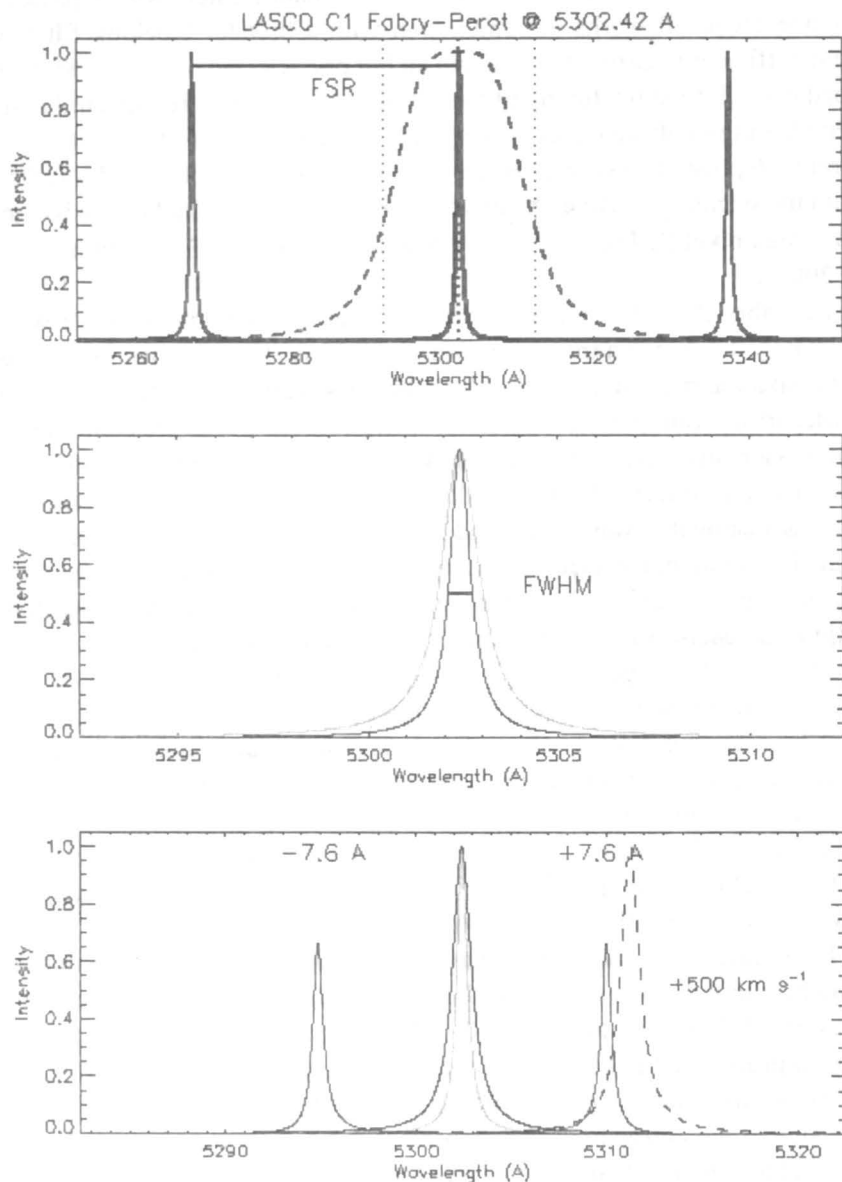


Fig. 1 – Upper panel: The FP transmission profile (the three peaks represented by continuous line) plus the blocking filter profile (dashed curve). The middle peak is the resultant filter transmission. Middle panel: Enlarged view of the FP transmission peak at the line center. The horizontal line: FWHM of the filter (0.7 Å). The outer profile is a Lorentzian profile with 1.2 Å FWHM. Lower panel: The filter tuned at 3 different wavelength positions. Blue and red (left hand and right hand profiles): -7.6 Å, and +7.6 Å, respectively from line center, middle profile: line center. The dashed curve shows the line shifted corresponding to 500 km/s (Courtesy of S. Mathew).

3. OBSERVATIONS AND DATA REDUCTION

The data analyzed here are green (Fe XIV) line images taken on 28–30 March 1998 (on the rising phase of the solar cycle). They consist of series of about 47 scans taken on both the west and east limb of the Sun. Each scan consists of 15 images at wavelengths ranging from 5 302 Å to 5 306.5 Å in steps of 0.3 Å, along with two off-line images at wavelengths of 5 311.4 Å and 5 299.9 Å (the wavelengths are given relative to vacuum). The images were taken with a cadence of 1 minute, the exposure time being 25 seconds. Thus, spectra are recorded in around 15 minutes.

A detailed description of data reduction is given in Mierla et al. (2005). To summarize, the following steps are applied:

1) *Bias subtraction*. The bias was artificially added when the signal was digitized in order to avoid negative values. The bias value for the C1 detector has been found to be slowly increasing with time (see Fig. 2). Calculation of the actual LASCO-C1 offset bias has been done using dark images for various days during the mission. The value of the offset bias for a particular dark image was taken to be the value assumed by the majority of pixels in the image.

2) *Cosmic-ray hits removal*. The bad pixel values due to high energetic particles were partially removed by using a median filter technique.

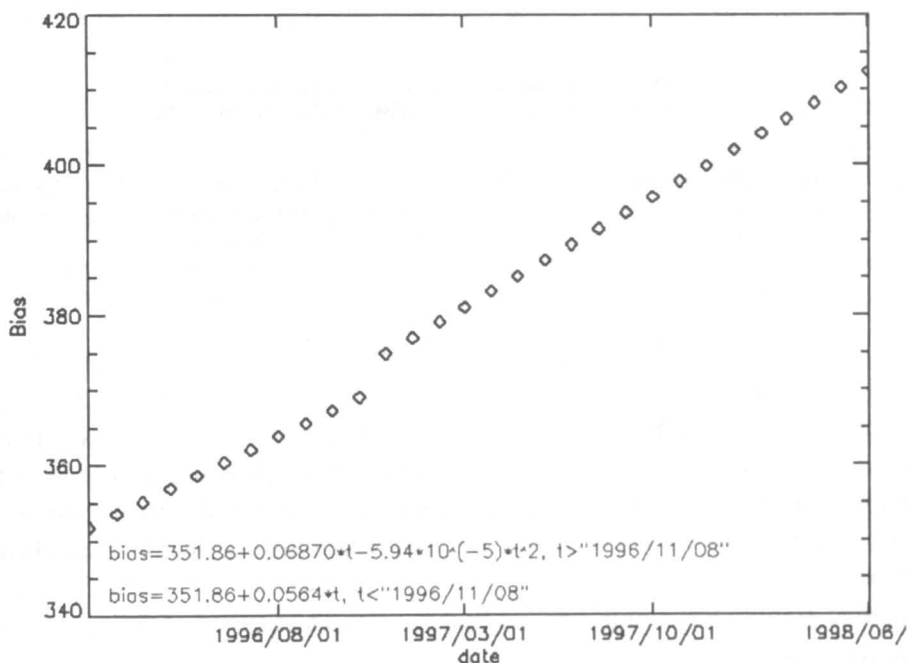


Fig. 2 – The variation of C1 bias in time. The bias was computed using the formulas shown in the figure.

3) *Exposure time normalization.* Each image in a data set was divided by the exposure time. This information is given in the header of each FITS file.

4) *Relative calibration of the wavelengths in the FOV of each image.* Due to different incident angles at which the light hits the FP detector, the wavelength is different at different locations in an image. Calibration was done by the LASCO-C1 team (see also http://lasco-www.nrl.navy.mil/~reiser/C1_wave.html).

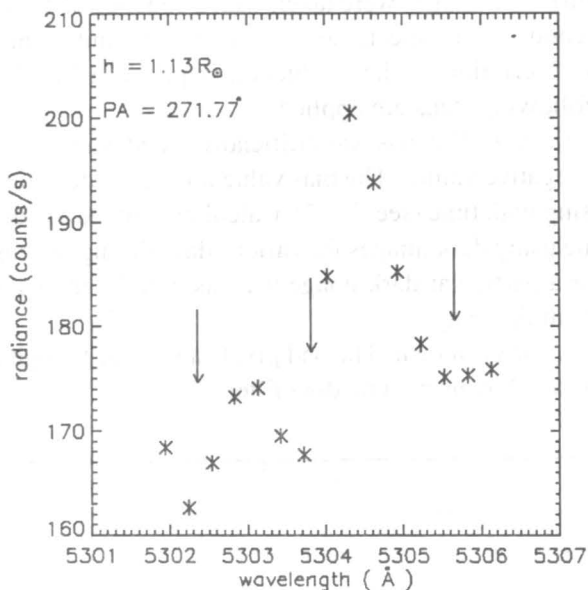


Fig. 3 – Spectrogram of the coronal region (PA 272° and $d = 1.13R_{\odot}$), as observed by LASCO-C1 on 28th of March 1998. The emission line is affected by three nearby absorption lines (indicated by the arrows), from scattered photospheric light. The statistical errors as determined by the formula described in

Subsection 5.2 are represented by vertical bars (in this case they are smaller than the symbol).

The polar angle (PA) is measured from North Pole, counterclockwise.

5) *Absolute wavelength calibration using the main Fraunhofer line at 5303.78 Å (Fe I) observed as scattered light into the instrument.* As it is observed in Fig. 3, Fe XIV profile is affected by three nearby absorption lines from the scattered light in the instrument (indicated by the arrows). In order to get the pure emission profile we have to take out the influence of the absorption lines. For getting the absorption lines we take those regions in the corona where the emission line is absent, like for example in the coronal holes above the poles. The measured radiances are fitted with 3-Gaussians (negative amplitudes) and a constant background. The method of fitting consists in minimization of χ^2 , using the Levenberg-Marquardt method. A profile of the absorption spectrogram above the North Pole, for a data scan recorded in March 1998,

is shown in Fig. 4. For comparison, the corresponding spectrogram from the photospheric atlas (NSO) is overplotted as a dashed line. The positions of the absorption lines are all shifted to overlay the value from the catalogue. The shifts are used to recalibrate the wavelengths for every data set.

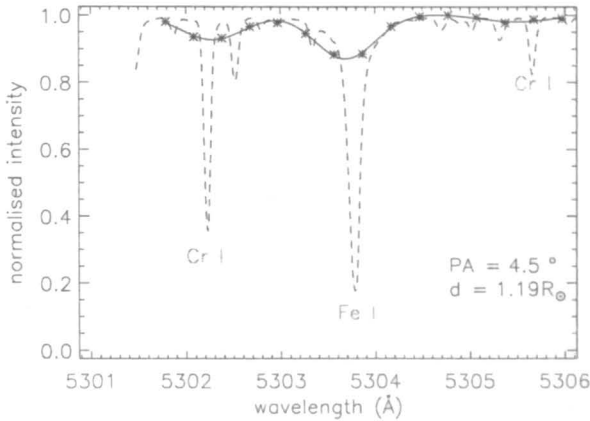


Fig. 4 – Spectrogram obtained for a region without green corona at north in the data of 28th of March 1998 (asterisks). The solid line shows 3-Gaussian fits to the data. The photospheric spectrogram (in vacuum wavelengths) in the region of interest from *An Atlas of the Spectrum of the Solar Photosphere* from 3 570 Å to 7 405 Å, Kitt Peak National Observatory, is overplotted in dashed line.

Once the parameters of the absorption lines are known each profile is fitted with the sum of four Gaussians (three absorption lines with the parameters deduced as explained above), plus one emission line and a constant background (for a detailed discussion see Mierla et al. 2005). From the line fit, emission parameters like line width, line-of-sight (LOS) speeds and radiances are deduced.

4. DATA INTERPRETATION AND DISCUSSION

The parameters of the emission line give us information about the ‘ion temperature’ (the width of the line) and velocities (the Doppler shift of the position of the line peak from the rest wavelength) in the corona, for each pixel in the image. The emission maps are built by replacing each pixel with the emission line parameters. Their interpretation is given in Subsection 4.1.

4.1. EMISSION MAPS DESCRIPTION

1) *Radiance maps*. In order to build the radiance map, each pixel of an image is replaced by the intensity at the peak of the line. Fig. 5 shows an example of radiance

map taken on 28th of March 1998, 05:46 UT. In order to see the entire corona, both east and west limb images are shown together.

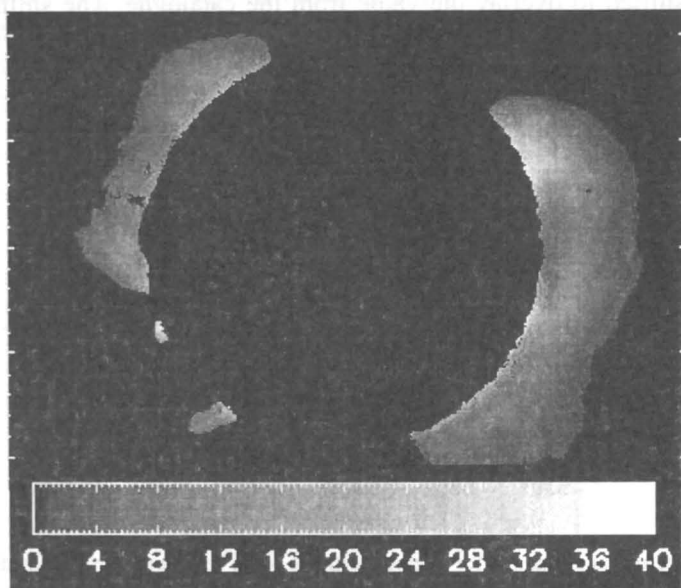


Fig. 5 – The radiance map for Fe XIV data on 28th of March 1998, at around 05:46 UT. The white color represents high radiance (counts/s).

The radiance maps give us information about the ion density distributions in the solar corona. The Fe XIV ions are confined mostly around the magnetic field lines usually in the shape of loops (see north west side of Fig. 5). Among other things, we find the well-known absence of green line emission above the polar coronal holes near activity minimum confirmed.

2) *Velocity maps.* From the shift of the central wavelength to the reference wavelength (Doppler shift), we can get the line-of-sight velocity:

$$\frac{\lambda_0 - \lambda}{\lambda_0} = \frac{v}{c}, \quad (1)$$

where: λ_0 is the rest (reference) wavelength, v the line-of-sight velocity and c is the speed of light. For $\lambda > \lambda_0$ the plasma moves away from the observer (the wavelengths are shifted to the red part of the visible spectrum). For $\lambda < \lambda_0$ plasma moves towards the observer (the wavelengths are shifted to the blue part of the visible spectrum).

Fig. 6 shows an example of a Doppler map for Fe XIV data recorded on 28th of March, 05:46 UT.

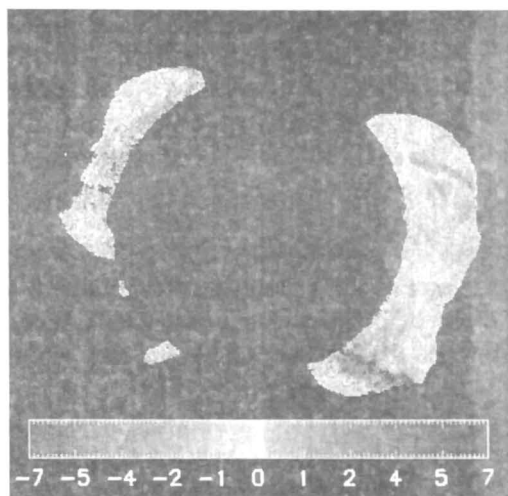


Fig. 6 – The velocity map for Fe XIV data, showing the plasma moving along the line of sight in the range -7 km/s to 7 km/s. The uncertainties in determining the speed values are of 2 km/s. The data were recorded on 28th of March, at around 05:46 UT.

One may notice the strong blue shift in the streamer at the south west limb. This was identified with the outflow at the base of the streamer observed in LASCO-C2 images (Mierla et al. 2006).

3) *FWHM maps*. The width map (Fig. 7) is a signature of plasma temperature and unresolved motions like e.g. turbulence.

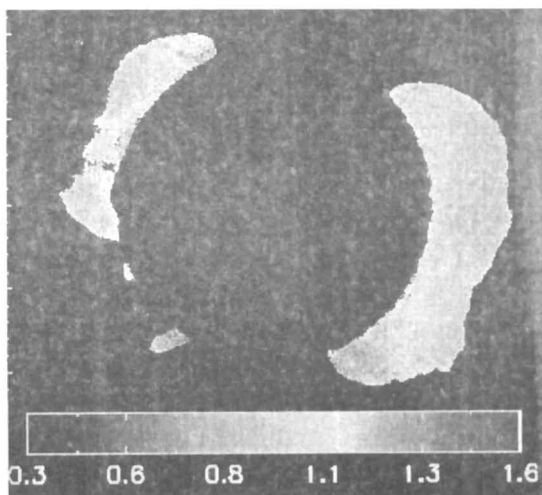


Fig. 7 – The FWHM map for Fe XIV data. The red color represents a large width (high effective ion temperature), and blue means a narrow line (low effective ion temperature). The data were recorded on 28th of March, at around 05:46 UT.

The FWHM of spectral lines represented by a Gaussian shape is given by:

$$FWHM = \left\{ 4 \ln 2 \cdot \frac{\lambda^2}{c^2} \cdot \left(\frac{2kT_{ion}}{M_{ion}} + \xi^2 \right) \right\}^{1/2} = \left\{ 4 \ln 2 \cdot \frac{\lambda^2}{c^2} \cdot \left(\frac{2kT_{ion,eff}}{M_{ion}} \right) \right\}^{1/2}, \quad (2)$$

where $T_{ion,eff}$ is defined as ‘effective ion temperature’, k is the Boltzmann constant and M_{ion} is the mass of the ion. It is seen that the FWHM in our data reaches up to 1.6 Å. This value corresponds to an effective temperature of around 8 MK. As 2 MK is the temperature at the maximum of ionic fraction for the Fe XIV line, higher values reflect the non-thermal contributions to the profile of the line.

4.2. DYNAMICS OF THE SOLAR CORONA – CASE STUDY FOR CORONAL MASS EJECTIONS

Coronal Mass Ejections are observed in LASCO-C2 and C3 coronagraphs (at distances higher than $2 R_{\odot}$) by the motion of the plasma in the sky plane. Due to their

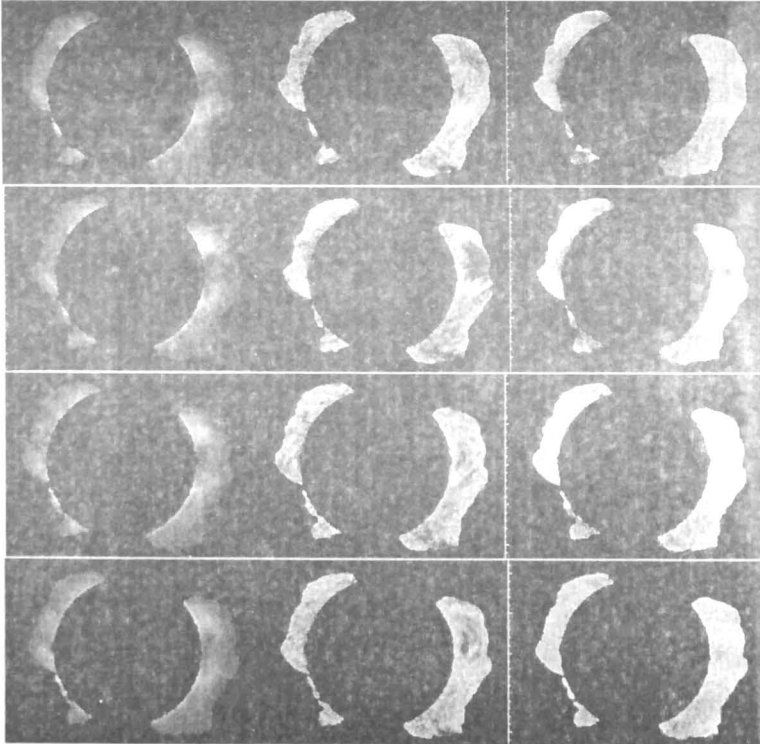


Fig. 8 – Radiance (left hand column of images), speed (middle images) and FWHM (right hand column of images) maps as recorded on 28th of March 1998 at 11:58 UT (upper row of images), 14:04 (the row right below), 15:46 and 16:10 (lower row).

high speeds (see, e.g., Schwenn et al. 2004) and due to the limited spectral range of the instrument the CMEs are very difficult to detect in C1 field of view (from 1.1 to 3 $R_{\odot}$). Nevertheless, in the period described here, we could catch the passage of three CMEs. They were identified in our spectral scans due to large red shifts in speed maps.

Fig. 8 shows the emission maps as recorded on 28th of March at four different times (see the caption of the figure). The right hand column represents the radiance map, the middle one – the speed maps and the left hand one – the FWHM maps. At around 14:04 UT a strong red shift is observed in the speed map, on the west side of the Sun (the second row, the middle image). Traces of the red shift are further noted up to around 16:00 UT (the last row, the middle image).

In the radiance maps (Fig. 8, left) no significant changes are observed. Looking at LASCO-C2 movie (http://cdaw.gsfc.nasa.gov/CME_list/), we see the CME rising at around 14:00. The height time (HT) diagram in Fig. 9 shows the points when the CME is observed in C2 and C3 field of views. The intersection of the rectangle with the line fits to the points shows the passage of the CME at a distance of 2 $R_{\odot}$, at around 14:00, time when the red shift is observed in our spectral data.

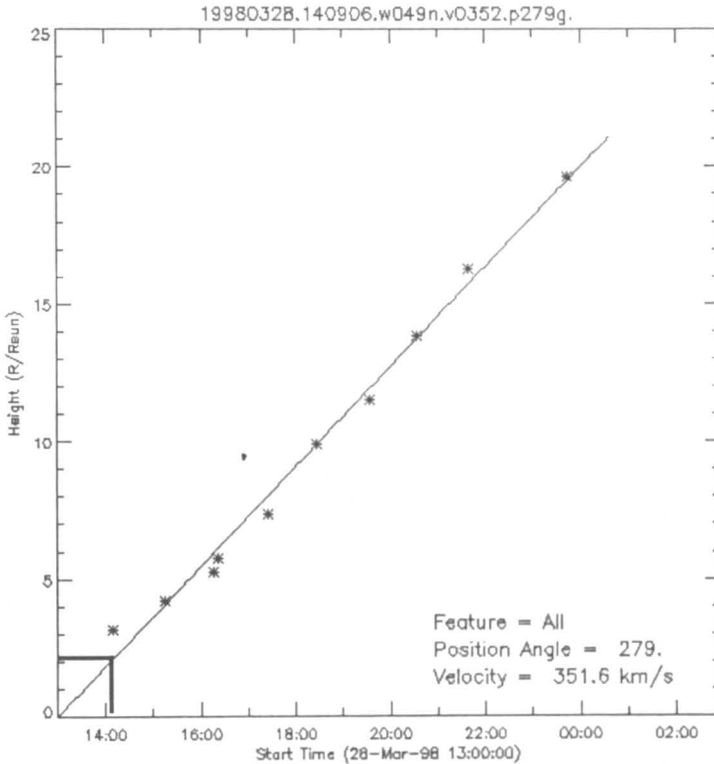


Fig. 9 – HT diagram of the CME from NRL CME catalogue, as seen in C2 and C3 field of views. The line shows the linear fit to the data points. It is extrapolated back in time in order to see the onset of CME in C1 field of view. The black rectangle shows the time of the CME at a distance of 2 $R_{\odot}$.

In the width maps (column 3 of Fig. 8) we find an increase of the FWHM (higher effective temperature) while the CME material was passing.

Later on, at around 22:41 another CME (a very faint halo CME) is passing the field of view of C2. This CME cannot be identified in our C1 spectral data. Instead, an outflow starting at about the same time (see Fig. 10) can be observed in speed maps as a red shift (see Fig. 11 – middle image).

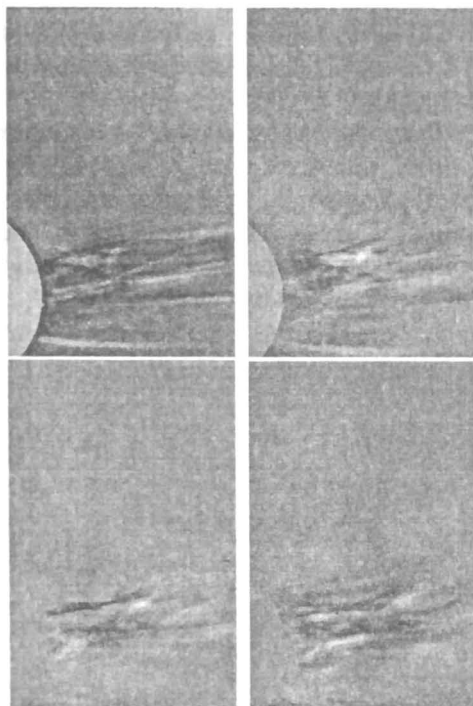


Fig. 10 – Running difference images showing the CME on the WNW limb of the Sun. The frames are recorded on 28–29 March 1998 at 21:35 UT (upper left), 22:41 UT (upper right), 23:36 UT (lower left) and 00:39 UT (lower right). The white regions followed by the black ones are signatures of the outflow material.

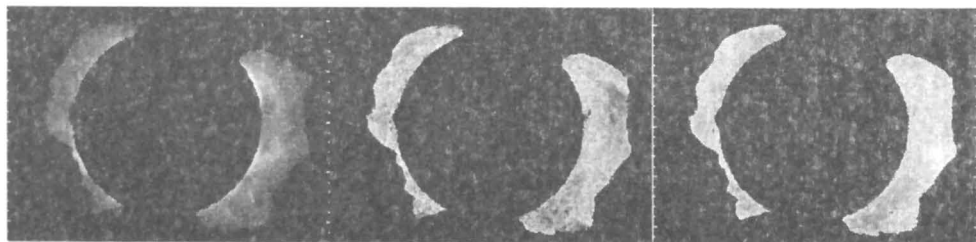


Fig. 11 – Radiance (left image), speed (middle image) and FWHM (right image) maps as recorded on 28th of March 1998 at 22:05 UT.

In this case, in the width map (right-hand image of Fig. 11), no significant changes are observed.

On 29th of March we observe in the C2 data a back-side halo CME (Schwenn et al. 2004) beginning at 03:48 UT. The front speed of this CME was almost 1400 km/s,

qualifying it as an extremely fast event (see Fig. 12). Extrapolating back the points seen in C2 and C3 field of views it observed that the CME front edge passes $2 R_{\odot}$ at around 03:00 (the intersection of the rectangle with the line fits to the points). In this period of time C1 spectral data are missing and we cannot say if the CME was observed or not in C1 field of view.

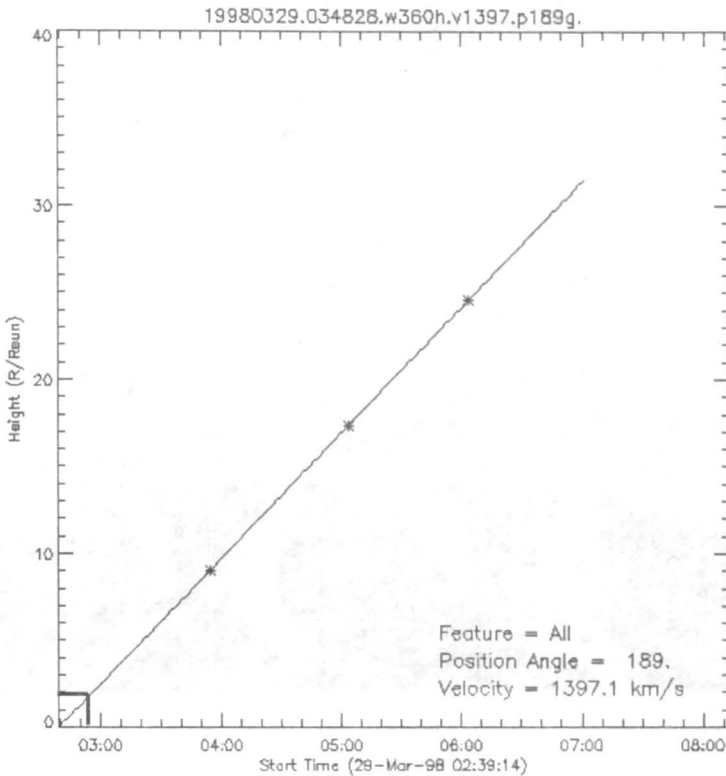


Fig. 12 – HT diagram of the CME from NRL CME catalogue, as seen in C2 and C3 field of views. The line shows the linear fit to the data points. It is extrapolated back in time in order to see the onset of CME in C1 field of view. The black rectangle shows the time of the CME at a distance of $2 R_{\odot}$.

Still, the signature of the plasma following the CME could be observed in the spectral data by the strong red shift on 29th of March 1998, 04:21 UT (third row, middle image – west limb of Fig. 13), long after the CME has passed the field of view of C1. In the left hand side images of Fig. 13 it is observed that the green corona does not changed significantly before (upper left image) and after (lower left image) the CME has passed.

Note that even if it was a halo CME, we could see its signatures only on the west side of the Sun. This is due to the fact that the CME does not expand with a uniform speed in all the directions and very high speed plasma must have moved considerably during the scanning time (15 minutes) of the C1 spectrum.

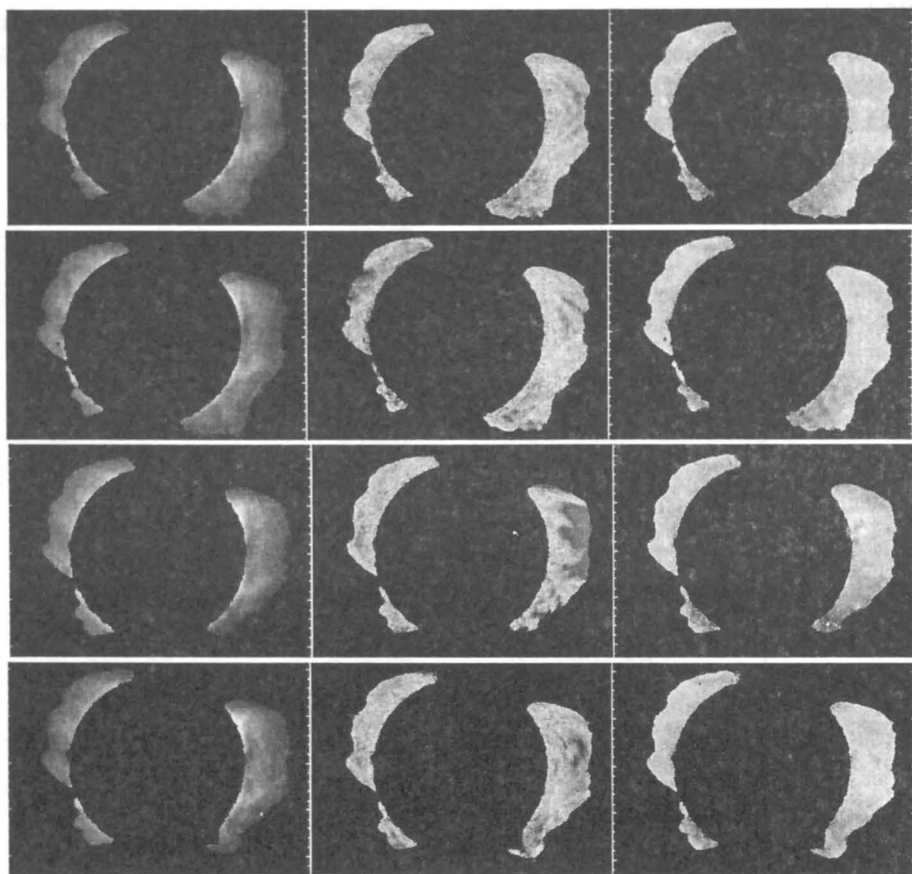


Fig. 13 – Radiance (left hand column of images), speed (middle images) and FWHM (right hand images) maps as recorded on 29th of March 1998 at 02:16 UT (upper images), 02:40 (the row right below), 04:21 and 06:30 (lower images).

5. SUMMARY

5.1. For the majority of data sets analyzed in this period speed values on the line of sight in the range of -7 to 7 km/s were observed. That allows studying the large-scale dynamics of the corona.

5.2. The line widths, if interpreted as ‘effective temperatures’ were found to be much larger than the typical temperatures at which Fe XIV ions form (2 MK). The excess is due to non-thermal motions in the plasma (like turbulences, wave motions, etc.).

5.3. CMEs can be identified by the large-scale line shifts, in the LASCO-C1 Doppler maps. The signature of the CME on 28 March 1998 (14:04 UT) was the strong red shift observed in the speed map right in the moment the CME plasma was passing

the field of view of C1. For the fast halo CME observed on 29 March, due to the time resolution of the spectral data, we could see only the plasma following the CME.

In summary, LASCO-C1 spectral analysis proved to be a valuable tool to study the dynamics of the inner corona.

Acknowledgments. The authors are thankful to Max Planck Institut für Sonnensystemforschung for the facilities to carry out this work. Our sincere thanks to Georgeta Mariş for the constructive comments and valuable discussions. We would like to thank the SOHO/LASCO/EIT consortium for providing the data and the software libraries. SOHO is a project of international cooperation between ESA and NASA.

REFERENCES

- Brueckner, G. E., Howard, R. A., Koomen, M. J. et al.: 1995, *Solar Phys.*, **162**, 357.
Hara, H., Ichimoto, K.: 1999, *Astrophys. J.*, **513**, 969.
Ichimoto, K., Ohtani, H., Ishigaki, T., Maemura, H., Noguchi, M.: 1995, *Publ. Astron. Soc. Japan*, **47**, 383.
Mierla, M.: 2005, PhD thesis, Copernicus GmbH, Göttingen.
Mierla, M., Schwenn, R., Teriaca, L., Stenborg, G., Podlipnik, B.: 2005, *Adv. Space Res.*, **35**, 2199.
Mierla, M., Schwenn, R., Teriaca, L., Stenborg, G., Podlipnik, B.: 2006, *Astron. Astrophys.* (to be submitted).
Schwenn, R., Dal Lago, A., Huttunen, E., Gonzalez, W. D.: 2005, *Ann. Geophys.*, **23**, 1.
Singh, J.: 1985, *Solar Phys.*, **95**, 253.

Received on 4 April 2005

HÉNON-HEILES' TWO-BODY PROBLEM.

II. MAIN FEATURES OF THE GLOBAL FLOW

DUMITRU PRICOPI, VASILE MIOC

*Astronomical Institute of the Romanian Academy
Str. Cuştitul de Argint 5, 040557 Bucharest, Romania
E-mail: dpricopi@aira.astro.ro, vmioc@aira.astro.ro*

Abstract. We tackle the two-body problem associated to Hénon-Heiles' famous potential. We fully describe the flows on the radial orbits and in their neighbourhood, and point out the main features of the global flow.

Key words: celestial mechanics – Hénon-Heiles' model – global flow

1. INTRODUCTION

Hénon-Heiles' (1964) potential reads $W(q_1, q_2) = (q_1^2 + q_2^2)/2 + q_1^2 q_2 - q_2^3/3$, where $(q_1, q_2) \in \mathbf{R}^2$ are the standard Cartesian coordinates. Many authors tackled this model, from the initial numerical experiment via Poincaré sections (Hénon and Heiles 1964) to quantitative and qualitative approaches (e.g., van der Merwe 1991; Antonov and Timoshkova 1993; Anisiu and Pal 1999; Mioc and Barbosu 2003a, b). For a much larger and deeper insight into the model, see Boccaletti and Pucacco (1996).

In Section 2, the basic equations are written down. In Section 3 we study the existence and the nature of the equilibrium points. We find three saddle points at the distance $r = 1$ from centre on the directions $\theta \in \{\pi/2, 7\pi/6, 11\pi/6\}$. The origin $(0,0)$ is a saddle, too. In Section 4, the main features of the global flow are pointed out.

In Section 5, we discuss the particle dynamics on the radial orbits. The motion in the neighbourhood of the collision and infinity manifold is described, too. The results agree well with our previous paper (Mioc and Pricopi 2005). Some considerations regarding the stability of radial orbits end the paper.

2. BASIC EQUATIONS

Consider the relative motion of a unit-mass particle with respect to the field-generating source. Its dynamics is associated to the planar Hamiltonian

$$H(\mathbf{q}, \mathbf{p}) = (p_1^2 + p_2^2)/2 - [(q_1^2 + q_2^2)/2 + q_1^2 q_2 - q_2^3/3], \quad (1)$$

in which $\mathbf{q} = (q_1, q_2) \in \mathbf{R}^2$ and $\mathbf{p} (= \dot{\mathbf{q}}) = (p_1, p_2) \in \mathbf{R}^2$ are the configuration vector and the momentum vector of the particle, respectively.

The equations of motion generated by (1) explicitly read

$$\begin{aligned} \dot{q}_1 &= p_1, & \dot{q}_2 &= p_2, \\ \dot{p}_1 &= q_1 + 2q_1 q_2, & \dot{p}_2 &= q_1^2 - q_2^2 + q_2. \end{aligned} \quad (2)$$

The Hamiltonian (1) is a constant of motion, i.e. $H(\mathbf{q}, \mathbf{p}) = h$, which provides the first integral of energy (h is the energy constant).

The angular momentum $L(\mathbf{q}, \mathbf{p})$ is not a constant of motion (except certain cases that will be pointed out in the next sections), hence we do not dispose of the corresponding first integral. This was to be expected, given the anisotropic structure of the potential.

3. EQUILIBRIUM POINTS

We find the equilibria by putting $\dot{q}_i = \dot{p}_i = 0, i = \overline{1, 2}$ in (2) and solving the resulting set of equations:

$$\begin{aligned} q_1(1 + 2q_2) &= 0, \\ q_1^2 + q_2 - q_2^2 &= 0. \end{aligned} \quad (6)$$

We find four equilibrium points:

$$(q_1^0, q_2^0) \in \{(0, 0), (0, 1), (\sqrt{3}/2, -1/2), (-\sqrt{3}/2, -1/2)\}. \quad (7)$$

The characteristic equation reads:

$$[\lambda^2 - 2(1/2 + q_2^0)][\lambda^2 - 2(1/2 - q_2^0)] + 4(q_1^0)^2 = 0. \quad (8)$$

For $(q_1^0, q_2^0) = (0, 0)$ the eigenvalues are $\lambda_1 = \lambda_2 = -1$ and $\lambda_3 = \lambda_4 = 1$. This equilibrium point is a saddle (the motion in the neighbourhood of the collision manifold is unpredictable: the test particle can collide or can escape to infinity). For $(q_1^0, q_2^0) = (0, 1)$, the eigenvalues are $\lambda_{1,2} = \pm i$ and $\lambda_{3,4} = \pm\sqrt{3}$. So, this is a saddle, too. For $(q_1^0, q_2^0) \in \{(-\sqrt{3}/2, -1/2), (\sqrt{3}/2, -1/2)\}$ the characteristic equation reads

$\lambda^4 - 2\lambda^2 + 3 = 0$. It is easy to show that two roots have positive real part and two have negative real part: $\text{Re } \lambda_{1,2,3,4} = \pm \sqrt{(1 + \sqrt{3})/2}$. These equilibria are saddles, too.

4. MAIN FEATURES OF THE GLOBAL FLOW

As usual in anisotropic fields, we pass to standard polar coordinates via the real analytic diffeomorphism

$$\begin{aligned} r &= |\mathbf{q}|, \quad \theta = \arctan(q_2 / q_1), \\ u &= \dot{r} = (q_1 p_1 + q_2 p_2) / |\mathbf{q}|, \quad v = r\dot{\theta} = (q_1 p_2 - q_2 p_1) / |\mathbf{q}|, \end{aligned} \quad (9)$$

which represents a McGehee-type transformation of the second kind (McGehee 1974). In these coordinates the motion equations read

$$\begin{aligned} \dot{r} &= u, \quad \dot{\theta} = v / r, \\ \dot{u} &= v^2 / r + r + (3 - 4 \sin^2 \theta) r^2 \sin \theta, \\ \dot{v} &= -uv / r + (4 \cos^2 \theta - 3) r^2 \cos \theta, \end{aligned} \quad (10)$$

whereas the energy integral acquires the form

$$(u^2 + v^2) / 2 - r^2 / 2 - [(3 - 4 \sin^2 \theta) r^3 \sin \theta] / 3 = h. \quad (11)$$

All the orbits are situated on the Hill's surface

$$f(r, \theta) = r^2 / 2 + [(3 - 4 \sin^2 \theta) r^3 \sin \theta] / 3 + h, \quad (12)$$

where $f(r, \theta) = (u^2 + v^2) / 2 \geq 0$ is the kinetic energy of the particle. From a detailed analysis of the Hill's surface we obtain the following results:

I. On the directions $\theta \in (\pi/3, 2\pi/3) \cup (\pi, 4\pi/3) \cup (5\pi/3, 2\pi)$:

1. In the negative-energy case, $h < 0$, we distinguish the following three situations:

a) $h < -1/6$. In this case there exist six directions $\tilde{\theta}_i, i = \overline{1,6}$ such that $f = 0$.

The motion is possible only on the directions

$$\theta \in (\pi/3, \tilde{\theta}_1] \cup [\tilde{\theta}_2, 2\pi/3) \cup (\pi, \tilde{\theta}_3] \cup [\tilde{\theta}_4, 4\pi/3) \cup (5\pi/3, \tilde{\theta}_5] \cup [\tilde{\theta}_6, 2\pi).$$

On these directions:

- the motion is bounded;
- there are no equilibrium points;
- the collisions are not possible.

b) $h = -1/6$. In this case:

- the motion is possible on all of the directions considered;
- the motion is bounded;
- the collisions are not possible;

There exist three equilibrium points on the directions $\theta \in \{\pi/2, 7\pi/6, 11\pi/6\}$ at the distance $r = 1$ from the centre. (They are saddle points, according to the discussion of Section 3).

c) $-1/6 < h < 0$. In this case:

- the motion is possible on all of the directions considered;
- the motion is bounded;
- the collisions are not possible;
- there are no equilibrium points.

2. In the case of nonnegative energy, $h \geq 0$ (see Fig. 1 for the zero-energy case):

- the motion is possible on all of the directions considered;
- the motion is bounded;
- the collisions are possible;
- there are no equilibrium points.

II. On the directions $\theta \in [0, \pi/3] \cup [2\pi/3, \pi] \cup [4\pi/3, 5\pi/3]$:

- the motion is unbounded (the particle can be captured from infinity or can escape to infinity);
- the collisions are possible;
- there are no equilibrium points.

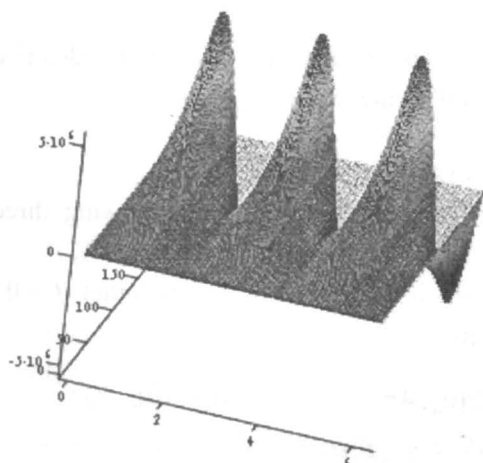


Fig. 1 – Hill's surface for the zero-energy case.

5. RADIAL ORBITS

From the equations of motion (10) we can write the angular momentum of the test particle $r^2\dot{\theta} = r\nu$. His variation in time is writing

$$\frac{d}{dt}(r^2\dot{\theta}) = r^3 \cos 3\theta. \quad (13)$$

It follows that the radial orbits exists on the directions

$$\theta \in \{\pi/6, \pi/2, 5\pi/6, 7\pi/6, 3\pi/2, 11\pi/6\}.$$

From the discussion about the global flow (see Section 4) it results that the radial orbits on the directions $\theta \in \{\pi/6, 5\pi/6, 3\pi/2\}$ are unbounded (capture/escape) and, those on the directions $\theta \in \{\pi/2, 7\pi/6, 11\pi/6\}$ are bounded (oscillatory orbits).

5.1. THE RADIAL ORBITS ON THE DIRECTIONS $\pi/6$, $5\pi/6$ AND $3\pi/2$

As we said, these radial orbits are unbounded, the test particle can be captured or can escape to infinity. In the sequel we shall discuss the motion on these orbits. For simplicity, we shall focus on the radial orbit with $\theta = \pi/6$. The motion on the other two radial orbits is completely similar.

For $\theta = \pi/6$ the equations of motion (10) read

$$\begin{aligned} \dot{r} &= u, \\ \dot{\theta} &= 0, \\ \dot{u} &= r + r^2, \\ \dot{\nu} &= 0. \end{aligned} \quad (14)$$

The first integral of energy is

$$\dot{r}^2 = r^2 + \frac{2}{3}r^3 + 2h, \quad (15)$$

and the particle is moving according to

$$t = \int \frac{dr}{\pm \sqrt{r^2 + \frac{2}{3}r^3 + 2h}}. \quad (16)$$

In particular, for the zero energy case $h = 0$, we can write the equation of motion

$$r(t) = 6 \frac{e^{\pm(t-t_0) + \ln\left[\left(\sqrt{r_0+3/2}-\sqrt{3/2}\right)/\left(\sqrt{r_0+3/2}+\sqrt{3/2}\right)\right]}}{\left[1 - e^{\pm(t-t_0) + \ln\left[\left(\sqrt{r_0+3/2}-\sqrt{3/2}\right)/\left(\sqrt{r_0+3/2}+\sqrt{3/2}\right)\right]}\right]^2}, \quad (17)$$

where we denoted $r_0 = r(t_0)$, t_0 being the initial instant. From (17) we can obtain the time when the particle reaches infinity:

$$t_\infty = t_0 \pm \ln\left[\left(\sqrt{r_0+3/2}-\sqrt{3/2}\right)/\left(\sqrt{r_0+3/2}+\sqrt{3/2}\right)\right]. \quad (18)$$

In addition we can observe that

$$t_\infty^+ = t_0 + \ln \frac{\sqrt{r_0+3/2}-\sqrt{3/2}}{\sqrt{r_0+3/2}+\sqrt{3/2}} < t_0 < t_0 - \ln \frac{\sqrt{r_0+3/2}-\sqrt{3/2}}{\sqrt{r_0+3/2}+\sqrt{3/2}} = t_\infty^-. \quad (19)$$

Thus, $\Delta t = -\ln\left[\left(\sqrt{r_0+3/2}-\sqrt{3/2}\right)/\left(\sqrt{r_0+3/2}+\sqrt{3/2}\right)\right]$ represents the time interval in which the particle is captured from infinity or escapes to infinity. Moreover, we see that $\Delta t \rightarrow \infty$ if $r_0 \rightarrow 0$ (the particle can touch the collision manifold or can escape from the collision manifold only in a infinite time).

If we take the “+” sign in (17), the particle departs from the centre ($t_\infty = t_\infty^+$), whereas if we take the “-” sign in (17) the particle comes towards collision ($t_\infty = t_\infty^-$) (but $r(t) \rightarrow 0$ for $t \rightarrow \infty$).

Remark 1. In the neighbourhood of the infinity manifold, equation (17) describes the radial motion on the directions $\theta \in \{\pi/6, 5\pi/6, 3\pi/2\}$ for $h \neq 0$, too. This is due to the fact that in the neighbourhood of the infinity manifold we have $r \gg 1$ and the dominant term in the first integral of energy is $r^2 + (2/3)r^3$.

Remark 2. Because the capture/escape is radial on the directions $\theta \in \{\pi/6, 5\pi/6, 3\pi/2\}$ (see Mioc and Pricopi 2005), the particle is moving according to (17) in the neighbourhood of the infinity manifold.

5.2. THE RADIAL ORBITS ON THE DIRECTIONS $\pi/2, 7\pi/6$ AND $11\pi/6$

As we said, the radial orbits on the directions $\theta \in \{\pi/2, 7\pi/6, 11\pi/6\}$ are bounded. In the sequel we shall discuss the motion on these orbits. For simplicity, we

shall focus on the radial orbit with $\theta = \pi/2$. The motion on the other two radial orbits is completely similar.

For $\theta = \pi/2$ the equations of motion (10) read

$$\begin{aligned}\dot{r} &= u, \quad \dot{\theta} = 0, \\ \dot{u} &= r - r^2, \quad \dot{v} = 0.\end{aligned}\quad (20)$$

The first integral of energy is

$$\dot{r}^2 = r^2 - 2r^3/3 + 2h. \quad (21)$$

From (21) is easy to show that:

- for $h < -1/6$ the radial motion on these directions is not possible;
- for $h = -1/6$ we have three equilibrium points at the distance $r = 1$ from centre and directions $\theta \in \{\pi/2, 7\pi/6, 11\pi/6\}$;
- for $h > -1/6$ there exist radial bounded orbits on the directions $\theta \in \{\pi/2, 7\pi/6, 11\pi/6\}$. On these orbits, the particle is moving accordingly to

$$t = \int \frac{dr}{\pm \sqrt{r^2 - \frac{2}{3}r^3 + 2h}}. \quad (22)$$

In particular, in the case of zero energy case, $h = 0$, we can write the equation of motion on these orbits:

$$r(t) = 6 \frac{e^{\pm(t-t_0) + \ln\left[\left(\sqrt{3/2} - \sqrt{-\eta_0 + 3/2}\right)/\left(\sqrt{3/2} + \sqrt{-\eta_0 + 3/2}\right)\right]}}{\left[1 + e^{\pm(t-t_0) + \ln\left[\left(\sqrt{3/2} - \sqrt{-\eta_0 + 3/2}\right)/\left(\sqrt{3/2} + \sqrt{-\eta_0 + 3/2}\right)\right]}\right]^2}. \quad (23)$$

Remark 3. If $r_0 \rightarrow 0$ then $r(t) \rightarrow 0$. That is the particle cannot leave the collision manifold.

Remark 4. If $t \rightarrow \infty$ then $r(t) \rightarrow 0$: the particle arrives on the collision manifold in an infinite time.

Remark 5. The orbits are bounded: $0 < r < 3/2$. If $r_0 \approx 3/2$ then

$$r(t) \approx 6 \frac{e^{\pm(t-t_0)}}{\left[1 + e^{\pm(t-t_0)}\right]^2}. \quad (24)$$

5.3. THE RADIAL MOTION IN THE NEIGHBOURHOOD OF THE COLLISION MANIFOLD

In the previous sections we showed that the collision is possible only for $h \geq 0$. For $h = 0$ and $\theta \in \{\pi/6, 5\pi/6, 3\pi/2\}$ the motion is described by (17) and for $\theta \in \{\pi/2, 7\pi/6, 11\pi/6\}$ the motion is described by (23).

Now, let us consider a radial orbit on the direction $\theta = \theta_0 \in [0, 2\pi)$ and $h \geq 0$. The first integral of energy reads

$$\dot{r}^2 = r^2 + \frac{2}{3}r^3 \sin 3\theta_0 + 2h. \quad (25)$$

Because in the neighbourhood of collision $r \ll 1$, we can neglect the term containing r^3 in (25). We obtain the implicit equation of motion:

$$\ln(\sqrt{r^2 + 2h} + r) = \pm(t - t_0) + \ln(\sqrt{r_0^2 + 2h} + r_0). \quad (26)$$

In the particular case of $h = 0$ the particle is moving in the vicinity of collision according to

$$r(t) = r_0 e^{\pm(t-t_0)}. \quad (27)$$

Taking the “+” sign in (27), the particle ejects from collision, and if we take the “-” sign in (27), the particle is moving towards collision (which will be attained in an infinite time: $\lim_{t \rightarrow \infty} r(t) = \lim_{t \rightarrow \infty} r_0 e^{-(t-t_0)} = 0$). If $r_0 = 0$, $r(t) = 0$, $(\forall)t$ (we showed in Section 3 that $(0,0)$ is an equilibrium point).

5.4. STABILITY OF RADIAL ORBITS

As we showed in a previous paper (Mioc and Pricopi 2005), the particle can escape to infinity or can be captured from infinity only on the directions $\theta \in \{\pi/6, 5\pi/6, 3\pi/2\}$. So, for $r \gg 1$ all the orbits come in a neighbourhood of one of radial orbits with $\theta \in \{\pi/6, 5\pi/6, 3\pi/2\}$; in other words, in the neighbourhood of the infinity manifold, all orbits that escape are stable.

Now, let us study the stability of radial orbits on the direction $\theta = \pi/2$ in the zero-energy case (for $h \neq 0$ the discussion is very similar). The motion is possible only in the range $0 < r < 3/2$. The set of equations of motion is (20). We shall consider an infinitesimal perturbation:

$$\begin{aligned}
 \tilde{r} &= r + r', \\
 \tilde{\theta} &= \frac{\pi}{2} + \theta', \\
 \tilde{u} &= u + u', \\
 \tilde{v} &= v + v';
 \end{aligned} \tag{28}$$

the linearized set is

$$\begin{aligned}
 \dot{r}' &= u', \\
 \dot{\theta}' &= \frac{v'}{r}, \\
 \ddot{u}' &= r'(1 - 2r), \\
 \ddot{v}' &= -\frac{uv'}{r} + r^2\theta'.
 \end{aligned} \tag{29}$$

Taking for perturbations a time-dependence of the form $\propto e^{\lambda t}$, we write the characteristic equation:

$$[\lambda^2 - (1 - 2r)][\lambda^2 + \frac{u}{r}\lambda - r] = 0. \tag{30}$$

I. Case $\lambda^2 - (1 - 2r) = 0$.

1. If $0 < r < 1/2$, then $\lambda_{1,2} = \pm\sqrt{1 - 2r}$: in the range $r \in (0, 1/2)$ we have a stable orbit and an unstable one.

2. If $r = 1/2$, then $\lambda_{1,2} = 0$, i.e., degenerate equilibrium.

3. If $r > 1/2$, then $\lambda_{1,2} = \pm i\sqrt{1 - 2r}$. Thus, in this range, we have oscillatory orbits around the radial orbit, with the period $T = 2\pi/\sqrt{2r - 1}$.

II. Case $\lambda^2 + \frac{u}{r}\lambda - r = 0$.

1. If $u/r = \sqrt{1 - 2r/3}$ (namely, the particle ejects from collision), then $\lambda_{3,4} = (1/2)(-\sqrt{1 - 2r/3} \pm \sqrt{1 + r/3})$. Thus, $\lambda_3 < 0$ (stable solution) and $\lambda_4 > 0$ (unstable solution).

2. If $u/r = -\sqrt{1 - 2r/3}$ (namely, the particle comes to collision), then $\lambda_{3,4} = (1/2)(\sqrt{1 - 2r/3} \pm \sqrt{1 + r/3})$. Thus, $\lambda_3 > 0$ (unstable solution) and $\lambda_4 < 0$ (stable solution).

From the cases I and II we conclude that the motion in the neighbourhood of the radial orbit is not predictable (chaotic motion).

Now, let us study the stability of radial orbits on the direction $\theta = \pi/6$ in the zero-energy case (for $h \neq 0$ the discussion is very similar). The equations of motion are (14). We shall consider an infinitesimal perturbation:

$$\begin{aligned}\tilde{r} &= r + r', \\ \tilde{\theta} &= \pi/6 + \theta', \\ \tilde{u} &= u + u', \\ \tilde{v} &= v + v';\end{aligned}\tag{28}$$

the linearized set is:

$$\begin{aligned}\dot{r}' &= u', \\ \dot{\theta}' &= \frac{v'}{r}, \\ \dot{u}' &= r'(1 + 2r), \\ \dot{v}' &= -uv'/r - r^2\theta'.\end{aligned}\tag{29}$$

Taking for perturbations a time-dependence of the form $\propto e^{\lambda t}$, we write the characteristic equation:

$$\left[\lambda^2 - (1 + 2r)\right] \left[\lambda^2 + \frac{u}{r}\lambda + r\right] = 0.\tag{30}$$

I. Case $\lambda^2 - (1 + 2r) = 0$. The solution is $\lambda_{1,2} = \pm\sqrt{1 + 2r}$.

II. Case $\lambda^2 + (u/r)\lambda + r = 0$. It is easy to show that $\text{Re } \lambda_{3,4} < 0$ if the particle ejects from collision ($u > 0$) and $\text{Re } \lambda_{3,4} > 0$ if the particle comes to collision ($u < 0$). This is in accord with the fact that the particle can escape to infinity only on the radial directions $\theta \in \{\pi/6, 5\pi/6, 3\pi/2\}$.

REFERENCES

- Anisiu, M.-C., Pal, A.: 1999, *Rom. Astron. J.*, **9**, 179.
 Antonov, V. A., Timoshkova, E. I.: 1993, *Astron. Rep.*, **37**, 138
 Boccaletti, D., Pucacco, G.: 1996, *Theory of Orbits*, Springer-Verlag, Berlin, Heidelberg, New York.
 Chenciner, A., Montgomery, R.: 2000, *Ann. Math.*, **152**, 881.
 Craig, S., Diaconu, F. N., Lacombe, E. A., Perez, E.: 1999, *J. Math. Phys.*, **40**, 1359.

- Delgado, J., Diacu, F.N., Lacomba, E.A., Mingarelli, E., Mioc, V., Perez, E., Stoica, C.: 1996, *J. Math. Phys.*, **37**, 2748.
- Devaney, R. L.: 1978, *Invent. Math.*, **45**, 221.
- Diacu, F. N., Mingarelli, A., Mioc, V., Stoica, C.: 1995, in R. P. Agarwal (ed.) *Dynamical Systems and Applications*, World Scientific Series in Applicable Analysis, Vol. 4, World Scientific, Singapore, p. 213.
- Diacu, F. N., Mioc, V., Stoica, C.: 2000, *Nonlinear Anal.*, **41**, 1029.
- Gutzwiller, M. C.: 1973, *J. Math. Phys.*, **14**, 139.
- Hénon, M., Heiles, C.: 1964, *Astron. J.*, **69**, 73.
- McGehee, R.: 1973, *J. Diff. Eq.*, **14**, 70.
- McGehee, R.: 1974, *Invent. Math.*, **27**, 191.
- Mioc, V.: 2002, *Rom. Astron. J.*, **12**, 193.
- Mioc, V., Barbosu, M.: 2003a, *Serb. Astron. J.*, **167**, 47.
- Mioc, V., Barbosu, M.: 2003b, *Spacetime Subst.*, **4**, 164.
- Mioc, V., Pricopi, D.: 2005, *Rom. Astron. J.*, **15**, 65.
- Mioc, V., Stavinschi, M.: 2000, *Rom. Astron. J.*, **10**, 71.
- Mioc, V., Stavinschi, M.: 2001, *Phys. Lett. A.*, **279**, 223.
- Mioc, V., Pérez-Chavela, E., Stavinschi, M.: 2003, *Celest. Mech. Dyn. Astron.*, **86**, 81.
- Mioc, V., Anisiu, M.-C., Barbosu, M.: 2005, *Celest. Mech. Dyn. Astron.*, **91**, 269.
- Saari, D. G.: 1974, *Celest. Mech.*, **9**, 55.
- Santoprete, M.: 2002, *J. Math. Phys.*, **43**, 2307.
- Stoica, C., Mioc, V.: 1997, *Astrophys. Space Sci.*, **249**, 161.
- Van der Merwe, P. du T.: 1991, *Phys. Lett. A.*, **156**, 216.
- Wintner, A.: 1941, *The Analytical Foundations of Celestial Mechanics*, Princeton University Press, Princeton, N.J.

Received on 23 July 2005

NOTE ON THE PHOTOGRAVITATIONAL RESTRICTED THREE-BODY PROBLEM

CIPRIAN CHIRUȚĂ¹, TIBERIU OPROIU²

¹ "Ion Ionescu de la Brad" University of Agricultural Sciences and Veterinary Medicine
Aleea M. Sadoveanu 3, 700490 Iași, Romania
E-mail: kyru@univagro-iasi.ro

² Astronomical Institute of the Romanian Academy
Astronomical Observatory Cluj-Napoca
Str. Cireșilor 19, 400487 Cluj-Napoca, Romania
E-mail: toproiu@email.ro

Abstract. We present the photogravitational restricted three-body problem for the radiation pressure model proposed by Constantin Popovici in 1923. We establish the differential equations of the relative motion of the body with a negligible mass.

Key words: restricted three-body problem – radiation pressure – orbital energy.

1. INTRODUCTION

As we already know, the photogravitational model of Constantin Popovici was proposed for the first time in 1923 and it has been presented in his works (Popovici 1923, 1939). It was independently used by Armellini (1939) in order to generalize the Newtonian law of the universal attraction.

Constantin Popovici's model has been reconsidered in the last decades by several authors (Anisiu 1995a,b, 2003; Bărbosu 2000; Mioc and Blaga 2001, 2002; Bărbosu and Oproiu 2004, etc.).

The photogravitational problems (two, three or many bodies) attracted much attention during the last years (see, e.g., Kunitsyn and Polyakhova 1995; Kunitsyn and Tureshbaev 1984; Kazazakis and Kalvouridis 2003).

In his work (Popovici 1923) there is given the result (valid for the photogravitational two-body problem):

"L'énergie ne se conserve plus. Cette quantité

$$E = \frac{v^2}{2} - \frac{k}{r},$$

que l'on appelle dans la Mécanique newtonienne l'énergie, varie dans le même sens avec le temps. La relation suivante, qui est en même temps l'équation du mouvement:

$$\frac{dE}{dt} = \alpha k \left(\frac{r'}{r} \right)^2, \quad r' = \frac{dr}{dt};$$

$$\frac{dE}{dt} > 0 \text{ attraction}; \quad \frac{dE}{dt} < 0 \text{ répulsion},$$

nous fait voir comment l'énergie est dépensée par le mécanisme de la propagation. "

We demonstrate here that the expression for the energy remains valid also for the restricted photogravitational three-body problem.

2. EQUATIONS OF ABSOLUTE MOTION

Let us consider the restricted circular problem of the three following bodies: P_1 (with the mass M_1), P_2 (with the mass M_2) and a third body P , with the mass m , negligible with respect to the masses of the first two.

P_1 and P_2 are moving along circular orbits around their common centre of mass, under the effect of gravitation.

The body with a negligible mass is subjected to the photogravitational action generated by P_1 and to the gravitational action generated by P_2 .

The differential equations, in a vectorial form, of the motion of the body P can be written as:

$$m\ddot{\vec{r}} = \vec{F}_1 + \vec{F}_2, \quad (1)$$

where

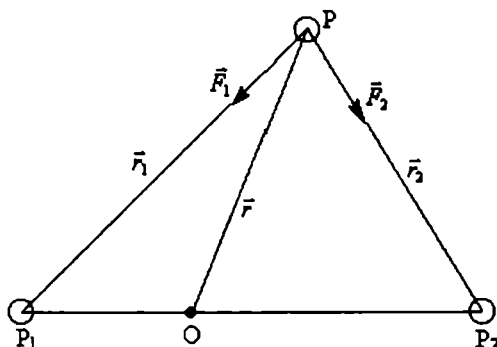
$$\vec{F}_1 = \vec{F}_g + \vec{F}_{rp} = -G \cdot \frac{m \cdot M_1}{r_1^2} \cdot \frac{\vec{r}_1}{r_1} + \frac{L_1}{4\pi c r_1^2} \cdot a \cdot \left(1 - \frac{\dot{r}_1}{c} \right) \cdot \frac{\vec{r}_1}{r_1} \quad (2)$$

is the photogravitational action due to the body P_1 , in which L_1 is the luminosity of the body P_1 (for example: the Sun), c is the speed of light, whereas a is the cross-sectional area on the P_1P direction, and

$$\vec{F}_2 = -G \cdot \frac{m \cdot M_2}{r_2^2} \cdot \frac{\vec{r}_2}{r_2} \quad (3)$$

is the attraction force of the body P_2 (Fig. 1).

Fig. 1 – The restricted three-body problem.



The relation (1), after divided by m , takes the form:

$$\ddot{\vec{r}} = -\frac{G \cdot M_1}{r_1^3} \cdot \left[1 - \frac{L_1}{4\pi c} \cdot \frac{1}{G \cdot M_1} \cdot \frac{1}{\sigma} \right] \cdot \vec{r}_1 - \frac{G \cdot M_2}{r_2^3} \cdot \vec{r}_2 - \frac{L_1}{4\pi c^2 r_1^2} \cdot \frac{\dot{r}_1}{r_1} \cdot \frac{1}{\sigma} \cdot \vec{r}_1. \quad (4)$$

The vectorial equation of absolute motion can be written in the form

$$\ddot{\vec{r}} = -\frac{A}{r_1^3} \cdot q_1 \cdot \vec{r}_1 - \frac{G \cdot M_2}{r_2^3} \cdot \vec{r}_2 - \frac{R}{c r_1^2} \cdot \frac{\dot{r}_1}{r_1} \cdot \frac{1}{\sigma} \cdot \vec{r}_1, \quad (5)$$

where the last term was introduced by Constantin Popovici in order to consider the fact that the radiative force due to the light propagates with a finite velocity unlike the attraction, which propagates instantaneously. This has a breaking effect, similar to the motion in a resisting medium (Antonacopoulos 1970).

The coefficients from the relation (5) have the expressions (Popovici 1923):

$$\begin{aligned} A &= G \cdot M_1, \\ R &= \frac{L_1}{4\pi c}, \\ q_1 &= 1 - \frac{R}{A} \cdot \frac{1}{\sigma}, \\ \sigma &= \frac{m}{a}, \end{aligned} \quad (6)$$

where A is the attraction of the body P_1 at unit distance, and R is the force due to the light pressure at unit distance.

We study the motion in an inertial system $O\xi\eta\zeta$. With respect to this system of reference, the three bodies have the following coordinates (Fig. 2): $P_1(\xi_1, \eta_1, \zeta_1)$, $P_2(\xi_2, \eta_2, \zeta_2)$ and $P(\xi, \eta, \zeta)$.

Let the origin of inertial coordinates system be taken at the center of mass of the primaries, and let the direction of the axes be so chosen that the $\xi\eta$ -plane is the plane of their motion, then $\zeta_1 = \zeta_2 = 0$.

The scalar equations of the motion in an inertial system are:

$$\begin{cases} \ddot{\xi} = -\frac{A}{r_1^3} \cdot q_1 \cdot (\xi - \xi_1) - \frac{G \cdot M_E}{r_2^3} (\xi - \xi_2) - \frac{R}{cr_1^3} \cdot \frac{\dot{r}_1}{\sigma} \cdot (\xi - \xi_1), \\ \ddot{\eta} = -\frac{A}{r_1^3} \cdot q_1 \cdot (\eta - \eta_1) - \frac{G \cdot M_E}{r_2^3} (\eta - \eta_2) - \frac{R}{cr_1^3} \cdot \frac{\dot{r}_1}{\sigma} \cdot (\eta - \eta_1), \\ \ddot{\zeta} = -\frac{A}{r_1^3} \cdot q_1 \cdot \zeta - \frac{G \cdot M_E}{r_2^3} \cdot \zeta - \frac{R}{cr_1^3} \cdot \frac{\dot{r}_1}{\sigma} \cdot \zeta. \end{cases} \quad (7)$$

3. EQUATIONS OF RELATIVE MOTION

Let us introduce the system $Oxyz$ with the same origins, in which the axes Ox and Oy rotate around the Oz -axis with the angular velocity equal to the unity. The Oz -axis coincides with the $O\xi$ -axis. The direction of the Ox -axis will be chosen so as the P_1 and P_2 to be situated on it and direction from P_1 to P_2 .

Let the unit of mass be so chosen that the sum of the masses of the finite bodies shall be unity ($M_1 + M_2 = 1$). Denoting $\mu = m/(1+m)$, where $m = M_2/M_1$ (do not confuse with the mass of the particle; see Section 2), we have $1-\mu$ for the mass of P_1 and μ for the mass of P_2 . Let the unit of distance be so chosen that the constant distance between the finite bodies shall be unity (Fig. 2).

Their coordinates will be $P_1(x_1, 0, 0)$ and $P_2(x_2, 0, 0)$, so that

$$x_2 - x_1 = 1.$$

The coordinates in the new system are defined as:

$$\begin{cases} x = \xi \cos t + \eta \sin t, \\ y = -\xi \sin t + \eta \cos t, \\ z = \zeta, \end{cases} \quad (8)$$

and

$$\begin{cases} \xi = x \cos t - y \sin t, \\ \eta = x \sin t + y \cos t, \\ \zeta = z, \end{cases} \quad (9)$$

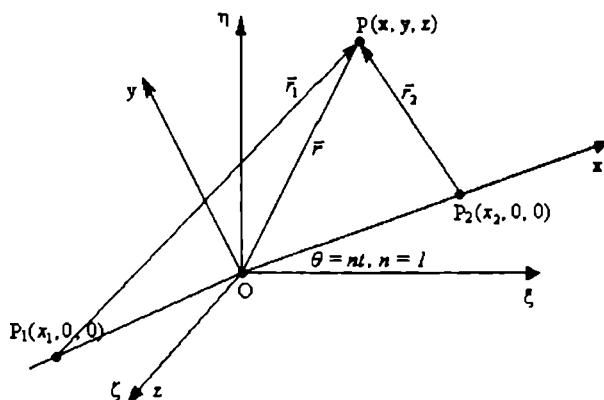


Fig. 2 – Reference system.

By calculating the second order derivatives of ξ, η, ζ from equations (9) and substituting them in (7), we find (cf. Moulton 1914):

$$\begin{cases} \ddot{x} - 2\dot{y} = x - \frac{1-\mu}{r_1^3} \cdot q_1 \cdot (x-x_1) - \frac{\mu}{r_2^3} \cdot (x-x_2) - \frac{R}{c \cdot r_1^3} \cdot \frac{\dot{r}_1}{\sigma} \cdot (x-x_1), \\ \ddot{y} + 2\dot{x} = y - \frac{1-\mu}{r_1^3} \cdot q_1 \cdot (y-y_1) - \frac{\mu}{r_2^3} \cdot (y-y_2) - \frac{R}{c \cdot r_1^3} \cdot \frac{\dot{r}_1}{\sigma} \cdot (y-y_1), \\ \ddot{z} = -\frac{1-\mu}{r_1^3} \cdot q_1 \cdot z - \frac{\mu}{r_2^3} \cdot z - \frac{R}{c \cdot r_1^3} \cdot \frac{\dot{r}_1}{\sigma} \cdot z. \end{cases} \quad (10)$$

Remark. If the last terms of the relations are zero, from the system (10), we get Radziewsky's case (Radziewsky 1950, 1953a, b).

If we denote: $U = \frac{x^2 + y^2}{2} + \frac{(1-\mu)q_1}{r_1} + \frac{\mu}{r_2}$, then the system (10) can be written in

the form

$$\begin{cases} \ddot{x} - 2\dot{y} = \frac{\partial U}{\partial x} - \frac{R}{c} \cdot \frac{\dot{r}_1}{\sigma} \cdot \frac{x-x_1}{r_1^3}, \\ \ddot{y} + 2\dot{x} = \frac{\partial U}{\partial y} - \frac{R}{c} \cdot \frac{\dot{r}_1}{\sigma} \cdot \frac{y-y_1}{r_1^3}, \\ \ddot{z} = \frac{\partial U}{\partial z} - \frac{R}{c} \cdot \frac{\dot{r}_1}{\sigma} \cdot \frac{z}{r_1^3}. \end{cases} \quad (11)$$

If these equations are multiplied by $\dot{x}$, $\dot{y}$, and $\dot{z}$, respectively, and added together, we obtain the relation

$$\ddot{x}\dot{x} + \ddot{y}\dot{y} + \ddot{z}\dot{z} = \frac{\partial U}{\partial x}\dot{x} + \frac{\partial U}{\partial y}\dot{y} + \frac{\partial U}{\partial z}\dot{z} - \frac{R}{c} \cdot \frac{\dot{r}_1}{\sigma} \cdot \left[\frac{x-x_1}{r_1^3} \cdot \dot{x} + \frac{y-y_1}{r_1^3} \cdot \dot{y} + \frac{z}{r_1^3} \dot{z} \right], \quad (12)$$

from which

$$\frac{1}{2} \frac{d}{dt} (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) = \frac{dU}{dt} - \frac{R}{c} \cdot \frac{\dot{r}_1}{\sigma} \cdot \frac{1}{r_1^3} \cdot \frac{1}{2} \frac{d}{dt} (r_1^2). \quad (13)$$

Denoting $E = \frac{1}{2} (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - U$, we are led to the following result:

THEOREM. *The variation of the energy of an infinitesimal particle in relative motion has the following property:*

$$\dot{E} = -\frac{R}{c} \cdot \frac{1}{\sigma} \cdot \left(\frac{\dot{r}_1}{r_1} \right)^2. \quad (14)$$

This relation generalizes Constantin Popovici's (1923) theorem.

Remark. If $\dot{E} = 0$ then we notice that the system allows the Jacobian first integral, which had been pointed out by Radziewsky (1950).

REFERENCES

- Anisiu, M.-C.: 1995a, *An. Șt. Univ. "Al. I. Cuza" Iași*, **16**, 115.
 Anisiu, M.-C.: 1995b, *Rom. Astron. J.*, **5**, 49.
 Anisiu, M.-C.: 2003, *Rom. Astron. J.*, **13**, 171.
 Antonacopoulos, G. A.: 1970, *Astrophys. Space Sci.*, **9**, 482.
 Armellini, G.: 1937, *Rendiconti, Accad. Naz. Lincei*, **26**, 609.
 Bărbosu, M.: 2000, in M. V. Alania, E. A. Grebenikov (eds), "*Mathematica*" System in Teaching and Research, Proc. 2nd Intl. Workshop on "*Mathematica*" System in Teaching and Research, January 28-30, 2000, Siedlce, Poland, p. 16.
 Bărbosu, M., Oproiu, T.: 2004, *Gen. Math.*, **12**, 19.
 Kazazakis, D., Kalvouridis, T. J.: 2003, *Earth, Moon Planets*, **93**, 75.
 Kunitsyn, A. L., Polyakhova, E. N.: 1995, *Astron. Astrophys. Trans.*, **6**, 283.
 Kunitsyn, A. L., Tureshbaev, A. T.: 1985, *Celest. Mech.*, **35**, 105.
 Mioc, V., Blaga, C.: 2001, *Rom. Astron. J.*, **11**, 45.
 Mioc, V., Blaga, C.: 2002, *Serb. Astron. J.*, **165**, 9.
 Moulton F. R.: 1914, *An Introduction to Celestial Mechanics*, 2nd rev.ed., Macmillan, New York.
 Popovici C.: 1923, *Bull. Astron.* **3**, 257.
 Popovici, C.: 1939, *C. R. Acad. Sci. Paris*, **208**, 2052.
 Radziewsky, V. V.: 1950, *Astron Zh.*, **27**, 250.
 Radziewsky, V. V.: 1953a, *Astron Zh.*, **30**, 265.
 Radziewsky, V. V.: 1953b, *Astron Zh.*, **30**, 378.

Received on 10 June 2005

OBSERVING NEAR EARTH ASTEROIDS WITH A SMALL TELESCOPE.

II. THE PROGRAM AT YORK UNIVERSITY OBSERVATORY

OVIDIU VĂDUVESCU^{1,2}

¹ *York University, Department of Physics and Astronomy
128 Petrie Science Building, 4700 Keele St., Toronto, M3J 1P3 Ontario Canada
Email: ovidiuv@yorku.ca*

² *Astronomical Institute of the Romanian Academy
Str. Cușitul de Argint 5, 040557 Bucharest, Romania
Email: ovidiuv@yahoo.com*

Abstract. Five Near Earth Asteroids have been observed during ten nights at York University Observatory, part of a NEA follow-up program initiated by the author in 2002. The astrometric and photometric data were sent to the Minor Planet Centre, following which an observatory code was assigned and four batches have been included in the NEODyS database and MPC Circulars. In the first paper we discussed the planning of the NEA observations. In this paper we present the program at York, the observations and data reduction. The results are applicable to any other small facility.

Key words: astrometry – NEAs – PHAs – CCD observations.

1. INTRODUCTION

Follow-up observations of Near Earth Asteroids are welcomed by the international astronomical community, in order to secure and improve their orbits, predict future close encounters with the Earth, and prevent any possible collision threat well ahead in advance.

In the first paper we reviewed the major discovery and follow-up programs, discussing how to plan systematic observations using a small facility. In the present paper we will discuss the data reduction methods, based on the number of reference stars available in the field. Also we will present such an observing program run by the author in the last two years at York University Observatory.

Giving the growing number of known Near Earth Asteroids, more work could be done at the bright end in the follow-up direction by modest facilities, such as college

and university observatories, also astronomy clubs and small amateur facilities, places where competition for observing time is not a problem, as it is on larger facilities which can be spared for deeper studies.

Using a telescope with a minimum diameter of only ~ 30 cm equipped with a CCD camera sub-sampling the seeing, the largest Near Earth Asteroids (NEAs) and Potentially Hazardous Asteroids (PHAs) can be observed at opposition, even from a light polluted place. Such a follow-up project has been initiated on the 60 cm telescope at York University Observatory in Toronto, in my spare time as a PhD student working full-time on an extragalactic topic. In the last two years, five NEAs from which three were PHAs have been observed during ten nights. Their reduced astrometry and photometry was submitted to Minor Planet Centre (MPC), and four of them have been accepted. An observatory code was assigned to York University Observatory, and four observing batches have been included in the NEODyS database (Milani et al. 2004a) and MPC Circulars (Văduvescu 2004b).

2. THE OBSERVING FACILITY

We have employed the 60-cm $F/D = 13.6$ Cassegrain telescope, the largest of the two equatorial reflectors at York University Observatory (York Observatory 2004). The two-dome facility is located at the Department of Physics and Astronomy of York University (Keele campus) in Toronto, Ontario Canada. Its coordinates are: longitude $79^{\circ}30'00''$ West, latitude $43^{\circ}46'30''$ North, altitude 196 m, MPC observatory code H79.

Initially, the telescope was equipped with a HPC1 512×512 pixels camera providing a field of view (FOV) of $4.86' \times 4.86'$ ($0.56''/\text{pixel}$). In August 2004, this CCD was retired and replaced with an SBIG ST-9 512×512 pixels camera, providing a FOV of $3.5' \times 3.5'$ ($0.41''/\text{pixel}$). Alternatively, this small FOV could be increased to $5.5' \times 5.5'$ or about $10' \times 10'$, using two focal reducer lenses. Nevertheless, despite of the advantage of having a larger field, we have some doubts about the possible distortions due to the field curvature introduced by an additional lens (e.g., Fierro and Calderon 2002).

The time stamp has been provided by WinSNTP software (Coetanian Systems 2001) which uses the Simple Network Time Protocol (SNTP) to adjust the PC clock via a Winsock TCP/IP Internet query of a NTP time server (`time.chu.nrc.ca`), once every minute. Accordingly to its manual, taking into account the network delay between client PC and the time server, WinSNTP provides an accuracy of about 0.1–0.2 s, sufficient for our purpose. We check this accuracy each night, using the USNO web master clock, also the time signal phone service provided by the National Research Council in Ottawa. The time stamp is recorded in the FITS image headers with an accuracy of 0.1 s by the HPC1 camera and 0.5 s by the ST-9 camera.

A Pentium II Intel MMX PC at 400 MHz with 200 MB RAM running Windows 95 and recently Windows 98 has supported the data acquisition. We have used the DEVON software to run the HPC1 camera, and more recently CCDOPS for the ST-9 camera.

For the photometry purposes, a regular Johnson BVRI system filter was used manually with the HPC1 camera, and an automate filter wheel with the ST-9 camera.

3. THE OBSERVATIONS

We observed 5 asteroids over 8 nights, namely PHA 35396, NEA 40267 (two nights), PHA 65803 Didymos, NEA 2002 CE 26, and PHA 4179 Toutatis (two nights). The first five nights were observed using the old HPC1 camera (asteroids 35396, 40267 and 65803), while the last three nights with the new ST-9 camera (asteroids 2002 CE26 and 4179 Toutatis).

Unfortunately, two batches including the NEA 40267 were rejected by MPC, due to their large O–C residuals (up to about 5"). Why did this happen? Following some afterward testing, we have realized that the time stamp was responsible. The HPC1 acquisition software "frozen" the PC clock for the period of exposure and reading process, thus the observing time was recorded delayed in the FITS headers. Because WinSNTP adjusts the correct time once every minute, the "freezing" effect delayed the PC clock by up to one minute. As we took exposures with a frequency less than one minute for most of the time that night, the large proper motion of the asteroid (8–9"/min) combined with the PC time "freezing" resulted in big residuals in the reduced positions. We were not aware before these two nights about this problem (happening only with the old HPC1 camera under Windows 95). Later, in order to correct this error for the HPC1, we have reset manually the PC time clock using WinSNTP after each image acquisition, in order to correctly reset the time before a new image. This unfortunate error is a warning to stress, once again, the importance of an accurate time stamp necessary to observe fast moving sources.

Table 1 gives the observing log, including the following columns:

- (1) the MPC name of the object and its class (NEA or PHA);
- (2) date in UT;
- (3) number of positions observed;
- (4) observed time interval in minutes;
- (5) number of astrometric reference stars used and their catalog source;
- (6) asteroid proper motion μ in arcseconds per second;
- (7) the apparent magnitude measured in R and quoted by HORIZONS ephemerides in V;
- (8) the observed bands;
- (9) the exposure time in seconds;
- (10) the MPC status of our batch (A = accepted, R = rejected, U = unknown).

Table 1

The observing log of our NEA observations

NEA/PHA (1)	Date (UT) (2)	Nr. pos (3)	Inter [min] (4)	Ref stars & catalogs (5)	μ ["s] (6)	Mag (7)	Bands (8)	Exp [s] (9)	MPC status (10)
PHA (35396)	09.11 2002	24	50	2 (1 HIP, 1 2MASS)	0.13	R 13.1 V 13.8	V, R, I	20	A
NEA (40267)	08.02 2003	9	54	2-5 (4 GSC 2.2, 1 Tycho-2)	0.16	R 13.1 V 13.3	R, I	20	R
NEA (40267)	10.02 2003	16	229	2-6 (5 2MASS, 1 Tycho-2)	0.13	R 13.2 V 13.4	V, R, I	20	R
PHA (65803) Didymos	21.11 2003	12	67	1-2 (Tycho-2)	0.12	R 12.4 V 12.7	R, I	30	A
PHA (65803) Didymos	04.12 2003	35	82	3 (GSC 2.2)	0.05	R 12.9 V 13.2	R, I	30	A
NEA 2002 CE26	22.08 2004	15	84	3-6 (USNO-A2)	0.06	R 15.1 V 15.5	R	60	U
PHA (4179) Toutatis	22.08 2004	21	131	2(Tycho-2)	0.01	R 12.4 V 13.1	R	20	A
PHA (4179) Toutatis	12.09 2004	20	55	2 (1 Tycho-2, 1 DENIS)	0.02	R 11.3 V 11.9	B,V, R, I	30	A

Additional to Table 1, two other two nights were spent on two fainter asteroids, namely NEA 25330 (1999 KV 4) on Dec 2/3, 2002 ($V=14.6$) and NEA 52387 (1993 OM 7) on Jan 6/7, 2003 ($V=14.9$). Both of them could not be seen. Including these two nights, the total number of nights spent on this project is ten.

4. DATA REDUCTION

4.1. IMAGE PRIORCESSING

The image processing consists in the classic method which includes corrections for the bias, dark and flat field. We perform this in IRAF 2.12.2 under Linux. IRAF is distributed by the National Optical Astronomy Observatories, which are operated by the Association of Universities for Research in Astronomy, Inc., under cooperative agreement with the National Science Foundation (NOAO 2004).

Dome flats were observed in the past, but various gradients were remarked, probably due to the small distance to the dome screen, also its small size. Given the excessive light pollution in Toronto, we obtained most of our flat fields by combining sets of 7-9 night sky images exposed 60-90 s, with successive fields nodded arbitrarily in order to eliminate the stars, combining the frames using the median mode. Nevertheless, twilight flat fields are to be preferred.

With the former HPC1 camera, sets of about 11–15 bias frames and 7–9 dark frames exposed the same as the science frames were taken usually at the end of observing nights. With the new ST-9 camera, we subtracted from each science image and flat field frame the dark taken the moment of their acquisition.

4.2. ASTROMETRY

The main problem to perform astrometric observations using a small facility in a light-polluted place is the relative small number of reference stars in most images. Based on the number of reference stars present in the science field, at least two reduction methods are possible.

4.2.1. Sky to Plate Transformation. Given two reference stars with equatorial coordinates (α_1, δ_1) , (α_2, δ_2) and CCD rectangular coordinates (x_1, y_1) , (x_2, y_2) , then the coefficients (a, b) expressing the rotation of the CCD with respect to the equator can be derived from the following equations:

$$\begin{aligned} a &= \frac{x_2 + y_2}{(x_2 - x_1)^2 + (y_2 - y_1)^2}, \\ b &= \frac{x(y_2 - y_1) - y(x_2 - x_1)}{(x_2 - x_1)^2 + (y_2 - y_1)^2}. \end{aligned} \quad (1)$$

In the above formula, x and y stand for two constants, given from the classic formula which expresses the sky-plate transformation (e.g., Montenbruck and Pflieger 1991):

$$\begin{aligned} x &= \frac{\cos \delta_2 \sin(\alpha_2 - \alpha_1)}{\cos \delta_1 \cos \delta_2 \cos(\alpha_2 - \alpha_1) + \sin \delta_1 \sin \delta_2}, \\ y &= -\frac{\sin \delta_1 \cos \delta_2 \cos(\alpha_2 - \alpha_1) - \cos \delta_1 \sin \delta_2}{\cos \delta_1 \cos \delta_2 \cos(\alpha_2 - \alpha_1) + \sin \delta_1 \sin \delta_2}. \end{aligned} \quad (2)$$

With (a, b) known, the asteroid equatorial coordinates $(\alpha_{\text{ast}}, \delta_{\text{ast}})$ can be found from the following equations:

$$\begin{aligned} \alpha_{\text{ast}} &= \alpha_* + \arctan\left(\frac{x_{\text{eq}}}{\cos \delta_* - y_{\text{eq}} \sin \delta_*}\right), \\ \delta_{\text{ast}} &= \delta_* + \arcsin\left(y_{\text{eq}} - x_{\text{eq}} \sin \delta_* \tan\left(\frac{\arctan(\alpha_{\text{ast}} - \alpha_*)}{2}\right)\right). \end{aligned} \quad (3)$$

Here (x_{eq}, y_{eq}) represent the rectangular coordinates of the asteroid in the rotated system:

$$\begin{aligned} x_{eq} &= a(x_{ast} - x_*) + b(y_{ast} - y_*), \\ y_{eq} &= -b(x_{ast} - x_*) + a(y_{ast} - y_*). \end{aligned} \quad (4)$$

4.2.2. Only One or Two Reference Stars in the Field. If only one or two reference stars are available in the field, at least one is recommended to be from Tycho-2 catalog. In this case, one can apply the sky to plate transformation as above.

In the rare case when only one Tycho-2 star is available in the field, a nearby “orientation field” which contains at least two Tycho-2 stars must be observed, from which we can get the plate solution, i.e., the parameters (a, b) . In order to define the same camera orientation, the orientation field must be chosen very close in space and time to the science frame. Then (a, b) can be used with the science field and the catalog star as the reference, in order to derive the asteroid position.

If two reference stars (preferably of Tycho-2) are available in the science field, then the asteroid coordinates are determined by averaging its coordinates derived using both stars, individually. In this case, (a, b) are determined from the same field.

Using this method and an ideal transformation equation (i.e. an absolute rectangular linear CCD perpendicular to the focal axis), then the expected errors in the final positions are given by the statistics of the stellar positions, quoted for each star in its catalog entry.

We used our own ORIENT and REL3 software (Văduvescu 2004a) to calculate the field orientation and to reduce the equatorial positions, respectively.

4.2.3. Minimum Three Reference Stars in the Field. In this case, one can use CCMAP and CCTRAN tasks in IRAF (NOAO 2004). First we employ CCMAP to compute the plate solution using the sky projection geometry using the default values. Second, we use CCTRAN to transform the asteroid coordinate list using the CCMAP plate solution.

Alternatively, one can use the same method from 4.2.2 to average the asteroid positions determined using all stars individually. In this case, the plate constants are derived from the same field, by averaging all available Tycho-2 pair results.

In both cases, the catalogs Tycho-2 and 2MASS positions are to be preferred first, followed by USNO-A2, USNO-B1 and GSC 2.2. Given the scarcity of stars in our small field and the low magnitude limit, we prefer to use star positions taken from different catalogs, provided all are given in the same ICRS/J2000 system. Should enough stars are available, the individual positions with residuals greater than 0.3" than the average can be rejected.

4.2. PHOTOMETRY

No attempt was made to observe photometric standard stars on any of our observing nights, because of some pointing problems, risk of loosing the target, some non-photometric conditions, also because of the lack of observers (I have observed alone for most of the time).

Differential photometry using the catalog stars in the field was sufficient to provide first order magnitude estimates for the NEAs. The stellar magnitudes were cross-correlated and weighted based on the best available catalogs providing photometry, e.g., searched using VizieR (CDS 2004b). The individual differences derived from all available stars in the field were averaged, in order to get the apparent magnitude of the asteroid. We used the PHOT task of IRAF to measure the photometry.

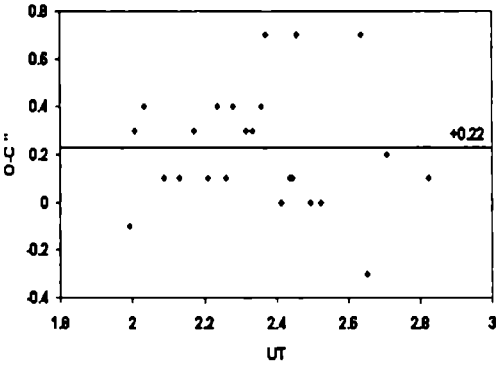
5. COMPARISON WITH THE EPHEMERIDES

One can use the classic “observed minus calculated” (O–C) approach to check the astrometry quality before sending each data batch to MPC. To do so, we employed HORIZONS ICRF/J2000 topocentric ephemerides run close to the date we produced our data (usually a few days following the observations). In order to calculate the O–C residuals, we used our own INTER software which interpolates linearly our observations using the HORIZONS one minute step ephemerides.

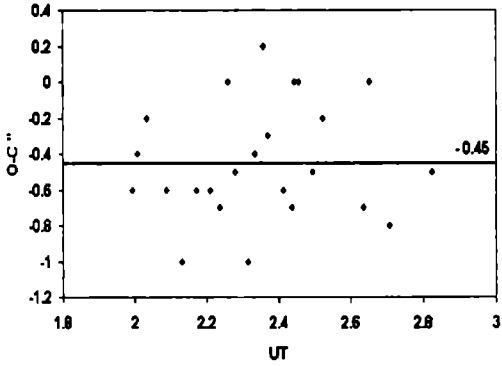
An alternative to using INTER would be an online server which calculates residuals of asteroid positions using OrbFit ephemerides (Scvarc 2004). This service expect the client/user to upload a file including the observation batch entered in the usual MPC format, then calculates the O–Cs, their averages, returning also the plots with the scatter of residuals and residual distributions.

We present in Fig. 1 the O–C values of our NEAs observations, run with INTER ephemerides interpolator based on the HORIZONS output. As one can see, most of the plots show residual values less than $0.3''$, with no systematic trend with the UT. Only PHA 40267 (the batch rejected by MPC), shows the largest residuals and some systematic trend, due to the time stamp problem. The second observation of PHA (4179) Toutatis shows smaller residuals than the first, possibly due to its improved orbit, as more data became available closer to opposition (about 350 observations between our two observing dates, cf. with NEODyS). As one can see, the errors of the average O–C residuals are comparable with the combined positional errors of the stars employed for astrometry, listed in catalogs.

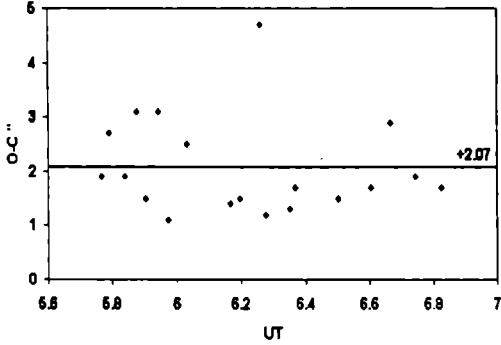
35396 O-C alpha



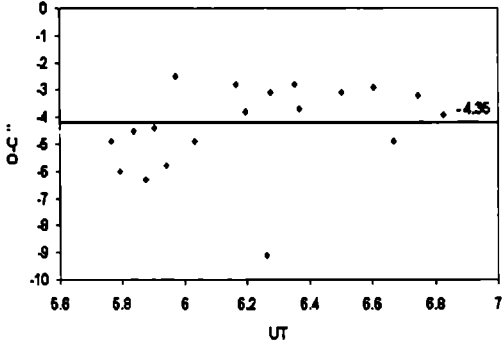
35396 O-C delta



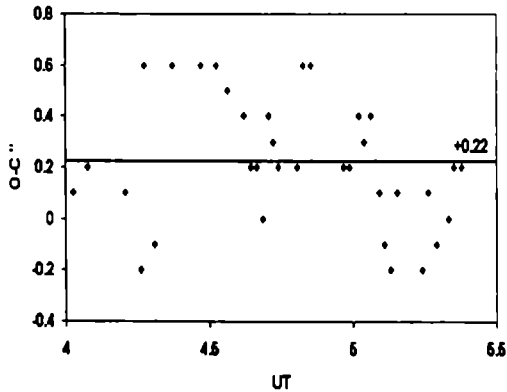
40267 O-C alpha



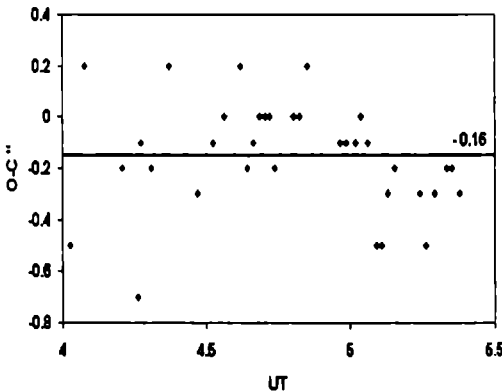
40267 O-C delta



65803 O-C alpha

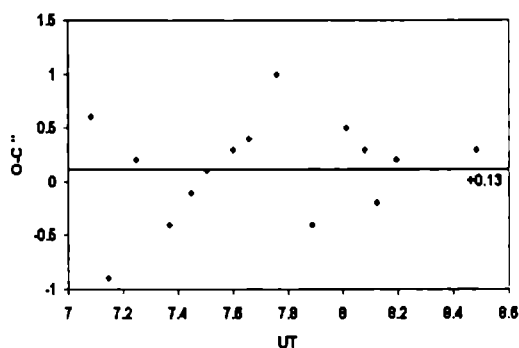
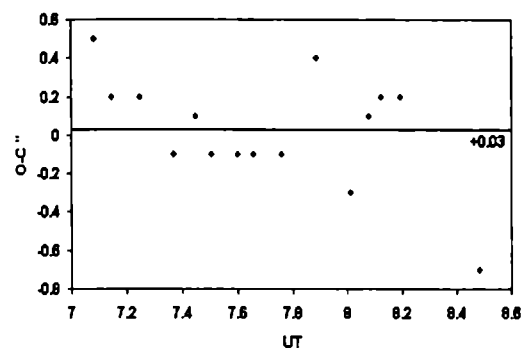


65803 O-C delta



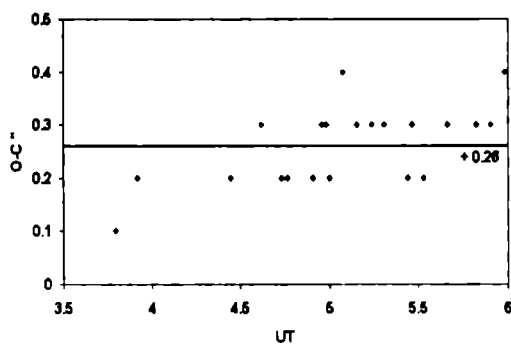
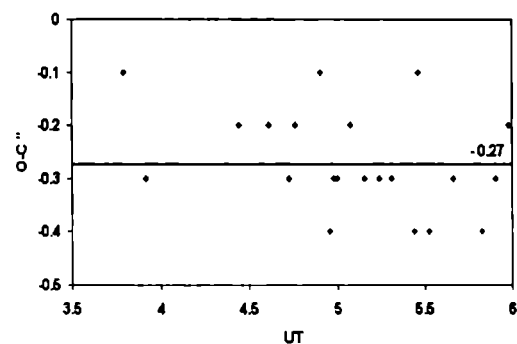
2002 CE26 O-C alpha

2002 CE26 O-C delta



4179 Aug 22 O-C alpha

4179 Aug 22 O-C delta



4179 Sep 12 O-C alpha

4179 Sep 12 O-C delta

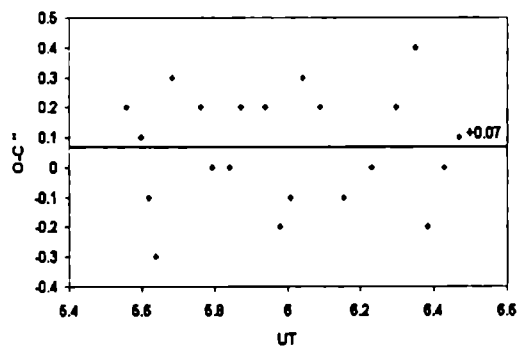
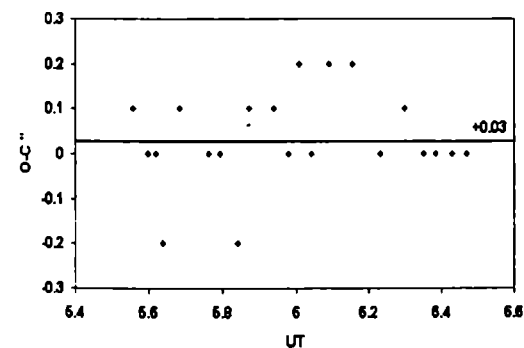


Fig. 1 – The Observed minus Calculated (O-C) values giving the residuals between our NEAs reduced data and the HORIZONS ephemerides, plotted in α (left panels) and δ (right panels).

The average values are plotted as a horizontal line and included to the right.

6. SENDING THE DATA TO THE MPC

After reducing the data and checking the O–C residuals, we email the observations to Minor Planet Centre, using their strict specific format (MPC 2004a). As an example, we give below such a batch (including 4179 Toutatis observed on August 21/22, 2004).

```
COD H79
CON O. Vaduvescu, York University, Department of Physics and Astronomy, 128
CON Petrie Science Building, 4700 Keele Street, Toronto, ON, M3J 1P3, Canada
CON [ovidiu@yorku.ca]
OBS O. Vaduvescu
MEA O. Vaduvescu
TEL 0.60-m f/13.6 Cassegrain + CCD SBIG ST-9
NET Tycho-2 (astrometry), USNO-B1 (photometry)
NUM 21
```

4179	C2004	08	22.15811	22	4	36.88	-12	41	12.2	12.5	R	H79
4179	C2004	08	22.16318	22	4	36.69	-12	41	13.7	12.4	R	H79
4179	C2004	08	22.18521	22	4	35.95	-12	41	20.8	12.4	R	H79
4179	C2004	08	22.19229	22	4	35.71	-12	41	23.0	12.4	R	H79
4179	C2004	08	22.19704	22	4	35.54	-12	41	24.6	12.4	R	H79
4179	C2004	08	22.19848	22	4	35.50	-12	41	25.1	12.4	R	H79
4179	C2004	08	22.20442	22	4	35.30	-12	41	27.0	12.4	R	H79
4179	C2004	08	22.20645	22	4	35.21	-12	41	27.6	12.4	R	H79
4179	C2004	08	22.20752	22	4	35.18	-12	41	27.9	12.4	R	H79
4179	C2004	08	22.20841	22	4	35.15	-12	41	28.2	12.4	R	H79
4179	C2004	08	22.21155	22	4	35.05	-12	41	29.0	12.4	R	H79
4179	C2004	08	22.21479	22	4	34.93	-12	41	30.2	12.4	R	H79
4179	C2004	08	22.21828	22	4	34.81	-12	41	31.3	12.4	R	H79
4179	C2004	08	22.22113	22	4	34.71	-12	41	32.2	12.4	R	H79
4179	C2004	08	22.22653	22	4	34.52	-12	41	33.9	12.5	R	H79
4179	C2004	08	22.22756	22	4	34.50	-12	41	34.2	12.5	R	H79
4179	C2004	08	22.23022	22	4	34.39	-12	41	35.1	12.4	R	H79
4179	C2004	08	22.23587	22	4	34.20	-12	41	36.8	12.4	R	H79
4179	C2004	08	22.24266	22	4	33.96	-12	41	38.9	12.4	R	H79
4179	C2004	08	22.24600	22	4	33.85	-12	41	39.9	12.5	R	H79
4179	C2004	08	22.24921	22	4	33.75	-12	41	40.8	12.4	R	H79

If there is a problem, the observer usually receives a brief reply, otherwise probably the batch is accepted (the procedure is not clear). If accepted, usually within one day the batch appears in the NEODyS online database (Milani et al. 2004a), but the protocol again is not clear. From some time, MPC archives online the front page and observation summary pages from each batch of their circulars, in order to reference the various observer contributions (MPC 2004b). We found such a circular in this archive, listing some of our reported data including 80 positions of 3 NEAs (Văduvescu 2004b).

Following the third asteroid batch we sent, York University Observatory has been assigned the observatory code H79 by the MPC. This probes both the quality of the data and the site location (as the reduced data sent to MPC is topocentric).

7. CONCLUSIONS

Even from a light polluted place, using a small telescope equipped with a CCD camera sub-sampling the seeing, the largest NEAs and PHAs with sizes in the range of

approximately 1 km or larger can be observed at opposition, close passing the Earth. An accurate time stamp must be available, this being especially important for the close encounters which produce very fast apparent motions.

Although a large field of view and a large CCD format are recommendable in order to have at least five reference stars in the field (preferably of Tycho-2 catalog), even a smaller FOV can serve the astrometry, employing even less than three stars having accurate J2000 positions, such as those taken from denser catalogs as Tycho-2, 2MASS, USNO-B2 or GSC 2.2.

Starting by fall 2002, we initiated a NEAs follow-up program using the 60-cm Cassegrain telescope at York University Observatory. We first used an HPC1 512×512 pixels camera (FOV 4.8'), which could reach a limiting magnitude $V \sim 14$ in one minute, under the very light polluted sky of Toronto. This was replaced recently by a more sensitive SBIG ST-9 camera (FOV 3.5' or 5.5' using a focal lens reducer) which can reach $V \sim 16$ in one minute. Between 2002 and 2004, we spent 10 nights on this project and got data for five NEAs at opposition, among which three objects were PHAs. The astrometry and differential photometry has been reduced using the available reference stars in the field, and has an estimated precision of about 0.1". We checked the observed minus calculated (O-C) residuals, then sent the reduced data to Minor Planet Centre (MPC). Four batches were accepted and included in NEODyS database and MPC circulars, and a code for York Observatory was assigned, H79.

Acknowledgments. Prof. Paul Delaney, the Director of York University Observatory, has assisted me in this project. Also, Paul and the undergraduate student Brenda Shaw helped me in two nights with the observations. During 2004, I have discussed this project and exchange some information with Alin Nedelcu of the Astronomical Institute of the Romanian Academy, who is involved in a similar NEA follow-up project using the double 38 cm diameter refractor in Bucharest. Dr. Jerome Berthier and Dr. Mirel Birlan of IMCCE in Paris, France helped me to compare my first data reduction batch versus MIDAS.

REFERENCES

- Coetanian Systems: 2004, *WinSNTF Software*.
 Fierro, I. H. B., Calderon, J. H.: 2002, *Rev. Mex. Astron. Astrofis.*, **38**, 215.
 Milani, A. et al: 2004a, *NEODyS Near Earth Objects Dynamic Site*, online at <http://newton.dm.unipi.it/cgi-bin/neodys/neoibo>.
 Milani A. et al: 2004b, *The OrbFit Software Package*, online at <http://newton.dm.unipi.it/~neodys/astinfo/orbfit>.
 Minor Planet Centre (MPC): 2004a, *Format for Optical Astrometric Observations of Comets, Minor Planets and Natural Satellites*, online at <http://cfa-www.harvard.edu/iau/info/OpticalObs.html>.
 Minor Planet Centre (MPC): 2004b, *On-line MPC Observation Summary Pages*, online at <http://cfa-www.harvard.edu/iau/MPCSummary/MPCSummary.html>.

- Montenbruck, O., and Pfleger, T.: 1991, *Astronomy on the Personal Computer*, Springer Verlag, Berlin.
- National Optical Astronomy Observatories (NOAO): 2004, *IRAF – Image Reduction and Analysis Facility*, online at <http://iraf.noao.edu>.
- Scvarc, J.: 2004, *Calculation of Residuals of Asteroid Positions*, online server, <http://astro.ago.uni-lj.si/asteroids/residuals.html>.
- Văduvescu O.: 2004a, *Astronomical Software by Ovidiu Văduvescu*, online at <http://www.geocities.com/ovidiu/astrsoft.html>.
- Văduvescu O.: 2004b, M.P.C. 52511, *Observations of Minor Planets between 2002/11/09–2004/08/22*, Aug 30 2004.
- York University Observatory, 2004: website <http://www.yorku.ca/observe>.

Received on 3 November 2004

ASTRONOMICAL REFRACTION CALCULATION FOR THE EARTH'S ELLIPSOIDAL ATMOSPHERE AT GREAT ZENITH DISTANCES

MINODORA LIPCANU

*University of Bucharest, Faculty of Mathematics and Computer Science
Str. Academiei 14, 010014 Bucharest, Romania
E-mail: mlipcanu@xnet.ro*

Abstract. We calculate the astronomical refraction for the Earth's ellipsoidal atmosphere, when the light ray situated in the meridian plane of the place is observed at a zenith distance between 70° and 81° . The calculation is made using the model of the terrestrial atmosphere TSA-60. It represents an extension of the calculation of the astronomical refraction for the ellipsoidal atmosphere at zenithal distances smaller than 70° . By comparing these results to the ones of the spherical model, it is ascertained that in this case also the refraction calculated for the ellipsoidal model is greater than the one for the spherical model, the difference increasing with the zenith distance and depending on the latitude of the place.

Key words: astrometry – astronomical refraction – atmospheric models.

1. INTRODUCTION

Even from the beginning of the 20th century, a model of terrestrial atmosphere has been searched for, which may allow the calculation of the refraction with a better approximation than in the case of the classical model, constituted by concentric spherical layers. An initial model was proposed by Harzer (1922, 1924), who took into consideration the Earth as being a revolution ellipsoid, and the atmosphere as being constituted of optical surfaces. This model was improved by authors as Sergienko (1979), Shabelnikov (1983), Yatsenko and Teleki (1985), Yatsenko (1995), the calculation still being time-taking. A different model, geometrically simple, was provided by Mihăilă (1973), who considered the atmosphere as an ellipsoidal layer.

In the following, making use of the refraction integral deduced by Mihăilă (1973), we present the calculation of the refraction for the ellipsoidal model, for zenith distances between 70° and 81° , when the refracted ray is situated in the meridian plane of the observational place. The obtained results will be compared to those from the spherical model. In a previous study this formula was applied for zenith distances

smaller than 70° (Mihăilă and Lipcanu 2004), and in this study the series of refraction will be extended with three more terms. For this purpose, the model of atmosphere TSA-60 will be applied (e.g., Dinulescu 1967), because in this case the refraction depends of the structure of the atmosphere. The model is given for the latitude of approximately 45° . The layers of equal density can be considered ellipsoidal layers. The method of Leone (Leone 1962; Dinulescu 1967) will be used, which considered that the TSA-60 model is spherical.

1. REFRACTION SERIES

It is assumed that the atmosphere of the Earth is constituted by ellipsoidal layers of constant density. The separation surfaces of the layers are homothetic ellipsoids, one of these being the ellipsoid of the Earth. The intersections of these surfaces with the meridian plane of the observational place will be homothetic concentric ellipses. For this model, Mihăilă (1973) obtained the following formula for the integral of refraction

$$R_\varphi = \int_1^{n_0} \frac{r_0 n_0 \sin z}{\sqrt{r^2 n^2 - r_0^2 n_0^2 \sin^2 z}} \frac{dn}{n}, \quad (1)$$

where z is the observed zenith distance, r is the radius of curvature of the ellipse that passes through the current point P on the light ray trajectory and n the refraction index of the atmospheric layer in which the considered point is situated. The subscript zero signifies the data from the observational place P_0 . The integral has the same form as in the case of the spherical model, with the simple difference that the radius of curvature comes instead of the geocentric distance. The relation (1) can also be written as

$$R_\varphi = \int_1^{n_0} \frac{\frac{r_0}{r} \frac{n_0}{n} \tan z}{\sqrt{1 + \left(1 - \frac{r_0^2}{r^2} \frac{n_0^2}{n^2}\right) \tan^2 z}} \frac{dn}{n}. \quad (2)$$

We shall introduce the following notation

$$2u_\varphi = 1 - \frac{n_0^2}{n^2} \frac{r_0^2}{r^2}. \quad (3)$$

For $z \leq 81^\circ$, $2|u_\varphi| \tan^2 z$ takes subunitary values, and the following expansion can be used

$$(1 + 2u_{\varphi} \tan^2 z)^{-\frac{1}{2}} = 1 - u_{\varphi} \tan^2 z + \frac{3}{2} u_{\varphi}^2 \tan^4 z - \frac{5}{2} u_{\varphi}^3 \tan^6 z + \frac{35}{8} u_{\varphi}^4 \tan^8 z - \dots \quad (4)$$

Performing the substitution in (2), we obtain

$$R_{\varphi} = \tan z \int_1^{n_0} \frac{r_0}{r} d\left(\frac{n_0}{n}\right) - \tan^3 z \int_1^{n_0} \frac{r_0}{r} u_{\varphi} d\left(\frac{n_0}{n}\right) + \frac{3}{2} \tan^5 z \int_1^{n_0} \frac{r_0}{r} u_{\varphi}^2 d\left(\frac{n_0}{n}\right) - \\ - \frac{5}{2} \tan^7 z \int_1^{n_0} \frac{r_0}{r} u_{\varphi}^3 d\left(\frac{n_0}{n}\right) + \frac{35}{8} \tan^9 z \int_1^{n_0} \frac{r_0}{r} u_{\varphi}^4 d\left(\frac{n_0}{n}\right) - \dots$$

From this series, only the first five terms have significance for $z \leq 81^\circ$, with a 0".001 error. Keeping only these terms, we have

$$R_{\varphi} = A_{0\varphi} \tan z - A_{1\varphi} \tan^3 z + \frac{3}{2} A_{2\varphi} \tan^5 z - \frac{5}{2} A_{3\varphi} \tan^7 z + \frac{35}{8} A_{4\varphi} \tan^9 z, \quad (5)$$

where

$$A_{i\varphi} = \int_1^{n_0} \frac{r_0}{r} u_{\varphi}^i d\left(\frac{n_0}{n}\right), \quad i = \overline{0, 4}. \quad (6)$$

The coefficients (6) will be determined following the procedure used by Leone (1962) in the case of the spherical model. For simplification purposes we shall use the notation corresponding to the case of the spherical atmosphere:

$$N = \frac{n}{n_0}, \quad Q = \frac{r}{r_0}, \quad (7) \\ N' = \frac{1}{n_0}, \quad Q' = \frac{r_{\max}}{r_0}.$$

The subscript "max" refers to the last ellipsoidal layer, significant for the refraction, situated at 80 km altitude. With (7), the coefficients (6) become

$$A_{i\varphi} = \int_{N'}^1 \frac{u_{\varphi}^i}{Q N^2} dN, \quad (8) \\ u_{\varphi} = \frac{1}{2} \left(1 - \frac{1}{N^2 Q^2} \right).$$

Thus, the coefficients can be written recursively

$$\begin{aligned}
A_{0\varphi} &= \int_{N'} \frac{1}{Q N^2} dN, \\
A_{1\varphi} &= \frac{1}{2} A_{0\varphi} - \frac{1}{2} \int_{N'} \frac{1}{Q^3 N^4} dN, \\
A_{2\varphi} &= -\frac{1}{4} A_{0\varphi} + A_{1\varphi} + \frac{1}{4} \int_{N'} \frac{1}{Q^5 N^6} dN, \\
A_{3\varphi} &= \frac{1}{8} A_{0\varphi} - \frac{3}{4} A_{1\varphi} + \frac{3}{2} A_{2\varphi} - \frac{1}{8} \int_{N'} \frac{1}{Q^7 N^8} dN, \\
A_{4\varphi} &= -\frac{1}{16} A_{0\varphi} + \frac{1}{2} A_{1\varphi} - \frac{3}{2} A_{2\varphi} + 2 A_{3\varphi} + \frac{1}{16} \int_{N'} \frac{1}{Q^9 N^{10}} dN.
\end{aligned} \tag{9}$$

We also consider

$$\beta = n_0 / Q'. \tag{10}$$

After the integration by parts in relations (9), the refraction coefficients in formula (5) can be written as

$$\begin{aligned}
A_{0\varphi} &= \beta - 1 + \int_1^{R'} \frac{dQ}{N Q^2}, \\
A_{1\varphi} &= \frac{1}{2} A_{0\varphi} - \frac{1}{6} (\beta^3 - 1) - \frac{1}{2} \int_1^{R'} \frac{dQ}{N^3 Q^4}, \\
A_{2\varphi} &= -\frac{1}{4} A_{0\varphi} + A_{1\varphi} + \frac{1}{20} (\beta^5 - 1) + \frac{1}{4} \int_1^{R'} \frac{dQ}{N^5 Q^6}, \\
A_{3\varphi} &= \frac{1}{8} A_{0\varphi} - \frac{3}{4} A_{1\varphi} + \frac{3}{2} A_{2\varphi} - \frac{1}{56} (\beta^7 - 1) - \frac{1}{8} \int_1^{R'} \frac{dQ}{N^7 Q^8}, \\
A_{4\varphi} &= -\frac{1}{16} A_{0\varphi} + \frac{1}{2} A_{1\varphi} - \frac{3}{2} A_{2\varphi} + 2 A_{3\varphi} + \frac{1}{144} (\beta^9 - 1) + \frac{1}{16} \int_1^{R'} \frac{dQ}{N^9 Q^{10}}.
\end{aligned} \tag{11}$$

The Gladstone relation will give for the considered model

$$\frac{n-1}{\delta} = \frac{n_0-1}{\delta_0} = \rho_0 = 0.2389388, \tag{12}$$

where $\delta_0 = 1.2250 \times 10^{-3}$ for TSA-60 and $n_0 = 1.0002927$ (Danjon 1959). By (7) and (12), if we limit to the terms of order 4 in δ and ρ_0 , we get

$$\begin{aligned}\frac{1}{N} &= n_0(1 + \rho_0\delta)^{-1} = n_0(1 - \rho_0\delta + \rho_0^2\delta^2 - \rho_0^3\delta^3 + \rho_0^4\delta^4), \\ \frac{1}{N^3} &= n_0^3(1 + \rho_0\delta)^{-3} = n_0^3(1 - 3\rho_0\delta + 6\rho_0^2\delta^2 - 10\rho_0^3\delta^3 + 15\rho_0^4\delta^4), \\ \frac{1}{N^5} &= n_0^5(1 + \rho_0\delta)^{-5} = n_0^5(1 - 5\rho_0\delta + 15\rho_0^2\delta^2 - 35\rho_0^3\delta^3 + 70\rho_0^4\delta^4), \\ \frac{1}{N^7} &= n_0^7(1 + \rho_0\delta)^{-7} = n_0^7(1 - 7\rho_0\delta + 28\rho_0^2\delta^2 - 84\rho_0^3\delta^3 + 210\rho_0^4\delta^4), \\ \frac{1}{N^9} &= n_0^9(1 + \rho_0\delta)^{-9} = n_0^9(1 - 9\rho_0\delta + 45\rho_0^2\delta^2 - 165\rho_0^3\delta^3 + 495\rho_0^4\delta^4).\end{aligned}\quad (13)$$

Plugging these expressions into the integrands in (11), we obtain the following results:

$$\begin{aligned}A_{0\varphi} &= n_0 - 1 - \rho_0 n_0 \int_1^{R'} \delta \frac{dQ}{Q^2} + \rho_0^2 n_0 \int_1^{R'} \delta^2 \frac{dQ}{Q^2} - \rho_0^3 n_0 \int_1^{R'} \delta^3 \frac{dQ}{Q^2} + \rho_0^4 n_0 \int_1^{R'} \delta^4 \frac{dQ}{Q^2}, \\ A_{1\varphi} &= \frac{1}{2} A_{0\varphi} - \frac{1}{6} (n_0^3 - 1) + \frac{3}{2} \rho_0 n_0^3 \int_1^{R'} \delta \frac{dQ}{Q^4} - 3\rho_0^2 n_0^3 \int_1^{R'} \delta^2 \frac{dQ}{Q^4} + \\ &\quad + 5\rho_0^3 n_0^3 \int_1^{R'} \delta^3 \frac{dQ}{Q^4} - \frac{15}{2} \rho_0^4 n_0^3 \int_1^{R'} \delta^4 \frac{dQ}{Q^4}, \\ A_{2\varphi} &= -\frac{1}{4} A_{0\varphi} + A_{1\varphi} + \frac{1}{20} (n_0^5 - 1) - \frac{5}{4} \rho_0 n_0^5 \int_1^{R'} \delta \frac{dQ}{Q^6} + \frac{15}{4} \rho_0^2 n_0^5 \int_1^{R'} \delta^2 \frac{dQ}{Q^6} - \\ &\quad - \frac{35}{4} \rho_0^3 n_0^5 \int_1^{R'} \delta^3 \frac{dQ}{Q^6} + \frac{35}{2} \rho_0^4 n_0^5 \int_1^{R'} \delta^4 \frac{dQ}{Q^6}, \\ A_{3\varphi} &= \frac{1}{8} A_{0\varphi} - \frac{3}{4} A_{1\varphi} + \frac{3}{2} A_{2\varphi} - \frac{1}{56} (n_0^7 - 1) + \frac{7}{8} \rho_0 n_0^7 \int_1^{R'} \delta \frac{dQ}{Q^8} - \frac{7}{2} \rho_0^2 n_0^7 \int_1^{R'} \delta^2 \frac{dQ}{Q^8} + \\ &\quad + \frac{21}{2} \rho_0^3 n_0^7 \int_1^{R'} \delta^3 \frac{dQ}{Q^8} - \frac{105}{4} \rho_0^4 n_0^7 \int_1^{R'} \delta^4 \frac{dQ}{Q^8},\end{aligned}\quad (14)$$

$$A_{4\varphi} = -\frac{1}{16} A_{0\varphi} + \frac{1}{2} A_{1\varphi} - \frac{3}{2} A_{2\varphi} + 2 A_{3\varphi} + \frac{1}{144} (n_0^9 - 1) - \frac{9}{16} \rho_0 n_0^9 \int_1^{R'} \delta \frac{dQ}{Q^{10}} + \\ + \frac{45}{16} \rho_0^2 n_0^9 \int_1^{R'} \delta^2 \frac{dQ}{Q^{10}} - \frac{165}{16} \rho_0^3 n_0^9 \int_1^{R'} \delta^3 \frac{dQ}{Q^{10}} + \frac{495}{16} \rho_0^4 n_0^9 \int_1^{R'} \delta^4 \frac{dQ}{Q^{10}}.$$

For the calculation of the integrals from the expressions above, we considered a division of the atmosphere, of the TSA-60 model, constituted of 80 concentric ellipsoidal layers, having each the thickness of 1 km. For the density values, the arithmetic mean of the extreme values was taken at each layer. Obviously, the calculation of the coefficients will be more precise, the greater the number of the atmospheric layers is. The index φ from the relations (14) appears also in the expression of R' . Thus, using the expression of the radius of curvature of a point on the ellipse

$$r = \frac{a(1-e^2)}{(1-e^2 \sin^2 \varphi)^{3/2}}, \quad (15)$$

where a is the semimajor axis of the ellipse, e its eccentricity, φ the latitude of the point, by using (7) we obtain

$$Q' = \frac{a_{\max}}{a_0} \left(\frac{1-e^2 \sin^2 \varphi_0}{1-e^2 \sin^2 \varphi_{\max}} \right)^{\frac{3}{2}} = \frac{a_{\max}}{a_0} \left[(1-e^2 \sin^2 \varphi_0) (1-e^2 \sin^2 \varphi_{\max})^{-1} \right]^{\frac{3}{2}}, \quad (16)$$

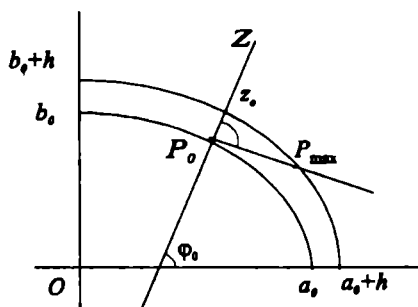
where $\varphi_{\max}$ stands for the geodetic latitude of the point $P_{\max}$, in which the radiation penetrates the terrestrial atmosphere at the upper limit of this one. This is obtained approximately, determining the position of the point situated at the intersection of the tangent in P_0 to the trajectory with the ellipse corresponding to the superior limit of the atmosphere. Only the atmospheric layer of 80 km thickness presents sensitive effect for the refraction.

The coordinates $(x_{\max}, y_{\max})$ of the point $P_{\max}$ of entering the atmosphere are given by the system

$$\begin{cases} y - y_0 = (x - x_0) \tan(\varphi_0 - z), & x_0 = \frac{a_0 \cos \varphi_0}{\sqrt{1-e^2 \sin^2 \varphi_0}}, & y_0 = \frac{a_0(1-e^2) \sin \varphi_0}{\sqrt{1-e^2 \sin^2 \varphi_0}}, \\ \left(\frac{x}{a_0+h} \right)^2 + \left(\frac{y}{b_0+h} \right)^2 = 1, \end{cases} \quad (17)$$

where a_0 and b_0 are the semi-axes of the terrestrial meridian (see Fig. 1).

Fig. 1 – The configuration in the meridian plane of the place.



If $(x_{\max}, y_{\max})$ is the solution of (17), then $\varphi_{\max} = \arctan \frac{y_{\max}}{x_{\max}} \left(\frac{a_0 + h}{b_0 + h} \right)^2$.

The product $e^2 \sin^2 \varphi_0$ is very small, between 0 and 6.4×10^{-3} . Therefore, expanding in series and keeping the first two terms, the result is

$$Q' = \frac{a_{\max}}{a_0} \left[1 + e^2 (\sin^2 \varphi_{\max} - \sin^2 \varphi_0) \right]^{\frac{3}{2}}. \quad (18)$$

Finally we get

$$Q' = \frac{a_{\max}}{a_0} \left[1 + e^2 \sin(\varphi_{\max} + \varphi_0) \sin(\varphi_{\max} - \varphi_0) \right]^{\frac{3}{2}}. \quad (19)$$

Because $\left| e^2 \sin(\varphi_{\max} - \varphi_0) \sin(\varphi_{\max} + \varphi_0) \right| \leq 6.4 \times 10^{-3}$, expanding in series and retaining the first two terms, we can write

$$Q' = \frac{a_{\max}}{a_0} \left[1 + \frac{3}{4} e^2 (\cos 2\varphi_0 - \cos 2\varphi_{\max}) \right]. \quad (20)$$

This is the expression that we used for the calculation of the coefficients given by the formulae (14).

3. RESULTS

In order to compare the results obtained in the case of the ellipsoidal model, using the formula (5), with the coefficients (14), the refraction in the case of the spherical model was calculated for the same zenithal distances between 70° and 81° . In this case the expression (5) becomes

$$R = A_0 \tan z - A_1 \tan^3 z + \frac{3}{2} A_2 \tan^5 z - \frac{5}{2} A_3 \tan^7 z + \frac{35}{8} A_4 \tan^9 z, \quad (21)$$

in which the coefficients do not depend on φ , while the variable Q becomes $Q = \frac{r}{r_0}$.

Here r_0 stands for the radius of the Earth and r represents the radius of the spherical layer. In this case we have $Q' = \frac{r_{\max}}{r_0}$, where $r_{\max}$ is the radius of the layer with the altitude $h = 80$ km.

In Table 1 we present the values of the refraction R (expressed in arcsec), calculated by means of the formula (21), for each degree, for a zenithal distance z between 70° and 81° .

Table 1

Refraction for the spherical model

z	R	z	R
70	164.218	76	237.466
71	173.413	77	255.714
72	183.556	78	276.749
73	194.810	79	301.263
74	207.370	80	330.191
75	221.485	81	364.842

Table 2 presents the deviations $\Delta(z, \varphi) = R_\varphi - R$ of the refraction values (expressed in arcsec), calculated for the ellipsoidal model, as compared to the corresponding values for the spherical model.

Table 2

The deviation $\Delta(z, \varphi)$ of the refraction value for the ellipsoidal model as compared to the value for the spherical model

$\varphi \backslash z$	0	10	20	30	40	45	50	60
70	0.000	0.067	0.126	0.169	0.192	0.195	0.192	0.169
71	0.000	0.082	0.154	0.207	0.235	0.239	0.236	0.208
72	0.000	0.101	0.190	0.256	0.291	0.296	0.292	0.257
73	0.000	0.126	0.237	0.320	0.364	0.370	0.365	0.321
74	-0.000	0.159	0.300	0.405	0.462	0.469	0.462	0.407
75	-0.000	0.204	0.385	0.520	0.593	0.602	0.593	0.523
76	-0.000	0.265	0.501	0.678	0.772	0.785	0.773	0.681
77	-0.010	0.349	0.663	0.898	1.024	1.041	1.026	0.904
78	-0.010	0.468	0.894	1.213	1.385	1.408	1.387	1.221
79	-0.020	0.641	1.231	1.674	1.913	1.945	1.917	1.687
80	-0.050	0.892	1.732	2.365	2.707	2.753	2.714	2.386
81	-0.110	1.261	2.492	3.423	3.929	3.998	3.941	3.458

We also present the graphic of the deviation $\Delta(z, \varphi)$ for $\varphi = 45^\circ$, latitude for which the deviation is more pronounced.

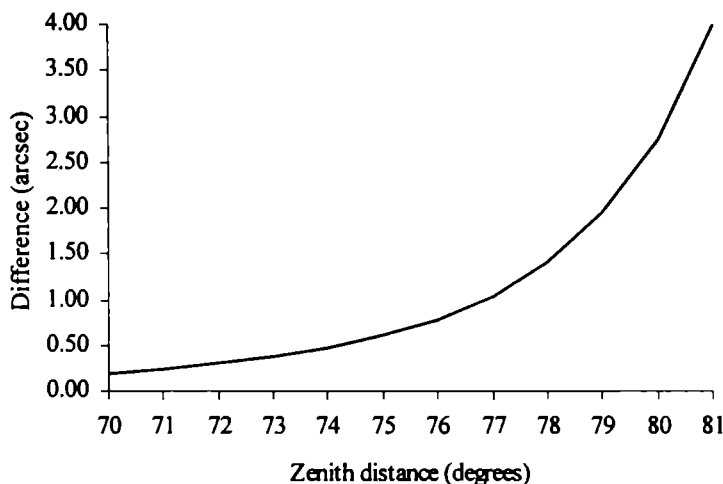


Fig. 2 – Difference of the refraction values for the latitude of 45° .

It is ascertained that the deviations have a systematic character. Therefore, the results obtained using the ellipsoidal model of the atmosphere prove that, at least for the meridian plane of the observational place, the refraction is greater than the one calculated for the spherical model. The results are in agreement with those obtained for zenithal distances smaller than 70° .

REFERENCES

- Danjon, A.: 1959, *Astronomie générale*, J. et R. Sennac, Paris.
- Dinulescu, N. I.: 1967, *Astronomical Refraction*, Ed. Acad. R.S.R., Bucharest (Romanian).
- Harzer, P.: 1924, *Publ. Sternwarte Kiel*, **14**, 1.
- Leone, S.: 1962, *Pubbl. Osserv. Astron. Palermo*, **10**, 19.
- Mihăilă, I.: 1973, *St. Cerc. Astron.*, **18**, 187.
- Mihăilă, I., Lipcanu, M.: 2004, *Rom. Astron. J.*, **14**, 187.
- Sergienko, V. I.: 1979, *Astron. Zh.*, **56**, 672.
- Shabelnikov, A. V.: 1983, *Pis'ma Astron. Zh.*, **9**, 695.
- Yatsenko, A. Yu.: 1995, *Astron. Astrophys. Suppl.*, **111**, 579.
- Yatsenko, A. Yu., Teleki, G.: 1985, *Bull. Obs. Astron. Belgrade*, **135**, 1.

Received on 27 April 2004

THE INTERNATIONAL SCIENTIFIC SESSION “SCIENTIFIC PROGRAMS AND ASTRONOMY EDUCATION IN SEE AND UKRAINE” Bucharest, Romania, 16–17 September 2005

In 2000, during the XXIVth General Assembly of the International Astronomical Union in Manchester, UK, a new working group of the Division 1 was founded. Named “The Future Development of Ground-Based Astrometry” (FDGBA), it appeared as a natural consequence of the new state of astrometry after the success of the Hipparcos space mission.

In the same year the South-Eastern Branch of the European Astronomical Society was founded. This also is a natural consequence of the similarity of problems the astronomy in this region of Europe faces.

Both organizations were created due to Romanian initiatives. Both are chaired by the Romanian representative (Dr. M. Stavinschi). Moreover, due to the same Romanian initiative, UNESCO-ROSTE funded in 2004 the project “Enhancing Astronomical Research and Observation in SEE and Ukraine”. The Sub-Regional European Astronomical Committee (SREAC) was founded within the framework of this project.

Due to the sponsorship of UNESCO-ROSTE, and aided by the three above structures, closely interrelated, the Astronomical Institute of the Romanian Academy organized the international scientific meeting “Astrometry with Small Telescopes” (Bucharest, 22–23 October 2004). A new meeting, “Scientific Programs and Astronomy Education in SEE and Ukraine”, was organized under the same aegis (Bucharest, 16–17 September 2005).

This meeting, much more eclectic than the first one, included five sessions.

The first session was dedicated to astrometry. Among the main problems discussed here, I mention: asteroid observational programs within a large international cooperation; CCD astrometry using ground-based telescopes; virtual observatories; artificial intelligence methods applied to astrometry.

Celestial mechanics and dynamical astronomy constituted the topics of the second session. Subjects of a high degree of generality were discussed; among them: the Gylden-type problem, the Hénon-Heiles-type problem, the photogravitational circular restricted three-body problem. Even if rather theoretical, these problems model a large number of concrete astronomical situations.

The third session included two subsessions quite different: one dedicated to cosmology and extragalactic astronomy, the other dedicated to solar physics. Within the latter one, interesting contributions concerning helioseismology or the relation between the solar activity and the geomagnetic field were presented.

Stellar astrophysics was the subject of the fourth session. There were remarked some presentations about a large cooperation in binary-star research (the Central-East-South European Binary Star Group) or concerning the stellar seismology within the COROT space mission.

Maybe the most interesting session was the fifth one, dedicated to astronomy education and history of astronomy. Contributions regarding concrete educational activities or general ideas about the

role of astronomers in public education must be emphasized. The contributions oriented towards the history of astronomy were very interesting, too.

A round table mainly intended to discuss astronomy education problems and perspectives in the SREAC member countries (and not only) closed the session.

Poster covering the topics of the first four sessions (and more: e.g., astronomical geodesy, geopotential representation, Venus' transit 2004, etc.) completed the meeting.

The contributions to this scientific meeting were signed by 86 authors belonging to 14 countries. 42 scientists from Armenia, Bulgaria, Greece, Romania, Russia, Serbia and Montenegro, Turkey, and Ukraine were present. Their contributions were co-authored by 44 astronomers from Australia, Czech Republic, Germany, Hungary, UK, USA.

Besides the intrinsic value of the presented contributions, the meeting pointed out a real and fruitful cooperation involving not only SREAC member countries, but much more countries all over the world. It is sufficient to mention that there were many contributions co-authored by scientists from two, three, and even five countries.

All the contributions (both oral presentations and posters) will be published soon in a special supplement of the *Romanian Astronomical Journal*.

Vasile Mioc

**THE THIRD MEETING OF THE SUB-REGIONAL
EUROPEAN ASTRONOMICAL COMMITTEE (SREAC)
within the framework of the UNESCO-ROSTE funded Project
“ENHANCING ASTRONOMICAL RESEARCH AND OBSERVATION
IN SEE AND UKRAINE”
Bucharest, Romania, 18 September 2005**

The Sub-Regional European Astronomical Committee (SREAC) was created within the framework of the project "Enhancing astronomical research and observation in South-East Europe and Ukraine" of the UNESCO Regional Bureau for Science in Europe (UNESCO-ROSTE).

Its main objective is to elaborate and implement a sub-regional strategy for the development of astronomy in South-East Europe and Ukraine and to strengthen astronomical co-operation in the sub-region, and between the region and countries outside.

On 18 September 2005, the third meeting of SREAC was held in Bucharest, following the session Scientific Programs and Astronomy Education. The countries, members of SREAC, represented during this meeting were: Bulgaria, Greece, Romania, Serbia & Montenegro, Turkey, and Ukraine. The SREAC chair, Magda Stavinschi, presented the report on the SREAC activity in the period October 2004 – September 2005.

The delegate of UNESCO-ROSTE, Alec Boksenberg, presented his suggestions concerning a new UNESCO support for astronomy, in the framework of a IBSP (International Basic Sciences Programmes) projects. The discussion was very lively (interventions from A. Boksenberg, M. Stavinschi, K. Panov, R. Konstantinova-Antova, G. Pinigin, H. Rovithis-Livaniou, A. Mickelian, G. Djurasevic, V. Mioc, Z. Aslan). Many aspects were tackled, as, for instance:

- continuation of cooperation in astronomy;
- broadening experience in astronomy;
- education and training in astronomy;
- the role of the meetings in astronomical cooperation;
- access to medium/large facilities;
- involvement of engineering in astronomical research;
- upgrading the major instruments in the region;
- role of the small telescopes.

Taking into account the rules of the SREAC statute, a new board was elected for the year 2006: Zeki Aslan, as chair, and Helen Rovithis-Livaniou, as secretary. SREAC included the position of Counselor of the Chair. Magda Stavinschi was appointed to this position. Armenia became a new associate country member.

Magda Stavinschi

**EDITAT CU SPRIJINUL
AGENȚIEI NAȚIONALE PENTRU CERCETARE ȘTIINȚIFICĂ**

NOTICE TO AUTHORS

ROMANIAN ASTRONOMICAL JOURNAL appears twice a year and is open to original contributions in Astronomy and related disciplines. The contributions – in English or French – can be accepted only if they were neither published before nor destined to another publication.

Manuscripts should, preferably, be submitted as e-mail attachments to roaj@aira.astro.ro, to the Editor in Chief (vmioc@aira.astro.ro), or to one of the members of the Editorial Board. They must be written in a well-known editor, preferably a recent version of Word. Authors who cannot submit an electronic version of their manuscript should send it to the Editorial Board. In this case two copies of the manuscript (along with the text of the article on floppy disk) are required. The first page should contain: article's title (brief and informative), author's name and affiliation, followed by an *Abstract* in English and *Key words*. The text should be clear and concise (it is recommended not to exceed 10 pages). The *Abstract* will present clearly the principal conclusions on the work, in no more than 10–15 lines.

Chapters and Paragraphs. Papers, except short notes, should be divided into chapters, numbered by Arabic numerals. Chapters may be divided into paragraphs denoted by the number of the chapter and the number of the paragraph; each chapter and each paragraph should have a short descriptive title (e.g. "3.2. Results").

Formulae have to be centered and numbered consecutively in Arabic numerals, too, but included in brackets on the right-hand side of the manuscript.

Tables should be numbered consecutively in Arabic numerals; they should be introduced in the text at their appropriate place.

Figures and Illustrations should be submitted separately, in such a form as to permit reproduction without retouching. Any lettering should be large enough to be legible after the figure has been reduced in size for printing. Captions should be introduced in the text at their appropriate place. All figures should be numbered consecutively in Arabic numerals and referred to in the text, e.g. Fig. 2 or Figs. 2–5. Photographs should be given only if essential and should be enlarged enough to permit clear reproduction.

References are indicated in the text by the author's name and year of publication. They should be listed in alphabetic and chronologic order at the end of the paper, as follows: name and initials of the author(s), year of publication, suitable abbreviation of the journal (or title of the book and editing house), its volume and page.

The *fonts* to be used are: 11 for the text proper, 13 for the paper title, 9 for author's name and affiliation, abstract, key words, titles of chapters and paragraphs, figure captions, tables, running titles, and references.

Detailed instructions for the preparation of manuscripts can be found at <http://www.astro.ro/~roaj>.

Please pay attention to these recommendations; the manuscripts that do not observe them will be returned to the authors.

ROMANIAN ASTRONOMICAL JOURNAL

Vol. 15, No. 2, 2005

CONTENTS

Ioannis Iraklis HARANAS, Pressure Calculation of a Constant-Density Star in the Dynamic Theory of Gravity.....	95
Ibrahim Mohamed SELIM, Inall Adham HASSAN, Hamed Abd-Alhamed ISMAIL, Magdy Mahros ALKATEEB, Virialized Hickson Compact Groups of Galaxies	107
Peter ROVITHIS, Alexandru DUMITRESCU, Helen ROVITHIS-LIVANIOU, Gabriela OPRESCU, Marian Doru ȘURAN, The Eclipsing Binary AW Uma: Observations at Athens and Bucharest.....	113
Nabil Shoukry AWADALLA, Monir Ahmed HAMDY, Magdy Abdel-Malek HANNA, Abdel-Fady Beshara MORCOS, Study of the Orbital Period Variation of the Eclipsing Binary System BV Dra	129
Marilena MIERLA, Rainer SCHWENN, Luca TERIACA, Guillermo STENBORG, Borut PODLIPNIK, Dynamics of the Solar Corona Using LASCO-C1 Spectral Data	137
Dumitru PRICOPI, Vasile MIOC, Hénon-Heiles' Two-Body Problem. II. Main Features of the Global Flow	153
Ciprian CHIRUȚĂ, Tiberiu OPROIU, Note on the Photogravitational Restricted Three-Body Problem.....	165
Ovidiu VĂDUVESCU, Observing Near Earth Asteroids with a Small Telescope. II. The Program at York University Observatory.....	171
Minodora LIPCANU, Astronomical Refraction Calculation for the Earth's Ellipsoidal Atmosphere at Great Zenith Distances	183

MISCELLANEA

The International Scientific Session "Scientific Programs and Astronomy Education in SEE and Ukraine", Bucharest, Romania, 16–17 September 2005 (Vasile MIOC)	193
The Third Meeting of the Sub-Regional European Astronomical Committee (SREAC) within the Framework of the UNESCO-ROSTE Funded Project "Enhancing Astronomical Research and Observation in SEE and Ukraine", Bucharest, Romania, 18 September 2005 (Magda Stavinschi)	194

ISSN 1220-5160

Rom. Astron. J., Vol. 15, No. 2, p. 93–196, Bucharest, 2005