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CONTENTS

Magda STAVINSCHI, Vasile MIOC, The Bucharest Observatory of the Astronomical Institute at Centenary: 100 Years and More of Romanian Astronomy	3
Ponsamy SRIRAMACHANDRAN, Ramachandran SHANMUGAVEL, Narayanan RAJAMANICKAM, Somashekar Pundalikarao BAGARE, Evaluation of Astrophysically Useful Parameters for LaO Molecule	21
Ovidiu C. FURDUI, On the Stability of Planetary Orbits with a Modified Hermite Integrator ..	29
Vasile MIOC, Rodica ROMAN, Libration Points in Fock's Restricted Three-Body Problem ...	41
Mohamed Adel SHARAF, Bielliptic Transfer with Plane Changes	51
Mohamed Adel SHARAF, Determination of a Celestial Plane Orientation: Application to Milky Way	61
Gheorghe BOCŞA, Milcho TSVETKOV, Precise Determination of Minor Planet Positions ...	69
Ivan KOTLIAROV, New Laws of Planetary Distances	77
Gerd HELMECKE, Ulrich HERKENRATH, From Synchronization of the Atom to Organization of the Universe via Gravitation	85

MISCELLANEA

The International Conference "Exploring the Solar System and the Universe", Bucharest, Romania, 8–12 April 2008 (Vasile Mioc, Cristiana Dumitrache, Nedelia Antonia Popescu)	93
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THE BUCHAREST OBSERVATORY OF THE ASTRONOMICAL INSTITUTE AT CENTENARY: 100 YEARS AND MORE OF ROMANIAN ASTRONOMY

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Abstract. On 1 April 1908 the Astronomical Observatory of Bucharest was set up. But the Romanian astronomy is much older. There are evidences that go back to the dawn of the first millennium. The 19th century brought the modernization of Romanian astronomy in European context. By coincidence, another observatory was set up the same year on a Romanian territory (today in the Republic of Moldavia), that of Nicolae Donici. This paper constitutes itself in a survey of the most important events and personalities of the Romanian astronomy, especially in the context of the new international cooperation wave in which Romania participates.

Key words: history of astronomy – Romanian astronomy – centenary of the Bucharest Astronomical Observatory.

1. BEGINNINGS

The sky is fascinating. The sky lifts and overwhelms us. The sky asks us questions. The sky helps man preserve the only planet he can inhabit for the time being. It is only natural that man should observe the sky, ask himself questions and do his best to answer them. This is how Astronomy appeared and developed in all places on Earth. This is also how Astronomy appeared and developed on the Romanian lands.

Although we have very old testimonies of its existence on our lands, we can talk about Romanian scientific observatories only beginning with the last century. We have in mind the Observatory in the Charles I Park in Bucharest and the one on the right bank of the river Nistru, at Dubăsarii Vechi, today in the Republic of Moldavia.

We can talk of astronomical traditions on our territories even beginning with the first millennium, when the Dacians built in the Meridional Carpathians, at Grădiștea Muncelului, a sanctuary which still preserves evidence of their astronomical knowledge and implicitly, of an extremely precise calendar for those times (Fig. 1).

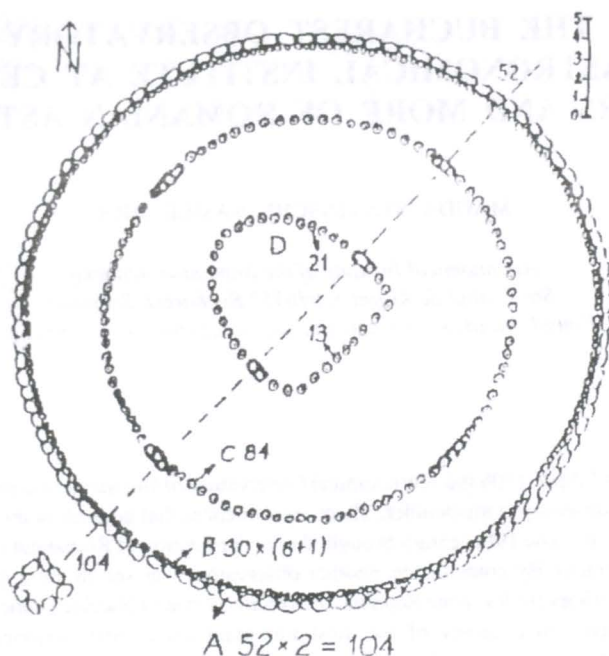


Fig. 1 – The big Dacian sanctuary at Grădiștea Muncelului.

Somewhat later, at the beginning of the 6th century, the Christian Church tried to establish a chronology, as adequate as possible for the religious celebrations, of Easter in particular, set up in keeping with various astronomical moments. Its author was Dionisie the Small (Dionysius Exiguus), a monk born in the territory of Dobrogea. In 528 AD he introduced in *Liber de Paschal* the counting of the years since the birth of Jesus Christ. His exceptional erudition (an impressive knowledge of astronomy, history, foreign languages, literature, etc.) enabled him to establish the date of the Saviour's birth after five centuries, with an accuracy of 4–7 years. That was an extraordinary fact if we take into consideration that no modern method has managed to come closer to the exact year of Christ's birth.

2. THE MIDDLE AGES

The centuries which followed led to the considerable development of astronomical knowledge, but its truly scientific foundation dates back only to the 16th and 17th centuries, that is why 2009 is the International Year of Astronomy.

In the period preceding Galileo there were various astronomical preoccupations on these lands, too. It is worth mentioning, for instance, the first astronomical observations made in this part of Europe by the bishop Ioan Vitez (Janos Vitez) (1408–1472), the teacher of prince Iancu of Hunedoara's children. The Observatory in Oradea, an

important centre of renaissance culture in that period, was created by Ioan Vitez in 1445, namely one century before the one set up at Uraniborg by Tycho Brahe. So that the goal of the Observatory should be fulfilled at the highest level, he invited the Viennese physicist Georg Peurbach to Oradea. The latter one also brought there his best student, Johannes Müller, also called Regiomontanus. Regiomontanus' observations, as well as those of a pupil of Ioan Vitez, became the basis of the famous *Tabula Varadiensis*, used by Columbus to discover Cuba, then by Tycho Brahe and Kepler.

A century later, Johannes Grass (Honterus) published notions of cosmography in *Rudimenta Cosmographica* (1568). The work had an important impact at the time, being issued in 26 successive editions, which were used for a long time in Germany as astronomy textbooks.

We should also mention *Coligatul de la Sibiu*, in which Conrad Haas (1509–1579) describes rockets in stages and uses this term for the first time.

The 16th and 17th centuries are epochs of important spreading of astronomical knowledge in the ever more numerous colleges which were set up in Transylvania, Moldavia and Wallachia.

From among the teachers of the epoch we shall mention only the name of Hrisant Notara, who was a collaborator of Giovanni Domenico Cassini (1625–1712), the first Director of Observatoire de Paris set up in 1667. He left us *Introductio ad geographiam et sphaeram* (published in Paris, 1716; see Fig. 2), the first Romanian

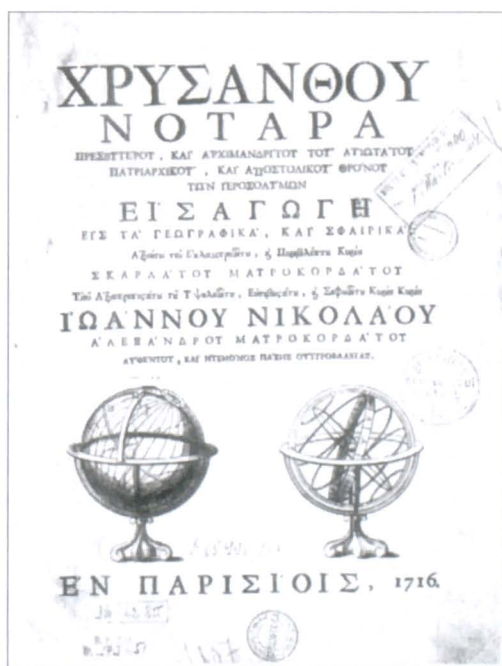


Fig. 2 – Notara's book.

scientific work with chapters dedicated to astronomy. The work also indicates for the first time the geographic coordinates of the towns of Bucharest and Târgoviște. After he was sent abroad by the ruler Constantin Brâncoveanu to improve his studies, Notara returned to Brâncoveanu's court, and then dedicated himself to clerical life, ascending the highest clerical positions up to that of Patriarch of Jerusalem.

It is also worth mentioning the first scientific astronomical observations performed by the erudite Constantin Cantacuzino, high steward and uncle of Brâncoveanu, by means of his own telescope.

The other observatories were built in Transylvania during the 18th century. Thus, the construction of the astronomical tower conducted by Maximilian Hell (1720–1790) was completed in 1759. The tower was destroyed by a fire in 1798 and restored in 1805.

The second observatory was set up in 1795 at Alba Iulia by the bishop Ignatius Batthyany – who founded the famous library that was to be called after his name. The Observatory ended its activity in 1860.

Meanwhile, astronomy also developed in the land across the Carpathians. Thus, in 1762, the Jesuit Ruggero Giuseppe Boscovich, one of the greatest astronomers of that time and founder of the Brera-Milan astronomical observatory, performed measurements of longitudes and latitudes, as well as other astronomical studies at Galați.

In 1773 the Russian astronomer Stephen Rumowski published *Determinatio Longitudinis et Latitudinis quorundam Moldaviae et Wallachiae locorum deducta ex Observationibus Johanne Islenieff institutis*, a work which he elaborated on the basis of the astronomical determination of the geographical coordinates for Bender, Akerman, Chilia Nouă, Ismail, Bucharest, Focșani, Jassy and Brăila, carried out by the Russian astronomer Ivan Isleniev.

3. THE 19TH CENTURY

At the beginning of the 19th century, the first amateur astronomer was recorded again in Moldavia. In 1823, the poet Costache Conachi bought himself a telescope in Vienna, which had a focal distance of 2 m and a diameter of 157 mm and was later used by students in their practical studies. This telescope was handed down by his heirs to the Observatory in Jassy.

During the period 1828–1832, astronomical observations for the determination of different geographical coordinates were performed. Ortemberg, Wrontchenko and others determined the coordinates of the Romanian towns of Jassy, Galați, Roman, Bucharest, Călărași, Turnu-Măgurele, Calafat, Craiova, Babadag, Constanța, Gurile Dunării.

Together with the development of the elementary and secondary educational systems, the first astronomy textbooks were issued, such as those by Gheorghe Asachi (1838), A. Marian (1829 – the first astronomy textbook written in Romanian) and August Treboniu Laurian (1859).

At the middle of the 19th century there were profound transformations, not only in Europe, but also on the Romanian lands. In 1859 Moldavia and Wallachia united and thus the first modern Romanian state was created. This fact called for not only a reform of the political and social life, but also one of education, directed to Western culture, to the French one in particular.

The foundation of the first Romanian university centres (Jassy, in 1860, and Bucharest, in 1864) also marked the beginning of academic instruction. The first professor of astronomy was Neculai Culianu (1832–1915) in Jassy – the author of a course in cosmography. In December 9, 1874, together with Professor Ștefan Micle (1820–1879), he observed the transit of Venus across the Sun's disk. The poor facilities available did not allow then the obtaining of good results such as those reported by T. von Oppolzer and E. Weiss, whose observations performed in Jassy proved to be even more accurate than the similar ones carried out in Vienna. Dimitrie Petrescu (1831–1896) was the first astronomy professor who delivered lectures at Bucharest University, where he was followed by Nicolae Coculescu.

Due to the development of astronomical academic education, a large number of young students were sent abroad to improve their training in astronomy. In this context, it is worth mentioning especially Constantin Căpităneanu (1849–1895) (Fig. 3), of whom M. Loewy used to say: "The Paris Observatory Annals for 1869 and 1870 include many pages under the signature of Căpităneanu which do great honour to the Romanian astronomer". After performing another training stage in Naples, he returned home (1873), where he was assigned a task of high responsibility, i.e., to draw up an accurate map of the country. A telescope (field glass) for latitudes, two refracting telescopes, a chronograph, chronometers, etc. were ordered for this purpose. These facilities helped



Fig. 3 – Constantin Căpităneanu.

him to construct in Jassy (1875) the first meridian dome on the Romanian territory. Seven years later a similar hall was also erected in Bucharest. In 1881 Căpităneanu and Kihnert published in Bucharest *A Determination of the longitudinal differences between Jassy and Cernăuți*. This was the first publication of astronomical observations carried out by a Romanian scientist in his own country.

4. THE FIRST DOCTORAL THESES IN ASTRONOMY

The first theoretical studies in astronomy were also reported at the end of the 19th century. Out of the first four Romanian scientists who took their doctor's degrees in mathematics at Sorbonne, in Paris, three devoted their theses to celestial mechanics: Spiru Haret, Constantin Gogu, and Nicolae Coculescu (Figs. 4, 5, and 6, respectively).



Fig. 4 – Spiru Haret.



Fig. 5 – Constantin Gogu.



Fig. 6 – Nicolae Coculescu.

Spiru Haret (1851–1912) dedicated his doctoral thesis (1878) to the study of the invariability of the major axes of planetary orbits. Félix Tisserand, professor of celestial mechanics at the Sorbonne, later resumed this study and reached the same conclusions and even recommended that young Haret's methods be extended to other astronomic calculations. At the same time, the well-known French mathematician Henri Poincaré highly appreciated Haret's thesis, concluding that it was a "great surprise". The same subject was resumed in 1955 by Jean Meffroy. Therefore, Spiru Haret can be considered as the first Romanian astronomer-theorist.

In 1882 another Romanian got his PhD in Paris, namely Constantin Gogu (1854–1897), who concentrated on long-periodic inequalities in the Moon's orbital motion, confirming the accuracy of Delaunay's calculations against Stockwell's errors. Following the publication of his doctoral thesis, Constantin Gogu became one of the most famous European specialists in the field of celestial mechanics and particularly in topics related to the Moon.

The third Romanian doctor in astronomy from the Sorbonne was Nicolae Coculescu (1866–1952), who gave a description of the interrelated motions of the three-body problem for a particular case. His PhD thesis (1895) concentrated on the perturbation function and supplied the approximate expressions of its higher-order terms, which were later mentioned in *Leçons de mécanique céleste* by Henri Poincaré (one of the letters of Coculescu to Poincaré was recently discovered). The doctorate commission included Paul Appell and Henri Poincaré among its members, and was chaired by Félix Tisserand.

5. FOUNDATION OF BUCHAREST OBSERVATORY

As the three scientists returned home, they could hardly go on with the work so brilliantly commenced abroad because of the lack of an adequately equipped astronomical observatory.

As early as 1870, Petru S. Aurelian, an enthusiastic scholar and statesman, ventured to "ask the Government and the Legislative Bodies: how long it would still take the Romanians to provide such an important institution of research as those which already exist in other countries, whose national revenue was ten times lower than Romania's, or even smaller?". In 1873, he challenged the authorities again by stressing the necessity of building-up an astronomical observatory in Romania, and wondering "...which of the ministers will have the honour of being regarded as founder of the Bucharest astronomical observatory?" The same questions were resumed later by Ștefan. C. Hepites (Fig. 7), particularly in his notes to the Academy, as well as in his papers related to the history of astronomy (he distinguished himself as the author of the first history of Romanian astronomy).



Fig. 7 – Ștefan C. Hepites.

It was in 1888 that the Ministry of Agriculture succeeded to acquire the land of Constantin Bosianu, ex-councilor of the first Romanian Prince Alexandru Ioan Cuza and for short time prime minister. This land is situated on the Filaret Hill (Căminul de Argint Street) and was used for the construction of the Meteorological Institute and the Weights and Measures Centre (1889). Typical for the beginnings of any science in a country, in Romania meteorology was also at first mingled with geodesy and astronomical studies. The first astronomical requirement consisted in an accurate determination of the mean time. Precise time determinations were regularly carried out by B.V. Vermont in his investigations of Jupiter's satellites. The right time determined through this procedure used to be made known at noon by firing a gun that unfortunately could only be heard as far as the Law Court. However, soon after that, the telephone came to be used, which facilitated a more rapid announcement.

6. 1 APRIL 1908 – THE SET UP OF THE OBSERVATORY IN BUCHAREST

The need for a systematic and sustained astronomical research and proper equipment stepped up the lobby for the creation of an astronomical institute. Nicolae Coculescu was the only one whose endeavor was truly rewarded.

After he returned to Romania, a master of an advanced scientific insight, greatly enriched by the total solar eclipse expedition which he had joined in Senegal (1893), he was appointed professor at the Bucharest University and managed to win the cooperative support of another astronomy supporter, Spiru Haret, who was at that time the minister of Public Instruction and Religious Affairs. Therefore, Nicolae Coculescu came to be remembered as founder of the first modern astronomical observatory in Romania – one of the earliest scientific establishments of the country.

However the set up of the astronomical Observatory on 1 April 1908 (including a meteorological department until 1920) marked the beginning and not the end of an unceasing strive for the development of this science in our country. Thus, during the same year (November 1908), in a relevant letter published in *Astronomische Nachrichten*, Coculescu (Director of the Bucharest Observatory) pointed out: "Since the existing meteorological institute, set up 24 years ago, is located in a most suitable area, it has been decided that an astronomy department be created on the same site. We only have a transit telescope of a simple design for keeping time and a 108 mm refracting telescope. In view of the astronomical research currently pursued at the Observatory, we have decided to order a twin refracting telescope provided with a 30 cm lens and 6.70 m focal distance. The mechanical part will be achieved by P. Gautier and the lens by the G. & S. Merz Optics Institute. The design and construction of the building provided by an 11 m diameter dome have been entrusted to A. Engels, an architect of the Bruxelles (Uccle) Observatory". The staff of the facility was made up of only two astronomy students: A. Teodosiu and Maria Teohari. The latter can be considered the first Romanian woman astronomer, although there are previous records of observations made by two other women, namely Elena B. Vermont and Felicia Boerescu, on 4 February 1906, at -17.7 degrees Celsius!

In the same year when Bucharest Observatory was set up, Gheorghe Demetrescu (1885–1969; see Fig. 8), who was to become the founder of the Romanian astronomical research, was sent to Paris Observatory for a training period. There he concentrated on the study of variable star photometry, photographic determination of the precise position of minor planets and comets, solar physics related to the photospheric phenomena, lenses of astronomical instruments, interpretation of seismographic data.

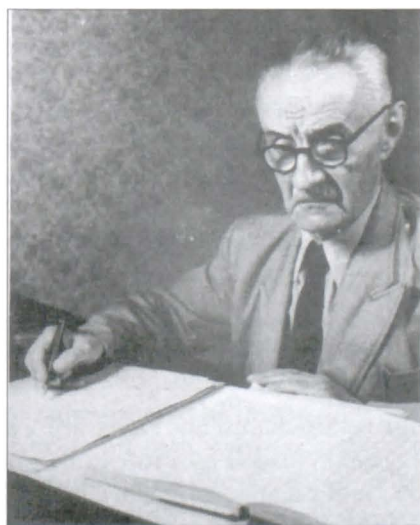


Fig. 8 – Gheorghe Demetrescu.

The instruments ordered by Nicolae Coculescu were installed after the main building was erected (1912). The first one was the refracting telescope, a Prin-Merz, mounted in 1912, tested in 1925, operational since 1930, and modified by Gheorghe Petrescu in 1935.

The Meridian circle Gauthier-Prin (mechanical part), Steinheil-Merz (optical part) was ordered in 1910, delivered in 1924, and installed in 1926 by Gheorghe Demetrescu.

7. OTHER OBSERVATORIES

Also in 1908, on the right bank of the river Nistru, at Dubăsarii Vechi, another astronomical observatory, this time a particular one, was created. It was headed for 32 years by Nicolae Donici (1884–1956?), one of the most remarkable personalities of Romanian and world astronomy (Fig. 9). His destiny was similar to that of the troubled lands where he was born. A founding member of the International Astronomical Union, an active participant in the first congresses of this union, an honorary member of the Romanian Academy, the author of important astrophysical works, the observer of no less than six total solar eclipses, the last years of his life are still surrounded by mystery, as there is no document left which should testify the date and place of his death.



Fig. 9 – Nicolae Donici.

After the Observatories in Bucharest and Dubăsarii Vechi, another observatory was soon set up in Jassy (1913), headed by Professor Constantin Popovici (1878–1956; see Fig. 10) until 1937 and yet another one in Cluj (1920), under the leadership of Gheorghe Bratu (1881–1941). The Observatory in Timișoara was built much later (1959) through the efforts of Professor Ioan Curea (1901–1977).



Fig. 10 – Constantin Popovici.

Constantin Popovici and his pupil and collaborator Vintilă Şiadbei (1898–1944) did their best to endow the Observatory in Jassy with a meridian refractor, a Ressel equatorial, two chronometers (for mean and sidereal time), a Graff photometer; all were largely deteriorated during WW2. Their work was continued by another outstanding astronomer, Victor Nadolschi (1911–1996), who was also the Director of the observatory between 1948 and 1966.

8. THE BUCHAREST OBSERVATORY UNTIL 1990

In 1922 Romania accepted formally the invitation to join the International Astronomical Union (IAU) set up in Brussels in 1918. Romania was represented in the first General Assembly of this organization (May 1922, Rome) by its delegate, Nicolae Donici, elected as a member of two IAU Commissions: for Solar Physics and Physical Observations on Planets, Comets and Satellites. Other Romanians became step by step members of the IAU and of its commissions. Romania is represented in IAU and, beginning with 1990, in the European Astronomical Society and other international organizations by the Romanian National Astronomical Committee (RNAC), set up in 1930. Its first president was Nicolae Coculescu. The topics of the first RNAC meeting, held on 4 April 1931, included the calendar and official time reform, as well as the participation of Bucharest Observatory in the worldwide campaign of longitude measurements to be initiated in 1933.

To prepare for this world campaign, two pendulum clocks (a sidereal Leroy-type and a mean time Riefler-type one) and a reception station for wireless telegraphy time signals were installed in the basement, under constant pressure and temperature conditions. The Romanian results were published in a volume edited by the IAU and

the International Geodesy and Geophysics Union (IGGU). They were regarded as some of the most accurate ascertainments.

During the next ten years the activities related to the programme for minor planets and comets concentrated on steadily obtaining accurate photographic positions. However, the magnitudes of the objects were limited to about 12.0. Bucharest Observatory largely contributed to the results acquired in this field. These results were published by the Astronomische Rechen-Institut (Berlin-Dahlem) and also appeared in journals such as *Astronomische Nachrichten*, or *Journal des Observateurs*.

To embark on the strenuous tasks of star cataloguing, the carrying out of investigations and data processing, as well as of theoretical studies, a new group was successively appointed during the years 1928–1930: Constantin Drâmbă (1907–1997; see Fig. 11), Gheorghe Petrescu (1905–1965), Călin Popovici (1910–1977; Fig. 12), Nicolae Dinulescu (1907–1989) and much later Ella Marcus (1909–1982; Fig. 13).



Fig. 11 – Constantin Drâmbă.



Fig. 12 – Călin Popovici.



Fig. 13 – Ella Marcus.

During 1937–1943 Constantin Popovici was appointed director of the Bucharest Observatory and worked at various studies on the effect of cosmic dust in the neighbourhood of stars, the effect of the solar radiation pressure upon planetary and cometary orbits and on equilibrium points of trajectories.

Another distinguished astronomer, Constantin Pârvulescu (1890–1945; Fig. 14) performed most original studies (also at the Bucharest Observatory) concerned with globular clusters. His work in this field was referred to in William M. Smart's book *Stellar Dynamics*.



Fig. 14 – Constantin Pârvulescu.

Following WW2, Romanian Astronomy was given a new impetus. All of the observatories were considerably provided with updated equipment.

Bucharest Observatory, which had been managed by the University until 1951, was subsequently taken over by the Academy and headed by Gheorghe Demetrescu (1943–1963), followed by Constantin Drămbă (1963–1977). In 1977 the Academy lost its institutions and the Observatory was integrated, along with other departments, into a Center of Astronomy and Space Research (headed by George Bebe Stănilă) within the Central Institute of Physics.

The closing in of the International Geophysical Year, which began in 1957, brought a series of new instruments: in 1952 a transit instrument (100/1000 mm), in 1957 a solar refractor (130/1950 mm) for visual and photographic observations of the solar photosphere in integral light. A H-alpha filter (6563 Å) of Halle-Lyot-Ohman type, mounted on a special refractor (80/1200 mm), is used for photographic observations of chromospheric formations (filaments, prominences, flares) and in 1964 a Cassegrain telescope (500/7500 mm) was bought for the photometry of variable stars.

The time service was first endowed with Belin, then with Rohde & Schwarz quartz clocks. However, the 80's brought a great recession, ranging from electric power or paper economies to the ceasing of the international relations, with the countries of the communist block included.

The publication of the Observatory's journal (*Studies and Researches in Astronomy and Seismology*, 1956–1962, then *Studies and Researches in Astronomy*) was stopped in 1974. The only publications which did not cease to be issued were those of tables, which did not raise any risk of interpretation, i.e., *Observations Solaires* and *Astronomical Yearbook*.

9. AFTER 1990

Immediately after the events of December 1989, Romanian astronomy, as well as the entire society, took an extraordinary turn. On 8 January the Institute's board of administration was changed and on 1 April the Astronomical Institute was set up under the aegis of the recently re-established Romanian Academy; the Institute was made up of three observatories: Bucharest, Cluj-Napoca, and Timișoara. It was headed first by Magda Stavinschi (1990–2005) and then by Vasile MIOC (beginning with 2005).

The Institute's journal was immediately set up again, under the name of *Romanian Astronomical Journal*, with two annual issues.

Collaborations extended to very many countries. Important international meetings were organized (or co-organized) by the Astronomical Institute. As some examples we mention: "CCD and Photometric Receivers Applied to the Observations of the Saturnian Satellites during the 1995–1996 opportunity" – PHESAT '95 (1994); NATO Advanced Research Workshop intended to prepare the 1999 total solar eclipse (1996); NATO Advanced Study Institute dedicated to the TSE '99 (1999); IAU International School for Young Astronomers (1999), International Seminar "Solar Researches in the South-Eastern European Countries: Present and Perspectives" (2001); Journées "Systèmes de référence spatio-temporels" (2002); IAU WG meeting "The Future Development of Ground-Based Astrometry" (2002); Conference "Professor Manev's Legacy: Contemporary Aspects of Astronomy, Theoretical and Gravitational Physics" (2004); Scientific Session "Astrometry with Small Telescopes" (2004); Second International Symposium on Space Climate "Long-Term Changes in the Sun and their Effects in the Heliosphere and Planet Earth" (2006); Conference "Fifty Years of Romanian Astrophysics" (2006), Workshop "Flows, Boundaries, Interactions" (2007), Young Scientists' International School "Heliosphere and Galaxy" (2007), and many others. The climax was the Conference "Exploring the Solar System and the Universe" (2008), occasioned by the celebration of the centenary of the Bucharest Observatory of the Astronomical Institute. This was the greatest astronomical conference ever organized in Romania.

Scientific and popularization books were published.

A remarkable event was the total solar eclipse of 11 August 1999, whose maximum was in Romania. On that occasion were organized the first NATO ARW before an eclipse and a NATO ASI right in the period when the eclipse took place (see above). The event was used also to obtain special funds from the government for the

consolidation and restoration of Bucharest Observatory buildings, and also for the construction of a special pavilion for a Planetarium (of 65 places and 8.5 m diameter), which is still without a projector.

Other important astronomical events involved the astronomers both in research programmes, as well as in those dedicated to astronomical education: Venus transit in 2004, mutual phenomena of the satellites of Jupiter, Saturn, or Uranus.

Romania was an important factor in the setting up of the South-Eastern European network, at first through the formation of the South-East Branch of European Astronomical Society, then through that of the Sub-Regional European Astronomical Committee – SREAC under the aegis of UNESCO-ROSTE and recently under that of UNESCO-BRESCE.

A special attention has been paid to astronomical education, especially in the conditions when the reduction of the school curricula led to the elimination of astronomy. At first it was the initiative of the resolution concerning the teaching of the astronomy (Sydney, Australia, 2003), Special sessions for Astronomy education in Europe during the JENAM meetings (starting in Budapest, 2003) were initiated, then the presidency of the IAU Commission 46 for 2006–2009.

Several young people, from the institute or from outside, obtained their doctoral degrees under the guidance of researchers of the institute. Some of them, at present, associated researchers of the Institute, work in important world research centers.

10. MAIN RESEARCH DIRECTIONS

Naturally, the scientific activity of Bucharest Observatory has continued in the first place the traditional one begun in 1908 and even before.

The presence of an instrument extremely good at the time, the meridian circle, has led to the development of a strong department of Meridian Astrometry.

In 1953 a collaboration agreement was concluded with the Russian colleagues on “The Set-Up of the Inertial Reference System of Stars”, a study concentrated on relating the positions and movements of stars to the distant extragalactic nebulae and the solar system.

Having gained experience in making up stellar catalogues, the Meridian Group was invited to bring its own contribution by drawing up a catalogue for the FKSZ main faint stars (645 stars) and a KSZ faint star (4000 stars) catalogues, The Romanian Academy highly appreciated the work and awarded the Meridian astrometry group – headed at that time by Ella Marcus – the prize “Gheorghe Lazăr” (1972). Many other catalogues followed.

In the recent times, the studies for the acquisition of stellar images continued with the elaboration of software for image processing by means of the new Apogee 47p CCD camera mounted on the 6000/380 Prin-Merz astrograph. The project of building an interface to be used for areas around extragalactic radio-sources was

accomplished. In order to include the accurate time coordinate in the computational process of CCD images, GPS time receivers were used. The studies concerning the reference stars to be used for extra-galactic radio-sources were continued. The Romanian contribution consists of the observation of the optical counterparts of ICRF sources and the elaboration of the intermediary reference catalogue.

Photographic astrometry was carried out many years. As far as the Solar system astrometry is concerned, the observations on Neptune and minor planets were completed. The computation program was build to improve orbital elements of the asteroids and an application of this program was finalized.

Lately, the cooperation with other observatories has been extended, especially with the Bulgarian colleagues, not only for the observation with other instruments, but also for the storing of the photographic plates gathered throughout the decades in Bucharest and Cluj-Napoca.

Together with Jean Kovalevsky, the working group "The Future Development of Ground-Based Astrometry" (2000–2006) was organized, replaced now by the IAU Division I WG "Astrometry by Small Ground-Based Telescopes".

Naturally, the beginnings of Romanian astronomy, marked by important theses and studies in the field of celestial mechanics, continued.

For several years, studies on terrestrial rotation were made. Under the supervision of Constantin Drămbă, the rotation of the Earth was studied within the more general framework of elastic deformations. Thus, on the basis of the Euler generalized equations, the existence of the Chandler ellipse described by the instantaneous pole of the Earth's rotation was theoretically established and so was the analytical expression of the Chandler period. Starting from the elasticity differential equations in relation to a system of rotating driven axes converted into global equations and applied to the Earth (small inertia products), the differential equations for the trajectory of the instantaneous rotation pole were determined.

The Time Department participated in the MERIT (Monitoring of the Earth Rotation and Intercomparison of the Techniques and Methods) international campaign, whose results actually led to the replacement of the classic ground-based techniques by modern space ones.

Starting from 1957, the studies about the motion of artificial Earth satellites (AES) gained a place of choice. Tracking stations were set up in Bucharest, Cluj, and Timișoara. Their observations were reported, along many years, to the data centers in Europe and USA. Parallely, theoretical studies of the AES motion under various perturbing factors were developed. The perturbing influences of such factors were tackled analytically, in various approximations.

Another field of choice was the motion of celestial bodies in post-Newtonian fields (relativistic or not). Many results were obtained in the two-body or the (general or restricted) three-body problems associated to the models of Einstein, Fock, Manev, Schwarzschild, Schwarzschild – de Sitter, zonal-satellite problem, etc. For most of

these models the qualitative methods of the theory of dynamical systems were used. This led to a general geometric characterization of all the orbits. This worked also in more general problems, as, for instance, Maxwell's $(n + 1)$ -body problem in Manev's or Schwarzschild's fields, or in photogravitational fields.

Lately an ever greater emphasis has been laid on studies of solar and stellar physics, as well as on extragalactic astronomy and cosmology.

As to solar physics, the research focused on the data analysis and interpretation using ground-based and space observations. The interest is directed to studying the active regions evolution and their implications in the chromospheric and coronal activity.

We search magnetic reconnections before and after flares or CMEs and the opening of the field lines during these events in 3D extrapolation of the coronal magnetic field from MDI magnetograms. We also study filaments and prominences activity in connection with their end in coronal mass ejections. The magnetic topologies during the evolution of an event reveal a coronal dynamics that allows us understand the solar active phenomena.

Another topic of our interest is the follow-up of a CMEs from the Sun to the interplanetary space. The halo CMEs from the solar source to the Earth's effects are also studied. The observational work is sustained by MHD 2D numerical simulations.

Our research framework belongs to the major scientific topics of the International Heliophysical Year.

Concerning the extragalactic astronomy and cosmology, at the Bucharest Observatory such studies begun in the early '80s, as a theoretical branch, directly related to the computational facilities available in our Observatory. Starting from a little Z8080 computer (early '80s) to a superscalar supercomputer of 44 processors (now), our cosmology team developed models, methods and techniques related to: the investigation of 2D and 3D catalogues of galaxies, clusters and superclusters; investigation of the log tails of the 2-points correlation functions; cosmological simulations (N-body + SPH) of the Large Scale Structure of the Universe (LSS); investigation of environmental effects in clusters of galaxies; application of neural methods in cosmology.

The use of such models and techniques allowed us to study problems related to: correlated signals in the long tail of the correlation functions for galaxies, clusters and superclusters (due to baryon oscillations); HD simulations of the LSS and of the evolution of the first and secondary Web structures; studies of the epochs of the formation of DM halos in a LCDM scenario (earlier than $z \sim 15$); studies of the evolution of halos and galaxies due to the parental merging phenomena; deceleration of the Butcher-Oemler and Oemler-Butcher effects in far or nearby clusters; studies of E+A galaxies; study of the synthetic spectra of galaxies and of the chemo-spectrophotometrical evolution of galaxies (for $z < 30$); photometric redshifts determination (for $z < 10$).

The observational study of variable stars and their theoretical modeling represents the scientific framework for the three groups from Bucharest, Cluj-Napoca and Timișoara. The main directions of investigation are focused on the observation and the light-curve analysis of eclipsing binary stars and Delta Scuti, RR Lyrae and Delta Cephei type stars, in order to determine their elements and evolutionary status. It was also built up a relevant data base for several types of variable stars, including close binaries, eclipsing binaries, interacting binary stars, late-type active stars, early-type O-B stars.

Other important directions of research are represented by the studies on stellar evolution, stellar pulsations, asteroseismology and searching for exoplanets, especially within the frame of the HELLAS and KASC Consortium and ESA/COROT, NASA/MOST, NASA/KEPLER space missions, in which we are actively involved. Important international collaborations in these fields were established with the Observatories of Paris-Meudon (France), Athens (Greece) and Belogradchik (Bulgaria).

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EVALUATION OF ASTROPHYSICALLY USEFUL PARAMETERS FOR LaO MOLECULE

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Abstract. Vibrational transition probabilities namely Franck-Condon (FC) factors and r -centroids for the band $C^2\Pi_{3/2}-A'^2\Delta_{5/2}$, $C^2\Pi_{1/2}-A'^2\Delta_{3/2}$, $C^2\Pi_{3/2}-X^2\Sigma'$, and $C^2\Pi_{1/2}-X^2\Sigma'$, systems of Lanthanum Oxide (LaO) molecule have been calculated. They are very closely related to transition probabilities, which have been evaluated by the more reliable numerical integration using a suitable potential. In each electronic state, wave functions are computed for all observed vibrational levels by numerical solution of the radial Schrödinger equation in which the potential energy function is calculated at equally spaced intervals of $0.01 \text{ \AA}$ in the range of nuclear separation range. The effect of high rotational quantum numbers on FC factors has been studied. The vibration-rotation interaction (VRI) on the FC factors of molecular band systems is negligible for relatively low rotational quantum numbers (J). Values of FC factors and r -centroids are given for all allowed bands of each system, and the available band wavelengths are also entered in the respective tables.

Key words: LaO, Franck-Condon factors, r -centroids.

1. INTRODUCTION

The molecular band spectroscopic methods for the determination of temperature of sources and for the study of physical processes occurring in various relative sources require an accurate theoretical knowledge of the corresponding potential energy curves and vibrational transition probabilities for the respective band heads in a band system. To a good approximation, the FC factors are proportional to these vibrational transition probabilities. The FC factor of a molecular band plays a controlling role in the determination of the relative band intensity. The theoretical prediction of intensity distribution in the spectra of many diatomic molecules, which are of interest in astrophysics, is necessary for an understanding of the physical-chemical conditions of the emitting sources. The knowledge of r -centroids is also useful in the discussion of the variation of the electronic transition moment with internuclear separation for each

vibrational band and for the study of other molecular properties. Consequently the transition probability parameters are required for diagnostic applications in astronomy, astrophysics and allied subjects.

The prominent bands of LaO were detected in the absorption spectrum of S-type stars by Keenan (1948) and Bidelman (1953). The presence of bands due to metal oxide LaO was reported in cool stars (an abundant molecule in O Ceti and X Cyg type stars) by Wöhl (1971). Metal oxide molecules were possibly detectable in sunspots on the basis of dissociation energies and metal abundances (Dirscherl et al., 1978). The estimates of relative abundances of these metallic species are important for understanding the evolutionary phases of the stars observed and are also the essential inputs in modeling the stellar atmospheres of the late type stars. Their relative abundances in interstellar medium give estimates of stellar activity like supernovae in the region observed. Similarly their abundance variations in cometary spectra can be used to study the interaction of solar wind with cometary material. Further, the relative abundances of these metallic species are useful in studying the cosmic recipe in these pristine objects.

As there is no report on the FC factor and r -centroid values for the band systems $C^2\Pi_{3/2} - A'^2\Delta_{5/2}$ and $C^2\Pi_{1/2} - A'^2\Delta_{3/2}$ of astrophysical molecule LaO in the literature, we have taken up the evaluation of reliable FC factor and r -centroid values for these systems. In the case of $C^2\Pi_{3/2} - X^2\Sigma^+$ and $C^2\Pi_{1/2} - X^2\Sigma^+$ systems of LaO molecule, Prahllad et al. (1985) reported a partial array of FC factors. Hence we have evaluated the FC factors and r -centroids for the experimentally observed vibrational levels using the molecular constants from the compilation of Huber and Herzberg (1979). FC factors are not only useful for calculating of individual values, and their band-by-band application, but are also useful in studies of radiative transfer in stellar and other astrophysical atmospheres which contain molecular species. As the molecule is of astrophysical interest, we have taken the present work to evaluate the values of FC factors and r -centroids for these band systems of the LaO molecule by the numerical integration method adopting a suitable potential.

2. FRANCK-CONDON FACTORS AND r -CENTROIDS

One of the parameters which control the intensity distribution in the emission of molecular bands is the FC factor, which is the square of the overlap integral between the upper state wave function and the lower state wave function:

$$q_{v',v''} = \left| \langle \psi_{v'} | \psi_{v''} \rangle \right|^2. \quad (1)$$

Here $\psi_{v'}$ and $\psi_{v''}$ are the vibrational wave functions for the upper and lower electronic states, respectively, between which the transition takes place.

The r -centroids $\bar{r}_{v',v''}$ represent the unique value of internuclear separation of a $v' \rightarrow v''$ transition in a diatomic molecular band system and are defined by

$$\bar{r}_{v',v''} = \langle \psi_{v'} | r | \psi_{v''} \rangle / \langle \psi_{v'} | \psi_{v''} \rangle. \quad (2)$$

For a proper understanding of the intensity distribution in the band systems of the molecules, it is necessary to choose a suitable potential. In this manner, Morse's (1929) potential function yields reliable FC factor and r -centroid values especially for a band in an electronic transition involving low vibrational quantum numbers (Sriramachandran et al., 2006). The computation of FC factor is made by Bates' (1949) method of numerical integration according to the detailed procedure provided by Rajamanickam et al (2001). Morse wave functions have been calculated at intervals of 0.01 Å for the range of r respectively from 1.63 Å to 2.11 Å, from 1.62 Å to 2.12 Å, from 1.59 Å to 2.16 Å and from 1.57 Å to 2.17 Å of observed vibrational band $C^2\Pi_{3/2} - A'^2\Delta_{5/2}$, $C^2\Pi_{1/2} - A'^2\Delta_{3/2}$, $C^2\Pi_{3/2} - X^2\Sigma^+$ and $C^2\Pi_{1/2} - X^2\Sigma^+$ systems of LaO molecule. Integrals in the equations (1) and (2) for the FC factors ($q_{v',v''}$) and r -centroids ($\bar{r}_{v',v''}$) are computed numerically and the results are tabulated in Tables 1 to 4 for the respective band systems of LaO molecule. The available wavelengths ($\lambda_{v',v''}$) data (Green 1971) are also included in the respective tables. The necessary molecular constants have been collected from the compilation of Huber and Herzberg (1979) and Carette (1990).

3. EFFECT OF ROTATIONAL QUANTUM NUMBERS ON FRANCK-CONDON FACTORS

The effect of vibration-rotation interaction (VRI) on the FC factors and band strengths of molecular band system is negligible for relatively low rotational quantum numbers (J). Shumaker (1969) obtained the expression for the minimum r_0 of an effective potential in the case of a vibrating rotator:

$$r_0 = r_e \left[1 + 4J(J+1)B_e^2 / \omega_e^2 \right], \quad (3)$$

where r_e , B_e , and ω_e are the usual spectroscopic constants. This is evident that the main effect of vibration-rotation interaction is the displacement of the minimum of the potential to larger r_0 values with increasing J . The FC factor, neglecting rotation, depends on the value of Δr_e , the difference of the minima of the two potentials for the two participating electronic states. For the vibrating rotator, the change in r_0 with J affects the FC factor and gives an idea of the effect of VRI on FC factor. Consequently, the value of $|\Delta r_0 - \Delta r_e|$ may serve as an indicator to represent the effect of VRI on FC factors. Therefore, the value of $|\Delta r_0 - \Delta r_e|$ is

$$|\Delta r_0 - \Delta r_e| = 4J(J+1) \left\{ \left| \frac{B_e'^2 r_e'^2}{\omega_e'^2} - \frac{B_e''^2 r_e''^2}{\omega_e''^2} \right| \right\}. \quad (4)$$

Murthy and Gowda (1977) have revealed that, the larger the value of $|\Delta r_0 - \Delta r_e|$ is, the greater the effect of VRI on FC factors will be.

4. RESULTS AND DISCUSSION

In the case of $C^2\Pi_{3/2} - X^2\Sigma^+$ and $C^2\Pi_{1/2} - X^2\Sigma^+$ band systems, the difference in the internuclear separation is very small (0.004 Å). It is seen from Tables 3 and 4 that the $\Delta v = 0$ sequence bands are most intense. Since only $\Delta v = 0$ sequence bands are experimentally observed, also the theoretical evaluation of these systems for the $\Delta v = \pm 1$, $\Delta v = 0 \pm 2$, etc. sequence band are not desirable, as indicated by the FC factors. Prahllad et al. (1985) reported the partial array of FC factors for these systems. Hence we have evaluated numerically FC factors and r -centroids for the experimentally observed vibrational levels.

The difference in internuclear separation for $C^2\Pi_{3/2} - A'^2\Delta_{5/2}$ and $C^2\Pi_{1/2} - A'^2\Delta_{3/2}$ transitions are small (0.019 Å). The magnitude of the $(q_{v'v''})$ values, as seen from Tables 1 and 2, indicate that the $\Delta v = 0$ sequence bands are most intense. These band systems should therefore be readily observable in sunspots and other astrophysical sources where the molecule is expected to be present under favorable physical conditions of temperature and abundance. The effective rotational temperature of $C^2\Pi_{1/2} - A'^2\Delta_{3/2}$ (0,0) of LaO molecule reported so far is 1897 K under suitable conditions (Sriramachandran et al., 2008). This report also discuss about the favorable conditions for the formation of LaO molecule in cool region of the umbral sunspots.

For a given r_e' and r_e'' , the r -centroid values depend only on the difference between v' and v'' , but not on their values taken individually. The sequence difference $\Delta \bar{r}_{v',v''} (= \bar{r}_{v'+1,v''+1} - \bar{r}_{v',v''})$ was found to be approximately constant (≈ 0.01 Å) for a given sequence, indicating that the potential curves are not wide (Kuzmanović et al., 2005). For the band systems $C^2\Pi_{3/2} - A'^2\Delta_{5/2}$, $C^2\Pi_{1/2} - A'^2\Delta_{3/2}$, $C^2\Pi_{3/2} - X^2\Sigma^+$, and $C^2\Pi_{1/2} - X^2\Sigma^+$ of LaO molecule, the sequence difference of r -centroid is found to be nearly a constant and is about 0.01 Å, which suggests that the potentials are not wide.

In the case of $C^2\Pi_{3/2} - X^2\Sigma^+$ and $C^2\Pi_{1/2} - X^2\Sigma^+$ band systems, since $r_e' > r_e''$, the presented data show that the r -centroid value has an increasing function of wavelength in each observed progression, which suggests that these bands are degraded to longer wavelength. For the $C^2\Pi_{3/2} - A'^2\Delta_{5/2}$ and $C^2\Pi_{1/2} - A'^2\Delta_{3/2}$ band systems,

since $r'_e < r''_e$, the shallower part of the excited state potential is probed, resulting in a sharper intensity distribution and hence the r -centroid value increases with decrease in wavelength, which is to be expected in violet-degraded band systems. These variations are of the expected nature, and the decrease or increase is found to be linear.

Generosa and Harris (1970) reported that the larger value of $|\Delta r_0 - \Delta r_e|$ is 27.60 mÅ ($J=100$) for the NO $B^2\Pi_r - X^2\Pi_r$ band system, and indicated an appreciable effect of high rotational quantum number on FC factors. Chakraborty et al. (1971) also reported that the relatively low value of $|\Delta r_0 - \Delta r_e|$ is 7.83 mÅ ($J=100$) for the N₂ $B^3\Pi_g - A^3\Sigma_u^+$ band system and shows a negligible effect on VRI. In the present study, from Table 5, the band systems $C^2\Pi_{3/2} - A'^2\Delta_{5/2}$, $C^2\Pi_{1/2} - A'^2\Delta_{3/2}$, $C^2\Pi_{3/2} - X^2\Sigma^+$, and $C^2\Pi_{1/2} - X^2\Sigma^+$ ($J=100$) of LaO have relatively low values of $|\Delta r_0 - \Delta r_e|$, and hence the effect of VRI here in the calculated FC factor values is negligible.

Table 1

FC factors, r -centroids for $C^2\Pi_{1/2} - A'^2\Delta_{3/2}$ of LaO molecule

		$v''=0$	$v''=1$	$v''=2$	$v''=3$	$v''=4$
$v'=0$	a)	0.945	0.050	0.005	0.001	*
	b)	1.843	1.669	1.650	1.604	
	c)	6607.76	—	—	—	
$v'=1$	a)	0.055	0.844	0.085	0.014	0.002
	b)	2.007	1.851	1.665	1.650	1.604
	c)	—	6597.05	—	—	—
$v'=2$	a)	*	0.111	0.727	0.146	0.014
	b)		2.009	1.857	1.699	1.573
	c)		—	6586.35	—	—
$v'=3$	a)	*	*	0.155	0.677	0.125
	b)			2.013	1.871	1.655
	c)			—	6575.71	—
$v'=4$	a)	*	*	*	0.199	0.608
	b)				2.017	1.883
	c)				—	6565.52
$v'=5$	a)	*	*	*	*	0.241
	b)					2.020
	c)					—
$v'=6$	a)	*	*	*	0.001	*
	b)				1.983	
	c)				—	
$v'=7$	a)	*	*	*	*	0.001
	b)					1.996
	c)					—

Table 2

FC factors, r -centroids for $C^2\Pi_{3/2} - A'^2\Delta_{5/2}$ of LaO molecule

		$v'' = 0$	$v'' = 1$	$v'' = 2$	$v'' = 3$	$v'' = 4$
$v' = 0$	a)	0.942	0.056	*	*	*
	b)	1.842	1.688			
	c)	6821.54	—			
$v' = 1$	a)	0.058	0.831	0.104	*	*
	b)	2.002	1.850	1.694		
	c)	—	6808.87	—		
$v' = 2$	a)	*	0.111	0.729	0.145	*
	b)		2.009	1.858	1.700	
	c)		—	6796.29	—	
$v' = 3$	a)	*	*	0.159	0.637	0.178
	b)			2.016	1.866	1.707
	c)			—	6783.71	—
$v' = 4$	a)	*	*	*	0.202	0.553
	b)				2.023	1.874
	c)				—	6771.16
$v' = 5$	a)	*	*	*	0.010	0.241
	b)				2.227	2.030
	c)				—	—
$v' = 6$	a)	*	*	*	*	0.014
	b)					2.235
	c)					—

Table 3

FC factors, r -centroids for $C^2\Pi_{3/2} - X^2\Sigma^+$ of LaO molecule

v', v''	$q_{v'v''}$ This work	$q_{v'v''}$ (Prahllad)	$\bar{r}_{v'v''}$ This work	$\bar{r}_{v'v''}$ (Prahllad)	$\lambda_{v'v''}$
0, 0	0.998	0.9975	1.831	1.8306	4371.87
1, 1	0.991	0.9923	1.838	1.8366	4375.71
2, 2	0.983	0.9869	1.845	1.8427	4379.61
3, 3	0.975	0.9813	1.852	1.8487	4383.53
4, 4	0.965	0.9755	1.860	1.8548	4387.49
5, 5	0.953	0.9695	1.867	1.8609	4391.47
6, 6	0.941	0.9633	1.875	1.8670	4395.69
7, 7	0.927		1.882		4400.05
8, 8	0.911		1.889		4404.78
9, 9	0.893		1.897		4409.64
10, 10	0.872		1.904		4414.65

Table 4

FC factors, r -centroids for $C^2\Pi_{1/2} - X^2\Sigma^+$ of LaO molecule

v', v''	$q_{v'v''}$ This work	$q_{v'v''}$ (Prahllad)	$\bar{r}_{v'v''}$ This work	$\bar{r}_{v'v''}$ (Prahllad)	$\lambda_{v'v''}$
0, 0	0.998	0.9982	1.830	1.8306	4418.13
1, 1	0.992	0.9924	1.838	1.8366	4423.06
2, 2	0.986	0.9865	1.845	1.8427	4427.99
3, 3	0.981	0.9803	1.852	1.8487	4432.95
4, 4	0.975	0.9738	1.859	1.8548	4437.89
5, 5	0.968	0.9661	1.866	1.8608	4442.88
6, 6	0.962	0.9592	1.873	1.8669	4447.92
7, 7	0.956		1.880		4453.06
8, 8	0.950		1.888		4458.36
9, 9	0.943		1.895		4464.06
10, 10	0.936		1.902		4471.20

In all the above tables the notation is: $q_{v'v''}$: FC factor; $\bar{r}_{v'v''}$: r -centroid (Å); $\lambda_{v'v''}$: wavelength (Å); *: $q_{v'v''} = 0$.

Table 5

Values of Δr_e and $|\Delta r_0 - \Delta r_e|$
for various band systems of LaO molecule

Band System	$\Delta r_e(\text{Å})$ $J = 0$	$ \Delta r_0 - \Delta r_e (10^{-3} \text{ Å})$	
		$J = 50$	$J = 100$
$C^2\Pi_{3/2} - A'^2\Delta_{5/2}$	0.019	0.1394	0.5520
$C^2\Pi_{1/2} - A'^2\Delta_{3/2}$	0.019	0.5453	2.1599
$C^2\Pi_{3/2} - X^2\Sigma^+$	0.004	0.1181	0.4676
$C^2\Pi_{1/2} - X^2\Sigma^+$	0.004	0.8579	3.3980

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ON THE STABILITY OF PLANETARY ORBITS WITH A MODIFIED HERMITE INTEGRATOR

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Abstract. We present the results of very-long-term numerical integrations of planetary orbital motions over 10^{10} yr time including all nine planets. A quick inspection of our numerical data shows that the planetary motion, at least in our simple dynamical model, seems to be quite stable even over this very long time-span. A closer look at the lowest-frequency oscillations, using a low-pass filter, shows us the potentially diffusive character of the terrestrial planets' motion, especially that of Mercury. However, there are no apparent secular increases of eccentricity or inclination in any orbital elements of the planets, which may be revealed by still longer-term numerical integrations. We have also performed a couple of trial integrations including the motions of the outer five planets over the duration of 5×10^{10} yr.

Key words: celestial mechanics – N -body problem – Solar System dynamics – stability.

1. INTRODUCTION

Planetesimal systems and planetary rings can be regarded as N -body systems. They evolve through mutual gravitational interactions and collisions between constituent bodies under the dominant Kepler potential of a central body. The question of the stability of our Solar System has been debated over several hundred years, since the era of Newton. The problem has attracted many famous mathematicians over the years and has played a central role in the development of non-linear dynamics and chaos theory. However, we do not yet have a definite answer to the question of whether our Solar system is stable or not. This is partly a result of the fact that the definition of the term “stability” is vague when it is used in relation to the problem of planetary motion in the Solar System. Actually, it is not easy to give a clear, rigorous and physically meaningful definition of the stability of our Solar System.

Among many definitions of stability, here we adopt the Hill definition (see, e.g., Gladman, 1993); actually, this is not a definition of stability, but of instability. We

define a system as becoming unstable when a close encounter occurs somewhere in the system, starting from a certain initial configuration (Chambers et al., 1996). A system is defined as experiencing a close encounter when two bodies approach one another within an area of the larger Hill radius. Otherwise the system is defined as being stable. Henceforward we state that our planetary system is dynamically stable if no close encounter happens during the age of our Solar System, about 5 Gyr. Incidentally, this definition may be replaced by one in which an occurrence of any orbital crossing between either of a pair of planets takes place. This is because we know from experience that an orbital crossing is very likely to lead to a close encounter in planetary and protoplanetary systems (Yoshinaga et al., 1999). Of course, this statement cannot be simply applied to systems with stable orbital resonances such as the Neptune-Pluto system.

1.1. PREVIOUS STUDIES AND AIMS OF THIS RESEARCH

In addition to the vagueness of the concept of stability, the planets in our Solar System show a motion character typical to dynamical chaos (Sussman and Wisdom 1988). The cause of this chaotic behavior is now partly understood as being a result of resonance overlapping (Murray and Holman, 1999; Lecar et al., 2001). However, it would require integrating over an ensemble of planetary systems including all nine planets for a period covering several 10 Gyr to thoroughly understand the long-term evolution of planetary orbits, since chaotic dynamical systems are characterized by their strong dependence on initial conditions.

From that point of view, many of the previous long-term numerical integrations included only the outer five planets (Sussman and Wisdom 1988; Kinoshita and Nakai 1996). This is because the orbital periods of the outer planets are so much longer than those of the inner four planets that it is much easier to follow the system for a given integration period.

The initial orbital elements and masses of planets are the same as those of our Solar System. Consequently, we found that the crossing time-scale of planetary orbits, which can be a typical indicator of the instability time-scale, is quite sensitive to the rate of mass decrease of the Sun. When the mass of the Sun is close to its present value, the Jovian planets remain stable over 10^{10} yr, or perhaps longer. Duncan and Lissauer (1998) also performed four similar experiments on the orbital motion of seven planets (Venus to Neptune), which cover a span of 10^9 yr. Their experiments on the seven planets are not comprehensive yet, but it seems that the terrestrial planets' orbits also remain stable during the integration period, maintaining almost regular oscillations.

On the other hand, in this accurate semi-analytical secular perturbation theory, we find that large and irregular variations can appear in the eccentricities and inclinations of the terrestrial planets, especially of Mercury and Mars, on a time-scale of several billion years.

In this paper we present preliminary results of six long-term numerical integrations on all nine planetary orbits, covering a span of several 10^9 yr, and of two other integrations covering a span of 5×10^{10} yr. One of the fundamental conclusions of our long-term integrations is that the Solar System planetary motion seems to be stable in terms of the Hill stability mentioned above, at least over a time span of 4 Gyr. Actually, in our numerical integrations the system was far more stable than what is defined by the Hill stability criterion: not only did no close encounter happen during the integration period, but also all the planetary orbital elements have been confined in a narrow region both in time and frequency domain, though planetary motions are stochastic. Since the purpose of this paper is to exhibit and overview the results of our long-term numerical integrations, we show typical example figures as evidence of the very long-term stability of Solar system planetary motion.

In Section 2 we briefly explain our dynamical model, numerical method, and initial conditions used in our integrations. Section 3 is devoted to a description of the quick results of the numerical integrations. Very-long-term stability of Solar System planetary motion is apparent in both planetary positions and orbital elements. A rough estimation of numerical errors is also given. Section 4 goes on to a discussion of the longest-term variation of planetary orbits using a low-pass filter and includes a discussion of angular momentum deficit. In Section 5, we present a set of numerical integrations for the outer five planets that spans 5×10^{10} yr. In Section 6 we also discuss the long-term stability of the planetary motion and its possible cause.

2. INITIAL CONDITIONS

We list our major six numerical integrations for all nine planets and two other integrations for the outer five planets in Table 1. We also show the mass of the planets and the Sun in Table 2. Note that our intention here is to show the very long-term stability of the outer Solar System as a typical example of point-mass dynamical systems, not to insist that the outer Solar system will really survive over this interval.

Table 1

Description of the long-term numerical integrations.

The upper are for all nine planets, and the last one are for the outer five planets (from Jupiter to Pluto). T denotes the length of integration

	Planets	T (years)	Remarks
N_{+1}	whole 9	5.0×10^9	
N_{+2}	whole 9	4.2×10^9	
N_{+3}	whole 9	3.9×10^9	
F_+	outer 5	5.0×10^{10}	Sun incorporates masses of inner planets

Table 2

The mass of the Sun M_S and the planetary masses (in solar mass units) used in the numerical integrations

Planet	Mass (in M_S)
m_1	1.652×10^{-7}
m_2	2.447×10^{-6}
m_3	3.003×10^{-6}
m_4	3.213×10^{-7}
m_5	9.544×10^{-4}
m_6	2.856×10^{-4}
m_7	4.384×10^{-5}
m_8	5.165×10^{-5}
m_9	6.006×10^{-9}
M_S	1.00

3. NUMERICAL METHOD

3.1. FORMULAE

We describe the time-symmetric Newmark method, and apply it to the integration of planetary orbits. The Newmark method formulae (Newmark 1959) take the form

$$x_1 = x_0 + v_0 \Delta t + \frac{1}{2} a_0 \Delta t^2 + \beta \dot{a}_0 \Delta t^3, \quad (1)$$

$$v_1 = v_0 + a_0 \Delta t + \gamma \dot{a}_0 \Delta t^2, \quad (2)$$

where x , v , and a are the position, velocity, and acceleration, respectively. Subscripts "0" and "1" indicate time, Δt is the time-step, and β and γ are the Newmark parameters. The time derivative of the acceleration a_0 is approximated as

$$\dot{a}_0 = \frac{a_1 - a_0}{\Delta t}. \quad (3)$$

In order to keep the time-symmetry, γ should be 1/2. The time-symmetric Newmark method formulae take the implicit form

$$x_1 = x_0 + \frac{1}{2} (v_1 + v_0) \Delta t + \left(\beta - \frac{1}{4} \right) (a_1 - a_0) \Delta t^2, \quad (4)$$

$$v_1 = v_0 + \frac{1}{2} (a_1 + a_0) \Delta t, \quad (5)$$

The $\beta = 0$ scheme corresponds to the leapfrog scheme, while for $\beta = \frac{1}{4}$ corresponds to the trapezoidal formula.

The accuracy of the Newmark method is second-order, as can be seen in equation (2). This means that the last term of the r.h.s. of equation (1) does not affect the order of accuracy. In other words, it is a free parameter. We can vary its value without changing the order of accuracy or losing the time-symmetry.

3.2. DERIVATION OF THE OPTIMAL β

Fig. 1 shows the error in the osculating orbital elements for the two-body Kepler problem solved by the time-symmetric Newmark method with $\beta = 0$, $\beta = 1/12$, and $\beta = 1/6$ for 10 orbital periods (20π time units). Time-symmetry is realized by applying three iterations of corrections. In other words, we use the $P(EC)^3$ scheme. The initial orbital elements are $a = 1$, $e = 0.1$, $\omega = \pi$, and $T = \pi$. The argument of pericenter is measured from the x -axis. The orbit is integrated with a constant time-step of $\Delta t = 2^{-5}$, in other words, about 250 integration steps per orbit. There are no secular errors for a and e , but periodic changes for all values of β . In time-symmetric integrators with a constant time-step, the periodic errors in a and e cancel in an orbit.

For ω , the result for $\beta = 1/12$ shows no secular error for this timescale, while those for $\beta = 0$ and $\beta = 1/6$ show linear drifts.

Fig. 2 shows the result of time integration in the same way as Fig. 1, but for 10^6 orbital periods with $\beta = 1/12$. In Fig. 2, $\Delta\omega$ is visible. However, the error is smaller than those by the $\beta = 0$ and $\beta = 1/6$ schemes. For $\beta = 0$ and $\beta = 1/6$, $|\Delta\omega|$ per orbit is of order 10^{-3} , while for $\beta = 1/12$ it is of order 10^{-7} . This magic coefficient, $\beta = 1/12$, can be derived analytically through evaluation of the leading term of the error.

The leading terms of the local truncation errors of the time-symmetric Newmark method are given by:

$$\Delta x = \left(\beta - \frac{1}{6} \right) \dot{a}_0 \Delta t^3, \quad (6)$$

$$\Delta v = \frac{1}{12} a_0^{(2)} \Delta t^3. \quad (7)$$

Because the eccentricity has no secular error, it is sufficient to evaluate the error of one component of the eccentricity vector. Here, we take the y -component of the eccentricity vector

$$e_y = e \sin \omega,$$

which is given by

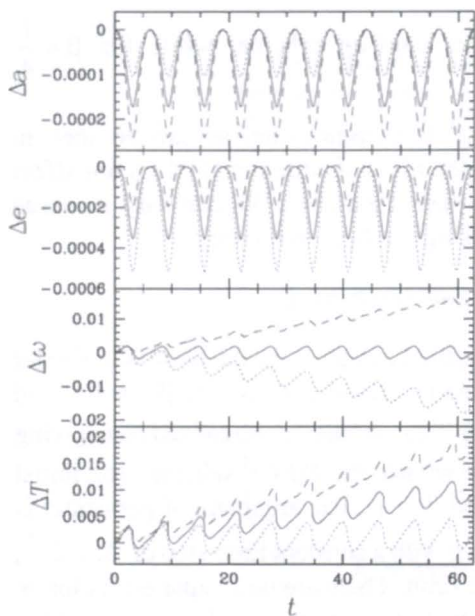


Fig. 1 – Errors of orbital elements a , e , ω , and T from top to bottom for the time-symmetric Newmark method with $\beta = 0$ (dotted curve), $\beta = 1/12$ (solid curve), and $\beta = 1/6$ (dashed curve) against time for 10 orbital periods. The Kepler period is 2π and the time-step is $\Delta t = 2^{-5}$.

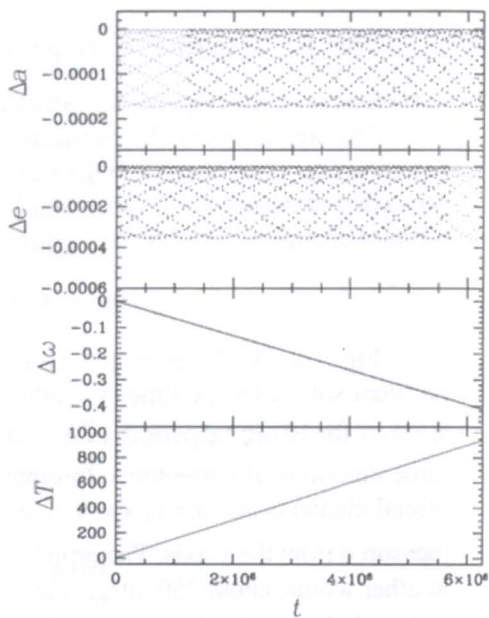


Fig. 2 – Same as Fig. 1, but for $\beta = 1/12$ for 10^6 orbital periods.

$$e_y = -v_x(xv_y - yv_x) - \frac{y}{r}, \quad (8)$$

where

$$r = \sqrt{x^2 + y^2}.$$

The leading term of the error of e_y is given by

$$\Delta e_y = \frac{\partial e_x}{\partial x} \Delta x + \frac{\partial e_y}{\partial y} \Delta y + \frac{\partial e_y}{\partial v_x} \Delta v_x + \frac{\partial e_y}{\partial v_y} \Delta v_y. \quad (9)$$

This leading term should cancel in an orbital period. Moreover, the symmetry of the Keplerian orbit requires that it should cancel in half an orbital period, namely

$$\int_0^{T_K/2} \Delta e_y dt = 0. \quad (10)$$

where T_K is the Keplerian period. Using equations (6)–(9), the above condition can be written as

$$\Delta t^3 \int_0^{T_k/2} \left[\left(\beta - \frac{1}{6} \right) \left(\frac{\partial e_y}{\partial x} \dot{a}_x + \frac{\partial e_y}{\partial y} \dot{a}_y \right) + \frac{1}{12} \left(\frac{\partial e_y}{\partial v_x} a_x^{(2)} + \frac{\partial e_y}{\partial v_y} a_y^{(2)} \right) \right] dt = 0. \quad (11)$$

If β satisfies this linear equation, the leading term of $\Delta\omega$ vanishes. We can see in equation (11) that β is not a function of Δt . Equation (11) leads to

$$\beta = -\frac{1}{12} \frac{\int_0^{T_k/2} \left(\frac{\partial e_y}{\partial v_x} a_x^{(2)} + \frac{\partial e_y}{\partial v_y} a_y^{(2)} \right) dt}{\int_0^{T_k/2} \left(\frac{\partial e_y}{\partial x} \dot{a}_x + \frac{\partial e_y}{\partial y} \dot{a}_y \right) dt}. \quad (12)$$

Substituting Keplerian solutions into equation (12) and integrating for half an orbit, we have

$$\beta = 1/12. \quad (13)$$

3.3. THE HERMITE INTEGRATOR

The time-symmetric Newmark method can be extended to higher order integrators. As example, we apply it to an implicit Hermite integrator, a fourth-order time-symmetric integrator. The Hermite integrator is based on the Taylor series up to the order of the third time-derivative of the acceleration, $a_0^{(3)}$, given by

$$x_1 = x_0 + v_0 \Delta t + \frac{a_0}{2} \Delta t^2 + \frac{\dot{a}_0}{6} \Delta t^3. \quad (14)$$

3.4. TOTAL ENERGY ERROR ESTIMATION

According to one of the basic properties of integrators, which conserve the physically conservative quantities well (total orbital energy and angular momentum), our long-term numerical integrations seem to have been performed with very small errors. The averaged relative errors of total energy (10^{-9}) have remained nearly constant throughout the integration period (Fig. 1). The special startup procedure, warm start, would have reduced the averaged relative error in total energy by about one order of magnitude or more.

Note that different operating systems, different mathematical libraries, and different hardware architectures result in different numerical errors, through the variations in round-off error handling and numerical algorithms. In the upper panel of Fig. 1, we can recognize this situation in the secular numerical error in the total angular momentum, which should be rigorously preserved up to machine-epsilon precision.

4. NUMERICAL RESULTS

In this section we briefly review the long-term stability of planetary orbital motion through some snapshots of raw numerical data. The orbital motion of planets indicates long-term stability in all of our numerical integrations: neither orbital crossings nor close encounters between any pair of planets took place.

First, we briefly look at the general character of the long-term stability of planetary orbits. Our interest here focuses particularly on the inner four terrestrial planets for which the orbital time-scales are much shorter than those of the outer five planets. As we can see clearly from the planar orbital configurations shown in Fig. 3 and Fig.4, orbital positions

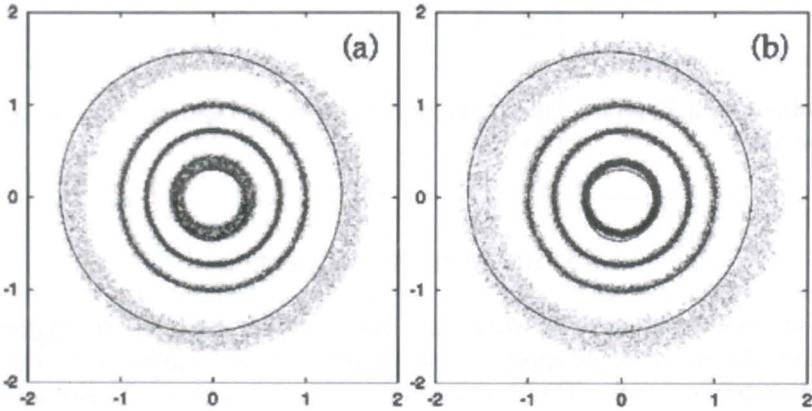


Fig. 3 – Vertical view of the four inner planetary orbits (from the z -axis direction) at the initial and final parts of the integrations N_{+1} . The unit of the axes is AU. The xy -plane is set to the invariant plane of Solar system total angular momentum. (a) Initial part of N_{+1} ($t = 0$ to 0.05×10^9 yr). (b) Final part of N_{+1} ($t = 4.95 \times 10^8$ to 4.98×10^9 yr). Solid lines denote the present orbits of the four terrestrial planets.

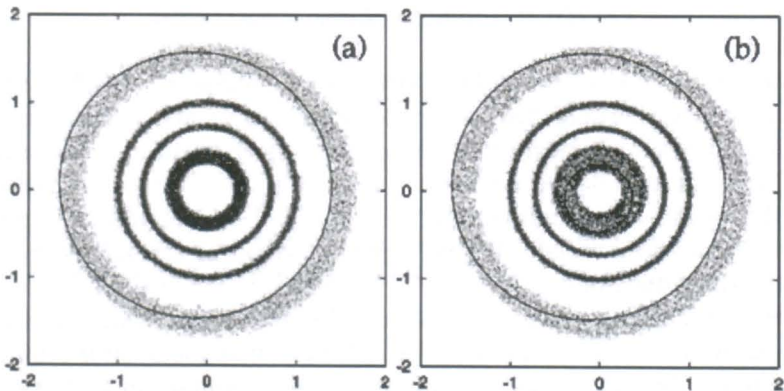


Fig. 4 – Same as in Fig. 3, but for N_{+2} . (a) Initial part of N_{+2} ($t = 0$ to 0.05×10^9 yr). (b) Final part of N_{+2} ($t = 4.98 \times 10^8$ to 5.03×10^9 yr).

of the terrestrial planets differ little between the initial and final part of each numerical integration, which spans several Gyr. The solid lines denoting the present orbits of the planets lie almost within the swarm of dots even in the final part of integrations. This indicates that throughout the entire integration period the almost regular variations of planetary orbital motion remain nearly the same as they are at present.

5. INTEGRATIONS OF OUTER PLANETARY ORBITS

Since the Jovian planets' masses are much larger than the terrestrial ones, we treat the Jovian planetary system as an independent system to study its dynamical stability. We added a couple of trial integrations that span 5×10^{10} yr, including only the outer five planets (the four Jovian planets plus Pluto). The results exhibit the rigorous stability of the outer planetary system over this time-span. Orbital configurations (Fig. 5), and variation of eccentricities and inclinations, show its very-long-term stability.

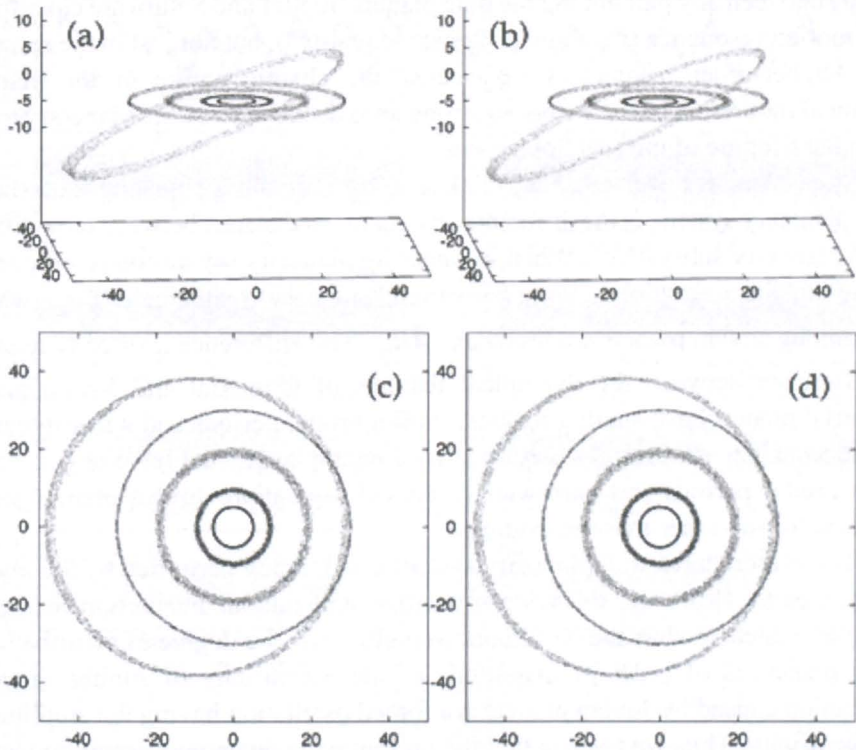


Fig. 5 – Oblique and vertical views of the outer five planetary orbits in the final parts of the integration F_+ . Unit of axes is AU. (a), (b): Final part of F_+ ($t = -4.819 \times 10^{10}$ to -5.000×10^{10} yr). (c), (d): Final part of F_+ ($t = 4.819 \times 10^{10}$ to 5.000×10^{10} yr). In each panel a total of 30,000 points are plotted with an interval of about 10^6 years over 2×10^9 yr.

6. DISCUSSION OF SOLAR SYSTEM STABILITY AND CONCLUSIONS

In Fig. 6 above we show eccentricities and inclinations of the five outer planetary orbits in the final parts of the integrations F_+ . We plotted the evolution of: (a)–(e) the eccentricities of Jupiter, Saturn, Uranus, Neptune and Pluto in the final part of the F_+ integration; (f)–(j) the inclinations of Jupiter, Saturn, Uranus, Neptune and Pluto in the final part of the F_+ integration; (k)–(o) the eccentricities of Jupiter, Saturn, Uranus, Neptune and Pluto in the final part of the F_+ integration; (p)–(t) the inclinations of Jupiter, Saturn, Uranus, Neptune and Pluto in the final part of the F_+ integration. The integration was performed in both the past ((a)–(j)) and the future ((k)–(t)) with respect to the initial instant.

What kind of dynamical mechanism maintains this long-term stability of the planetary system? We can immediately think of two major features that may be responsible for the long-term stability.

First, there seem to be no significant lower-order resonances (mean motion and secular) between any pair among the nine planets. Jupiter and Saturn are close to a 5:2 mean motion resonance (the famous “great inequality”), but not just in the resonance zone. Higher-order resonances may cause the chaotic nature of the planetary dynamical motion, but they are not so strong as to destroy the stable planetary motion within the lifetime of the real Solar System.

The second feature, which we think is more important for the long-term stability of our planetary system, is the difference in dynamical distance between terrestrial and Jovian planetary subsystems. When we measure planetary separations by the mutual Hill radii (R_H), separations among terrestrial planets are greater than $26R_H$, whereas those among Jovian planets are less than $14R_H$. This difference is directly related to the difference between the dynamical features of terrestrial and Jovian planets. Terrestrial planets have smaller masses, shorter orbital periods and wider dynamical separation. They are strongly perturbed by Jovian planets that have larger masses, longer orbital periods and narrower dynamical separation. Jovian planets are not perturbed by any other massive bodies.

The present terrestrial planetary system is still being perturbed by the massive Jovian planets. However, the wide separation and mutual interaction among the terrestrial planets renders the perturbations ineffective; the degree of perturbation by Jovian planets is of order of magnitude of the eccentricity of Jupiter, since the perturbation caused by Jovian planets is a forced oscillation having the amplitude of the eccentricity. Thus we assume that the present wide dynamical separation among terrestrial planets ($> 26R_H$) is probably one of the most significant conditions for maintaining the stability of the planetary system over a 10^{10} time. Our detailed

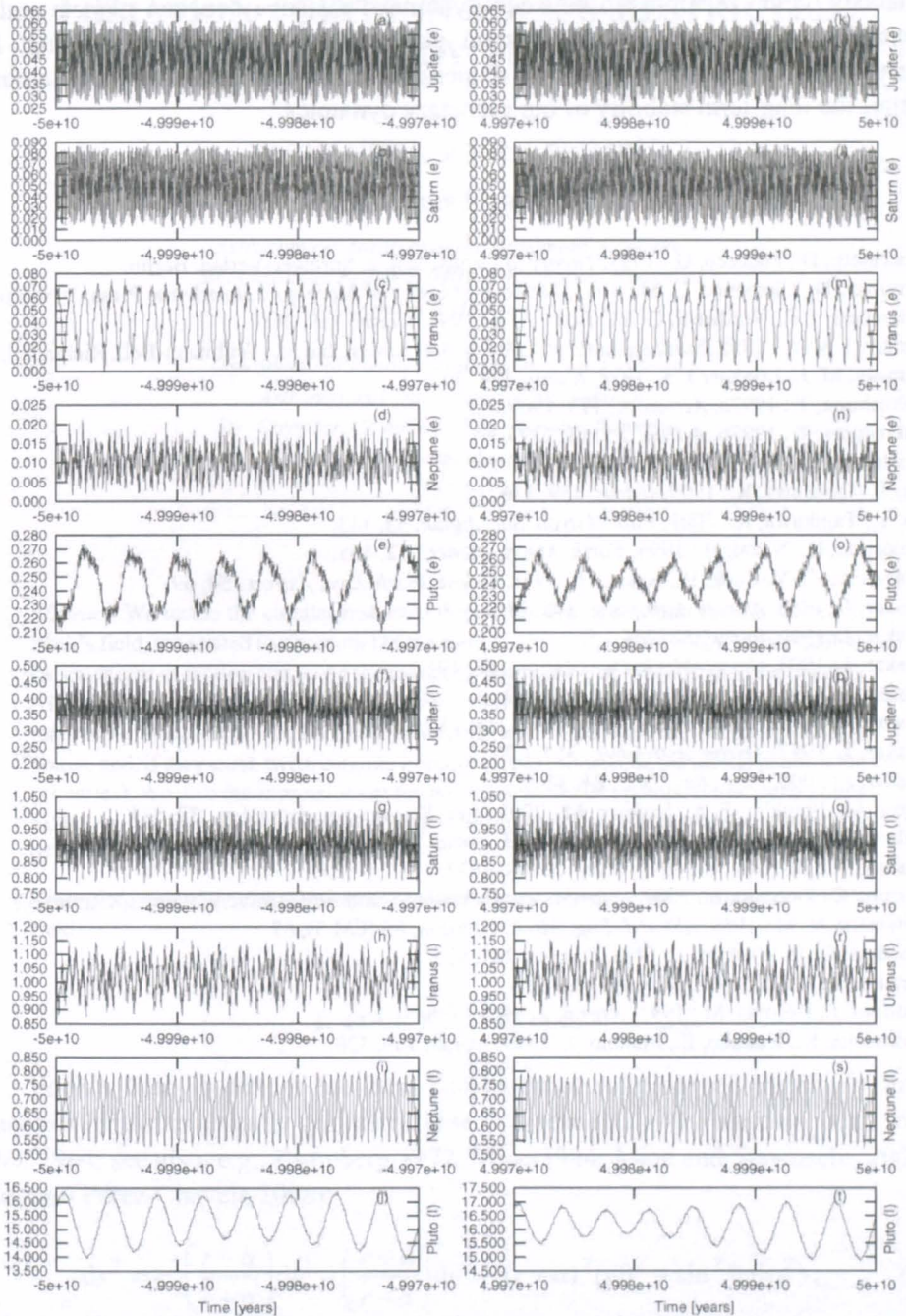


Fig. 6 – Eccentricities and inclinations of the five outer planetary orbits in the final parts of the integrations F_+ .

analysis of the relationship between dynamical distance between planets and the instability time-scale of Solar System planetary motion is now on-going. It is necessary to perform more and more numerical integrations to confirm and examine in detail the long-term stability of our planetary dynamics.

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LIBRATION POINTS IN FOCK'S RESTRICTED THREE-BODY PROBLEM

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Abstract. We tackle the circular restricted three-body problem within the framework of Fock's field, associated to a potential of the form $A_1/r + A_2/r^2 + A_3/r^3 + A_4/r^4$. We establish the equations of motion and prove the existence of an analog of the Jacobian first integral. After emphasizing some features of the zero-relative-velocity surfaces, we approach the main topic: libration points. As in the Newtonian case, there are three collinear libration points, and, if they exist, two triangular libration points (situated in the orbital plane of the primaries). We give the expressions of the positions of all these points. If triangular points exist, they may not form equilateral triangles; the triangles are isosceles for equal masses of the primaries, and scalene else.

Key words: celestial mechanics – restricted three-body problem – libration points – Fock's field.

1. FOCK'S POTENTIAL

Fock's field is one of the best known exact solutions of Einstein's field equations for the spherically symmetric case. It is featured by the metric (Fock 1959, 1960, 1964; see also, e.g., Brumberg 1972; Mioc 1994; Mioc and Stavinschi, 2000; Mioc and Pérez-Chavela 2008):

$$ds^2 = c^2 \left(\frac{r-\alpha}{r+\alpha} \right) dt^2 - \left(\frac{r+\alpha}{r-\alpha} \right) dr^2 - (r+\alpha)^2 (d\theta^2 + \sin^2 \theta d\varphi^2), \quad (1)$$

in which t = temporal coordinate, (r, θ, φ) = spherical spatial coordinates, c = speed of light, $\alpha = \frac{\mu}{c^2}$. Here $\mu = GM$, where G = Newtonian gravitational constant and M = mass; μ represents the gravitational parameter of the field-generating body.

Using a standard canonical formalism, the metric (1) provides a Binet-type equation:

$$\frac{d^2\left(\frac{1}{r}\right)}{d\varphi^2} + \frac{1}{r} = \frac{\alpha c^2(2E^2 - 1)}{L^2} + 6\left(\frac{\alpha c E}{L}\right)^2 \frac{1}{r} + \left(\frac{3\alpha^3 c^2(2E^2 + 1)}{L^2}\right) \frac{1}{r^2} + 2\alpha^2 \left(\frac{\alpha^2 c^2(2E^2 + 1)}{L^2} + 1\right) \frac{1}{r^3}.$$

Here the constants L and E are, respectively, given by

$$L = (r + \alpha)^2 \frac{d\varphi}{d\tau},$$

and

$$E = (r^2 - \alpha^2) \frac{dt}{d\tau},$$

where τ denotes the proper time of the particle. L can be interpreted as the constant of angular momentum (per unit mass), and E is connected with the total energy χ (per unit mass) of the particle with the relation: $E = 1 + \frac{\chi}{c^2}$. The force (per unit mass) corresponding to the Binet-type equation above has the expression:

$$F(r) = -\frac{\mu}{r^2} - \frac{2\mu(E^2 - 1)}{r^2} - \frac{6\mu E^2}{c^2 r^3} - \frac{3\mu^3(2E^2 + 1)}{c^2 r^4} - \frac{2\mu^3 \left[\frac{L^2}{\mu} + \frac{\mu(E^2 + 1)}{c^2} \right]}{c^4 r^5}. \quad (2)$$

Since this force is conservative, it can be considered as deriving from a potential function that reads:

$$U = \sum_{n=1}^4 \frac{A_n}{r^n},$$

where r denotes the distance between particles, while the expressions of the positive parameters A_n , $n = \overline{1,4}$, are

$$A_1 = \mu \left(1 + \frac{4\chi}{c^2} + \frac{2\chi^2}{c^4} \right),$$

$$A_2 = \frac{3\mu}{c^2} \left(1 + \frac{\chi}{c^2} \right)^2,$$

$$A_3 = \frac{\mu^2}{c^4} \left(3 + \frac{4\chi}{c^2} + \frac{2\chi^2}{c^4} \right),$$

$$A_4 = \frac{\mu^2}{2c^4} \left[\frac{L^2}{\mu} + \frac{\mu}{c^2} \left(2 + \frac{2\chi}{c^2} + \frac{\chi^2}{c^4} \right) \right].$$

2. EQUATIONS OF MOTION IN FOCK'S FIELD

Consider the circular restricted three-body problem, with primaries of masses m_1 and m_2 , in Fock's gravitational field (see Fig. 1).

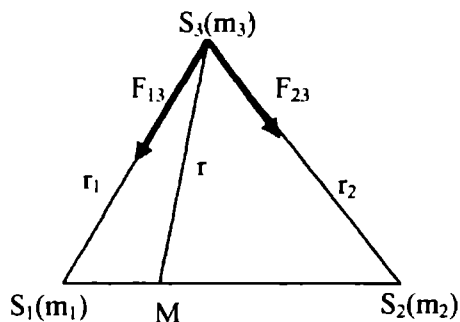


Fig. 1 – The gravitational forces acting on the test particle.

The gravitational forces derive from the potentials:

$$U_1 = \frac{a_1}{r_1} + \frac{a_2}{r_1^2} + \frac{a_3}{r_1^3} + \frac{a_4}{r_1^4}, \quad a_{1,2,3,4} > 0;$$

$$U_2 = \frac{b_1}{r_2} + \frac{b_2}{r_2^2} + \frac{b_3}{r_2^3} + \frac{b_4}{r_2^4}, \quad b_{1,2,3,4} > 0.$$

It is clear that $a_n, n = \overline{1,4}$, and $b_n, n = \overline{1,4}$, have the expressions of $A_n, n = \overline{1,4}$, corresponding to the primaries m_1 and m_2 , respectively.

Adopting, as usual, a rotating frame (with Keplerian angular velocity ω_k) in the motion plane of the primaries, originated in the common mass centre of m_1 and m_2 , the scalar equations of motion of the infinitesimal mass read:

$$\begin{cases} \frac{d^2 X}{dt^2} - 2\omega_k \frac{dY}{dt} = \omega_k^2 X - \frac{1}{m_3} \left(\frac{a_1}{r_1^3} + \frac{2a_2}{r_1^4} + \frac{3a_3}{r_1^5} + \frac{4a_4}{r_1^6} \right) (X + R_1) - \\ \quad - \frac{1}{m_3} \left(\frac{b_1}{r_2^3} + \frac{2b_2}{r_2^4} + \frac{3b_3}{r_2^5} + \frac{4b_4}{r_2^6} \right) (X - R_2), \\ \frac{d^2 Y}{dt^2} + 2\omega_k \frac{dX}{dt} = \omega_k^2 Y - \frac{1}{m_3} \left(\frac{a_1}{r_1^3} + \frac{2a_2}{r_1^4} + \frac{3a_3}{r_1^5} + \frac{4a_4}{r_1^6} \right) Y - \\ \quad - \frac{1}{m_3} \left(\frac{b_1}{r_2^3} + \frac{2b_2}{r_2^4} + \frac{3b_3}{r_2^5} + \frac{4b_4}{r_2^6} \right) Y, \\ \frac{d^2 Z}{dt^2} = -\frac{1}{m_3} \left(\frac{a_1}{r_1^3} + \frac{2a_2}{r_1^4} + \frac{3a_3}{r_1^5} + \frac{4a_4}{r_1^6} \right) Z - \frac{1}{m_3} \left(\frac{b_1}{r_2^3} + \frac{2b_2}{r_2^4} + \frac{3b_3}{r_2^5} + \frac{4b_4}{r_2^6} \right) Z. \end{cases} \quad (3)$$

where (X, Y, Z) are the spatial coordinates in the above specified frame, (r_1, r_2) are the distances of the infinitesimal mass from (m_1, m_2) , $R_1 = S_1M$, $R_2 = S_2M$. Here

$$\begin{aligned} r_1 &= \sqrt{(X + R_1)^2 + Y^2 + Z^2}, \\ r_2 &= \sqrt{(X - R_2)^2 + Y^2 + Z^2}. \end{aligned} \quad (4)$$

Theorem 2.1. *Fock's circular restricted three-body problem admits an analog of the Jacobian first integral.*

Proof. Considering the function $\Psi: \mathbf{R} \rightarrow \mathbf{R}$,

$$\begin{aligned} \Psi(X, Y, Z) &= \frac{1}{2} \omega_k^2 (X^2 + Y^2) + \frac{1}{m_3} \left(\frac{a_1}{r_1} + \frac{a_2}{r_1^2} + \frac{a_3}{r_1^3} + \frac{a_4}{r_1^4} \right) + \\ &+ \frac{1}{m_3} \left(\frac{b_1}{r_2} + \frac{b_2}{r_2^2} + \frac{b_3}{r_2^3} + \frac{b_4}{r_2^4} \right), \end{aligned} \quad (5)$$

multiplying equations (3) by $\dot{X}$, $\dot{Y}$, $\dot{Z}$, respectively, adding the resulting expressions together, and integrating with respect to time, we find

$$\frac{V^2}{2} = \Psi(X, Y, Z) - \text{const.},$$

where $V = \sqrt{\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2}$ stands for the velocity of the test particle. In this way we got an analog of the Jacobian integral for the Fock circular restricted three-body problem. $\square$

Theorem 2.2. *In the circular restricted three-body problem in Fock's field, the zero relative velocity surfaces are equipotential surfaces, too.*

Proof. For $V = 0$ it results $\Psi(X, Y, Z) = \text{const.}$ $\square$

The equipotential surface's equation is:

$$\frac{\omega_k^2}{2}(X^2 + Y^2) + \frac{1}{m_3} \left(\frac{a_1}{r_1} + \frac{a_2}{r_1^2} + \frac{a_3}{r_1^3} + \frac{a_4}{r_1^4} \right) + \frac{1}{m_3} \left(\frac{b_1}{r_2} + \frac{b_2}{r_2^2} + \frac{b_3}{r_2^3} + \frac{b_4}{r_2^4} \right) = \text{const.} \quad (6)$$

3. LIBRATION POINTS

Let us now pass to the S_1xyz coordinate system, originated in the mass m_1 , via the transformations:

$$\begin{aligned} x &= R_1 + X, \\ y &= Y, \\ z &= Z, \end{aligned} \quad (7)$$

under which we have

$$\begin{aligned} r_1 &= \sqrt{x^2 + y^2 + z^2}, \\ r_2 &= \sqrt{(x - R_1 - R_2)^2 + y^2 + z^2}. \end{aligned} \quad (8)$$

This coordinate system will be useful below for a better positioning of the libration points. Next, we choose the following system of units: $R_1 + R_2 = 1$ as unit of length, $m_1 + m_2 = 1$ as unit of mass, ω_k^{-1} as unit of time. Therefore we have $G = 1$ and $P = 2\pi$, where P stands for the orbital period. This system of units leads to

$$\begin{aligned} m_1 &= 1/(1+q), \\ m_2 &= q/(1+q), \\ q &= m_2/m_1; \\ r_1 &= q/(1+q), \\ r_2 &= 1/(1+q). \end{aligned}$$

These expressions make (6) become

$$\begin{aligned} \frac{1}{2}(x^2 + y^2) - \frac{q}{1+q}x + \frac{1}{2}\left(\frac{q}{1+q}\right)^2 + \frac{\tilde{A}_1}{1+q}\frac{1}{r_1} + \frac{\tilde{A}_2}{1+q}\frac{1}{r_1^2} + \frac{\tilde{A}_3}{(1+q)^2}\frac{1}{r_1^3} + \\ + \left[\frac{\tilde{D}_1}{1+q} + \frac{\tilde{N}_1}{(1+q)^3}\right]\frac{1}{r_1^4} + \frac{q\tilde{B}_1}{1+q}\frac{1}{r_2} + \frac{q\tilde{B}_2}{1+q}\frac{1}{r_2^2} + \frac{q^2\tilde{B}_3}{(1+q)^2}\frac{1}{r_2^3} + \\ + \left[\frac{q\tilde{D}_2}{1+q} + \frac{q^3\tilde{N}_2}{(1+q)^3}\right]\frac{1}{r_2^4} = C . \end{aligned} \quad (9)$$

where $\tilde{A}_i$, $\tilde{B}_i$, $\tilde{D}_j$, $\tilde{N}_j$ ($i = \overline{1,3}$ and $j = \overline{1,2}$) are obtained via transformation of a_i , b_i in the new system of units.

Now we are in the position to tackle the main topic of our note: the libration points and their location. Recall that they are double points situated on the zero-relative-velocity surfaces, their position being determined by the condition

$$\frac{\partial\psi}{\partial x} = \frac{\partial\psi}{\partial y} = \frac{\partial\psi}{\partial z} = 0 .$$

Theorem 3.1. *In the circular restricted three-body problem, in Fock's gravitational field, the geometrical properties of the equipotential surfaces are not determined only by the mass ratio q , as in the corresponding problem in the Newtonian field.*

Proof. The expressions of coefficients marked with tilde contain the speed of light c , which depends on the orbital period and on the distance between the primaries (because of the system of units). $\square$

Theorem 3.2. *In Fock's circular restricted three-body problem, the libration points are located in the orbital plane of the primaries.*

Proof. Equation $\frac{\partial\psi}{\partial z} = 0$ leads straightforwardly to

$$\begin{aligned} -z \left\{ \frac{\tilde{A}_1}{1+q}\frac{1}{r_1^3} + \frac{2\tilde{A}_2}{1+q}\frac{1}{r_1^4} + \frac{3\tilde{A}_3}{(1+q)^2}\frac{1}{r_1^5} + 4\left[\frac{\tilde{D}_1}{1+q} + \frac{\tilde{N}_1}{(1+q)^3}\right]\frac{1}{r_1^6} + \frac{q\tilde{B}_1}{1+q}\frac{1}{r_2^3} + \right. \\ \left. + \frac{2q\tilde{B}_2}{1+q}\frac{1}{r_2^4} + \frac{3q^2\tilde{B}_3}{(1+q)^2}\frac{1}{r_2^5} + 4\left[\frac{q\tilde{D}_2}{1+q} + \frac{q^3\tilde{N}_2}{(1+q)^3}\right]\frac{1}{r_2^6} \right\} = 0 , \end{aligned}$$

and from here it results $z = 0$, which proves the theorem. $\square$

Theorem 3.3. *In Fock's circular restricted three-body problem, there are three collinear libration points (L_1, L_2, L_3) .*

Proof. Starting from eq. $\frac{\partial \psi}{\partial x} = 0$, and imitating the classical proof used in the Newtonian case, we obtain straightforwardly the result. $\square$

Theorem 3.4. *The coordinates of the collinear libration points in Fock's circular restricted three-body problem are given by:*

$$\begin{aligned} & \underbrace{(x_{L1} - \tilde{A}_1 x_{L1}^{-2}) - q \left[(1 - x_{L1}) - \tilde{B}_1 (1 - x_{L1})^{-2} \right]}_{\text{}} - 2\tilde{A}_2 x_{L1}^{-3} - \frac{3\tilde{A}_3}{1+q} x_{L1}^{-4} - \\ & - 4 \left[\tilde{D}_1 + \frac{\tilde{N}_1}{(1+q)^2} \right] x_{L1}^{-5} - 2q\tilde{B}_2 (x_{L1} - 1)^{-3} + \frac{3q^2 \tilde{B}_3}{1+q} (x_{L1} - 1)^{-4} - \\ & - 4 \left[q\tilde{D}_2 + \frac{q^3}{(1+q)^2} \tilde{N}_2 \right] (x_{L1} - 1)^{-5} = 0 ; \end{aligned} \quad (10)$$

$$\begin{aligned} & \underbrace{(x_{L2} - \tilde{A}_1 x_{L2}^{-2}) + q \left[(x_{L2} - 1) - \tilde{B}_1 (x_{L2} - 1)^{-2} \right]}_{\text{}} - 2\tilde{A}_2 x_{L2}^{-3} - \frac{3\tilde{A}_3}{1+q} x_{L2}^{-4} - \\ & - 4 \left[\tilde{D}_1 + \frac{\tilde{N}_1}{(1+q)^2} \right] x_{L2}^{-5} - 2q\tilde{B}_2 (x_{L2} - 1)^{-3} - \frac{3q^2 \tilde{B}_3}{1+q} (x_{L2} - 1)^{-4} - \\ & - 4 \left[q\tilde{D}_2 + \frac{q^3}{(1+q)^2} \tilde{N}_2 \right] (x_{L2} - 1)^{-5} = 0 ; \end{aligned} \quad (11)$$

$$\begin{aligned} & \underbrace{(x_{L2} - \tilde{A}_1 x_{L2}^{-2}) + q \left[(x_{L2} - 1) - \tilde{B}_1 (x_{L2} - 1)^{-2} \right]}_{\text{}} - 2\tilde{A}_2 x_{L2}^{-3} - \frac{3\tilde{A}_3}{1+q} x_{L2}^{-4} - \\ & - 4 \left[\tilde{D}_1 + \frac{\tilde{N}_1}{(1+q)^2} \right] x_{L2}^{-5} - 2q\tilde{B}_2 (x_{L2} - 1)^{-3} - \frac{3q^2 \tilde{B}_3}{1+q} (x_{L2} - 1)^{-4} - \\ & - 4 \left[q\tilde{D}_2 + \frac{q^3}{(1+q)^2} \tilde{N}_2 \right] (x_{L2} - 1)^{-5} = 0 . \end{aligned} \quad (12)$$

Proof. The (Eulerian) collinear libration points have $y = 0$ and the following location: L_1 between m_1 and m_2 ; m_2 between L_2 and m_1 ; m_1 between L_3 and m_2 . So, by (8), we have to put: $r_1 = x_{L_1}$, $r_2 = 1 - x_{L_1}$ for L_1 ; $r_1 = x_{L_2}$, $r_2 = x_{L_2} - 1$ for L_2 ; $r_1 = -x_{L_3}$, $r_2 = 1 - x_{L_3}$ for L_3 . In this way, equations $\frac{\partial \psi}{\partial x} = \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial z} = 0$ lead straightforwardly to formulae (10), (11), and (12), respectively. (The terms in curly brackets are the expressions corresponding to the Newtonian field.) $\square$

As regards the triangular libration points (whose existence will be discussed elsewhere), for which $y \neq 0$, we state

Theorem 3.5. *If Fock's circular restricted three-body problem admits triangular libration points, the properties of these ones are as follows:*

- (i) *their coordinates depend on q ;*
- (ii) *there is only one couple of triangular libration points;*
- (iii) *the respective triangles are isosceles for $q = 1$, and scalene otherwise.*

Proof. We shall suppose that triangular libration points *do exist*.

(i) The equations which give the position of L_4 and L_5 are:

$$(1+q)^2 r_2^6 - \tilde{B}_1(1+q)^2 r_2^3 - 2\tilde{B}_2(1+q)^2 r_2^2 - 3\tilde{B}_3 q(1+q)r_2 - 4[\tilde{D}_2(1+q)^2 + q^2 \tilde{N}_2] = 0 ; \quad (13)$$

$$(1+q)^3 r_1^6 - \tilde{A}_1(1+q)^2 r_1^3 - 2\tilde{A}_2(1+q)^2 r_1^2 - 3\tilde{A}_3(1+q)r_1 - 4[\tilde{D}_1(1+q)^2 + \tilde{N}_1] - q(1+q)^2 r_1^6 = 0 . \quad (14)$$

Therefore, the coordinates of the triangular libration points depend on q .

(ii) Using Descartes' rule of signs, it is easy to see that both equations, (13) and (14), admit exactly one positive root each. This means that there are exactly two triangular libration points:

$$L_4: (x, y, z) = (x_{L_4}, y_{L_4} > 0, 0),$$

$$L_5: (x, y, z) = (x_{L_5}, y_{L_5} < 0, 0).$$

(iii) If $q = 1$, it follows that $\tilde{A}_i = \tilde{B}_i$, $i = \overline{1, 3}$, $\tilde{D}_1 = \tilde{D}_2$ and $\tilde{N}_1 = \tilde{N}_2$; this would mean $r_1 = r_2$, and triangles become isosceles. If $q = 1$ and the condition $4 - 4\tilde{B}_1 - 8\tilde{B}_2 - 6\tilde{B}_3 - 16\tilde{D}_2 - 4\tilde{N}_2 = 0$ is fulfilled, then the triangles become equilateral. But, taking into account the expressions of the coefficients marked with tilde, a straightforward calculation shows that the left-hand side of the above equality is strictly positive, a contradiction. So, $q = 1$ entails scalene triangles. $\square$

This offers a first insight into the libration points of the circular restricted three-body problem associated to Fock's relativistic gravitational field. Of course, further investigations are needed.

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BIELLIPTIC TRANSFER WITH PLANE CHANGES

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Abstract. The most powerful technique that has been devised for optimal plane change maneuvers is known as the third-body driven technique. This technique can be viewed as classical bielliptic plane changes (or restricted three-impulse plane change) with the apoapsis maneuver suppressed by the use of the third-body forces. In the present paper, full analysis of the bielliptic plane changes maneuver (BPCM) is developed, as well as its importance as the basis of the third-body driven technique. As to the computationally developments, we established new formula to estimate the initial guess for the iterative solutions of the fundamental equations of BPCM. This formula not only secures the convergence of the solutions, but also it could be used to accelerate the convergence. Finally, some numerical examples are included.

Key words: astrodynamics – orbital transfer maneuvers – trajectory optimization.

1. INTRODUCTION

The classical approach to orbital transfers is to assume an underlying two-body model and investigate the different ways of transferring from one conic orbit to another (e.g., Chobotov, 1996) between two successive impulsive burns. Coplanar maneuvers enable a satellite to alter two orbital elements: semimajor axis a , and eccentricity e , by using in-plane changes. To do an intercept or rendezvous, we may need to change two other elements: inclination, i , and the longitude of the ascending node, Ω . This requires applying ΔV out of the orbital plane – a noncoplanar change. We need noncoplanar maneuvers mainly because (Vallad, 2001):

- (1) A launch-site location restricts the initial orbit inclination for the satellite.
- (2) The direction of the launch can influence the amount of velocity the booster must supply, so certain orientation may be desirable.
- (3) Timing constraints may dictate a launch window that is not the best from which we must make changes.

Due to the relative expense of the orbital-plane-change maneuver, when it is accomplished by means of impulsive thrust, other techniques are to be used. In this respect, one of the most powerful techniques that has been devised today, is, the third-body driven (e.g., Villac and Scheeres, 2004).

Third-body driven technique can be viewed as classic bielliptic plane changes (or restricted three-impulse plane change) with the apoapsis maneuver suppressed by the use of the third-body forces. On the other hand, the resulting equations are highly transcendental and could be solved by iterative methods, which in turn need: (a) initial guess, (b) an iterative scheme. In fact, these two points are not separated from each other, but there is a full agreement that even accurate iterative schemes are extremely sensitive to initial guess. Moreover, in many cases the initial guess may lead to a drastic situation between divergent and very slow convergent solutions (Sharaf et al., 2007).

Due to the importance of third-body driven technique and due to its full resemblance with the bielliptic plane changes maneuver (BPCM), the present paper is devoted to the analytical and computational developments of BPCM. For the analytical developments, the fundamental equations are derived, while for its computational developments, we establish a new formula to estimate the initial guess for the iterative solutions of the fundamental equations. This formula not only secures the convergence of the solutions, but also it could be used to accelerate the convergence.

2. NONCOPLANAR BIELLIPTIC TRANSFER

First consider the transfer mode of the coplanar bielliptic maneuver (Fig. 1):

– From a circular orbit of radius r_1 an impulse ΔV_1 is applied and transfer along an elliptical orbit of semi major axis a_1 is initiated. The elliptical orbit ascends to an apoapsis farther removed from the dynamical center than the final circular orbit which is to be achieved.

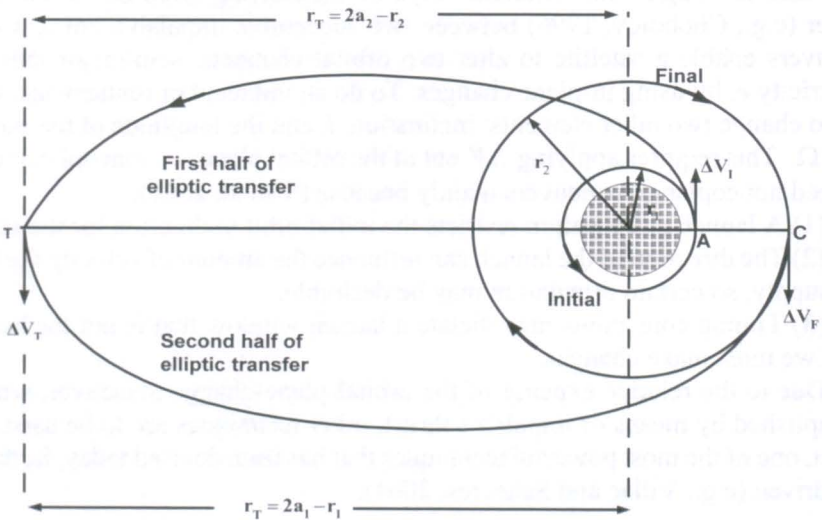


Fig. 1 – Coplanar bielliptic transfer maneuver.

- At apoapsis a second impulse ΔV_T is applied, and the run leg of a new elliptic orbit of semi major axis a_2 is initiated.
- A tangentially impulse ΔV_F at the periapsis of the second half of the elliptic orbit returns to the desired final orbit of radius r_2 . The bielliptic maneuver is a *three-impulse* transfer technique.

2.1. TRANSFER MODES FOR NONCOPLANAR BIELLIPTIC TRANSFER

Such a transfer is illustrated in Fig. 2. The steps are the following ones:

- A departure is made from *orbital plane 1* with plane change angle θ_1 .
 - At apoapsis of the resulting *elliptic orbit* a second impulse is applied to change the orbit plane through an angle θ_2 .
 - Finally, at the apoapsis of the resulting *second ellipse* a final plane change through an angle θ_3 places the vehicle in the final orbit, which is the *orbital plane 2*.
- The initial and final orbits are taken to be circular. The resulting bielliptic maneuver just described is general enough that appropriate choice of the θ_1 and radius magnitudes will include Hohmann transfers, if these modes of transfer are desired.

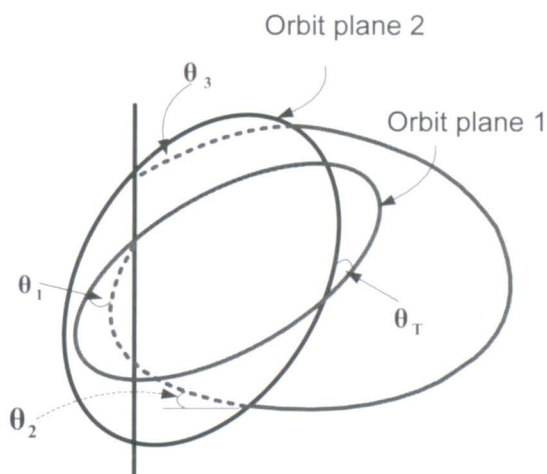


Fig. 2 – Bielliptic transfer with plane change.

2.2. VELOCITY REQUIREMENTS WITH PLANE CHANGE

To derive the velocity increments at each plane change, let us consider the vector diagram in Fig. 3.

This vector diagram gives the geometric relationships between, for example, the *initial* velocity, V_x , the velocity on the *transfer* orbit, V_y , and the *total* velocity

change ΔV . Adopting the previous notation, the velocity increment at all three plane changes can be obtained directly from the law of cosines:

$$\Delta V_j = (V_x^2 + V_y^2 - 2V_x V_y \cos \theta_j)^{1/2}, \quad j = \overline{1,3}. \quad (1)$$

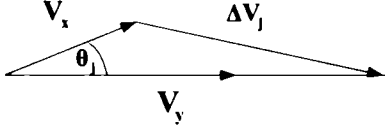


Fig. 3 – Velocity triangle.

Using the previous notations for the coplanar bielliptic transfer maneuver, we get:

$$\begin{aligned} \frac{\Delta V}{V_1} = & (1 + H_1^2 - 2H_1 \cos \theta_1)^{1/2} + H_2(1 + H_3^2 - 2H_3 \cos \theta_2)^{1/2} + \\ & + H_4(1 + H_5^2 - 2H_5 \cos \theta_3)^{1/2}, \end{aligned} \quad (2)$$

where

$$\begin{aligned} H_1 &= \left(\frac{2r_T}{r_1 + r_T} \right)^{1/2}; \\ H_2 &= \left(\frac{2r_1 r_2}{r_T(r_2 + r_T)} \right)^{1/2}; \\ H_3 &= \left(\frac{r_1(r_2 + r_T)}{r_2(r_1 + r_T)} \right)^{1/2}; \\ H_4 &= \left(\frac{r_1}{r_2} \right)^{1/2}; \\ H_5 &= \left(\frac{2r_T}{r_2 + r_T} \right)^{1/2} = \frac{H_1 H_4}{H_3}. \end{aligned} \quad (3)$$

A convenient parameter to specify is θ_T , which characterizes the total plane change, that is:

$$\theta_T = \theta_1 + \theta_2 + \theta_3. \quad (4)$$

Due to the conditions of symmetry, equation (2) can equally be used well for both situations below:

- outward transfer, that is $r_1 > r_2$, or
- inward transfer, that is $r_2 > r_1$.

Of course, it is obvious that the case in which $\theta_1 = \theta_2 = \theta_3 = 0$ reduces to the coplanar bielliptic transfer.

2.3. OPTIMUM SPLIT OF PLANE CHANGE

The manner in which we have to perform the splitting of the total plane change in the general transfer function defined by equation (2) is not obvious at all. Evidently, it would be desirable to *minimize* the total velocity requirement as a function of the angles $\theta_1, \theta_2, \theta_3$, subjected to the *constraint* of equation (4). Accordingly, the function to be minimized is:

$$M = \frac{\Delta V}{V_1} + \lambda(\theta_1 + \theta_2 + \theta_3 - \theta_T).$$

Consequently

$$\frac{\partial M}{\partial \theta_j} = \frac{\partial}{\partial \theta_1} \left(\frac{\Delta V}{V_1} \right) + \lambda = 0, \quad j = \overline{1, 3}.$$

Therefore

$$(1 + H_1^2 - 2H_1 \cos \theta_1)^{-1/2} H_1 \sin \theta_1 - [1 + H_5^2 - 2H_5 \cos(\theta_T - \theta_1 - \theta_2)]^{-1/2} \times \\ \times H_4 H_5 \sin(\theta_T - \theta_1 - \theta_2) = 0; \quad (5)$$

$$(1 + H_3^2 - 2H_3 \cos \theta_2)^{-1/2} H_2 H_3 \sin \theta_2 - [1 + H_5^2 - 2H_5 \cos(\theta_T - \theta_1 - \theta_2)]^{-1/2} \times \\ \times H_4 H_5 \sin(\theta_T - \theta_1 - \theta_2) = 0. \quad (6)$$

Equations (5) and (6) represent a nonlinear system of two equations with two unknowns, θ_1 and θ_2 , whose solution yields the appropriate optimum plane changes for a specified θ_T .

2.4. HOHMANN TRANSFERS WITH PLANE CHANGE

The resulting bielliptic maneuver described above is general enough that appropriate choice of the θ_1 and the radii magnitudes will include Hohmann transfers with plane changes. See some examples:

- Outward Hohmann transfer:

$$r_T = r_2; \\ \theta_2 = \theta_T - \theta_1, \quad \theta_3 = 0,$$

with θ_1 arbitrary.

– Inward Hohmann transfer:

$$\begin{aligned} r_T &= r_1; \\ \theta_2 &= \theta_T - \theta_3, \quad \theta_1 = 0, \end{aligned}$$

with θ_3 arbitrary.

– Modified Hohmann transfer:

$$\begin{aligned} r_T &= r_2; \\ \theta_1 &= \theta_2 = 0, \quad \theta_3 = \theta_T. \end{aligned}$$

The last case is an interesting and useful case of transfer, known as *modified Hohmann transfer*. It implies: a regular Hohmann transfer in the initial orbit plane and then a final plane change. If $\theta_3 = \theta_T$, this would correspond to a final plane change at the line of intersection of both orbit planes.

3. COMPUTATIONAL DEVELOPMENTS

Equations (5) and (6) are highly transcendental and could be solved by iterative methods, which in turn need:

- (a) initial guess;
- (b) an iterative scheme.

As we have mentioned in the introductory section, that these two points are not separated from each other, but there is a full agreement between them, even accurate iterative schemes are extremely sensitive to an initial guess. Moreover, in many cases the initial guess may lead to drastic situation between divergent and very slow convergent solutions.

Consequently, the first step in the computational developments of the problem is the proper choice of the initial guess, for the roots of equations (5) and (6).

3.1. INITIAL GUESS

In what follows we shall establish exact relations intended to determine the initial guess of equations (5) and (6).

Let η represent a percentage of the initial plane change angle θ_1 and $\theta_1 + \theta_2 = v\theta_T$, so

$$\begin{aligned} \theta_1 &= \eta\theta_T, \\ \theta_2 &= (v - \eta)\theta_T, \end{aligned} \tag{7}$$

where v is an arbitrary positive real number, such that

$$0 < v \leq 1. \tag{8}$$

Now our criterion is to minimize the function

$$G = 1 + H_1^2 - 2H_1 \cos \eta \theta_T + H_2^2 [1 + H_3^2 - 2H_3 \cos(v - \eta) \theta_T] .$$

Consequently

$$\frac{\partial G}{\partial \eta} = 0 \Rightarrow \sin \eta \theta_T = W \sin(v - \eta) \theta_T , \quad (9)$$

$$\text{where } W = \frac{H_2^2 H_3}{H_1} .$$

Expanding the right-hand side of equation (9), then collecting terms, and rearranging results, we get an exact relation for the determination of η :

$$\eta = \frac{1}{\theta_T} \tan^{-1} \left(\frac{W \sin v \theta_T}{1 + W \cos v \theta_T} \right) . \quad (10)$$

3.2. ITERATIVE SCHEME

The second point (point (b), mentioned above) for the computational developments of the problem is the iterative scheme. For this scheme we shall use Newton method summarized as follows:

Consider the two simultaneous nonlinear equations

$$\begin{aligned} U(x, y) &= 0, \\ V(x, y) &= 0, \end{aligned} \quad (11)$$

in two unknowns, x and y .

Let (x_0, y_0) be an approximation to a root (α, β) of the system of equations (11). Successive approximations are generated via the recursion formulae:

$$\begin{aligned} x_{i+1} &= x_i - \left[\frac{U \frac{\partial V}{\partial y} - V \frac{\partial U}{\partial y}}{J(U, V)} \right]_i = x_i - \delta x, \quad i = 0, 1, 2, \dots ; \\ y_{i+1} &= y_i - \left[\frac{V \frac{\partial U}{\partial x} - U \frac{\partial V}{\partial x}}{J(U, V)} \right]_i = y_i - \delta y, \quad i = 0, 1, 2, \dots , \end{aligned}$$

where the Jacobian of the system is $J(U, V) = \frac{\partial U}{\partial x} \frac{\partial V}{\partial y} - \frac{\partial V}{\partial x} \frac{\partial U}{\partial y}$;

all the functions involved are to be evaluated at (x_i, y_i) .

When the iteration converges, it converges quadratically. A set of conditions sufficient to ensure convergence is the following:

- (1) U and V and all their derivatives to second order are continuous and bounded in a region R containing (α, β) ;
- (2) the Jacobian $J(U, V)$ does not vanish in R ;
- (3) the initial approximation (x_0, y_0) is chosen sufficiently close to the root (α, β) .

The iterative process may be stopped by the condition that:

$$\max(|U|, |V|, |\delta x|, |\delta y|) \leq Tol.$$

where Tol stands for the given tolerance.

3.3. NUMERICAL EXAMPLES

For all examples we take $Tol = 10^{-6}$, and $\theta_T = 28^\circ.5$, while the corresponding number of iterations is denoted by N . In what follows we shall consider some inward ($r_2 > r_1$) and outward ($r_1 > r_2$) transfers.

3.3.1. Inward Transfers

Example 1:

– Input:

$$r_1 = 6671.53 \text{ km};$$

$$r_2 = 42163.9 \text{ km};$$

$$r_T = 24417.7 \text{ km};$$

$$v = 1.$$

– Output:

$$s = 0.134895;$$

$$\theta_1 = 2^\circ.68587, \quad \theta_2 = 18^\circ.8249, \quad \theta_3 = 6^\circ.98923;$$

$$\frac{\Delta V}{V_1} = 0.617651;$$

$$N = 4.$$

Example 2:

– Input:

$$r_1 = 22106.6 \text{ km};$$

$$r_2 = 34550.4 \text{ km};$$

$$r_T = 44213.2 \text{ km};$$

$$v = 0.9.$$

– Output:

$$s = 0.221038;$$

$$\theta_1 = 5^\circ.1313, \quad \theta_2 = 20^\circ.2183, \quad \theta_3 = 3^\circ.15036;$$

$$\frac{\Delta V}{V_1} = 0.481011;$$

$$N = 3.$$

Example 3:

Consider example 2, but with $v = 0.1$. The output is the same as the output of Example 2, but the number of iterations becomes $N = 10$! This example shows the effect of v on the rate of convergence.

3.3.2. Outward transfers

Example 4:

– Input:

$$r_1 = 34862.9 \text{ km};$$

$$r_2 = 15416.0 \text{ km};$$

$$r_T = 69725.8 \text{ km};$$

$$v = 0.9.$$

– Output:

$$s = 0.155046;$$

$$\theta_1 = 3^\circ.12189, \quad \theta_2 = 20^\circ.2183, \quad \theta_3 = 3^\circ.15036;$$

$$\frac{\Delta V}{V_1} = 0.481011;$$

$$N = 3.$$

Example 5:

– Input:

$$r_1 = 41241.0 \text{ km};$$

$$r_2 = 21794.1 \text{ km};$$

$$r_T = 69725.8 \text{ km};$$

$$v = 1.$$

– Output:

$$s = 0.182094;$$

$$\theta_1 = 3^\circ.5635, \quad \theta_2 = 21^\circ.1511, \quad \theta_3 = 3^\circ.15036;$$

$$\frac{\Delta V}{V_1} = 0.761085;$$

$$N = 3.$$

4. SUMMARY

In the present paper, a full analysis of the bielliptic plane-change maneuver (BPCM) was developed, in view of its importance for the bases of the most powerful techniques that have been devised for optimal plane-change maneuvers. Also, we established a new formula intended to estimate the initial guess for the iterative solutions of the fundamental equations of BPCM. This formula not only secures the convergence of the solutions, but it could be also used to accelerate the convergence.

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DETERMINATION OF A CELESTIAL PLANE ORIENTATION: APPLICATION TO MILKY WAY

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Abstract. We develop a general analytical method for the determination of any celestial plane orientation of objects with respect to a fundamental plane at a certain epoch. This plane is defined as the plane of greatest condensation of these objects. The method is applied for the determination of: the equatorial coordinates of the galactic pole and the inclination of the Milky Way plane to the celestial equator and the right ascension of its ascending node, all for the epoch 2000.0.

Key words: astrometry – ephemerides – galactic structure.

1. INTRODUCTION

In many applications of position astronomy (e.g., Meeus, 1991), the determination of the orientation of a celestial plane with respect to a fundamental plane at a certain epoch is of extreme importance, not only in coordinate transformations, but also for some problems concerning the relation between the physical properties of celestial objects with their condensation towards a plane of certain orientation (it suffices to exemplify by the Milky Way condensation of young Population I objects).

In the present paper, a general analytical method is developed for the determination of any celestial plane orientation of objects with respect to a fundamental plane at a certain epoch. This plane is defined as the plane of greatest condensation of these objects.

Consequently, the idea used in our analysis is to minimize the sum of the squared distances of the chosen group of objects from the plane.

We shall apply the proposed method to the determination of: (1) the equatorial coordinates of the galactic pole for the epoch 2000.0; (2) the inclination of the Milky Way plane to the celestial equator and the right ascension of its ascending node at the same epoch.

2. FORMULATION

Consider Fig. 1: P_1 is the pole of the reference plane (fundamental plane) and P_2 is the pole of the required plane. We define a celestial rectangular frame by specifying its origin O , the fundamental plane (XY -plane) and the preferred direction. In addition, we must specify the sense, or the positive direction. Finally, we shall consider a right-handed system, that is, the positive directions of the axes form an orthogonal triplet (X, Y, Z). The spherical coordinates that define the required plane are (θ_1, θ_2) , where θ_1 is measured above (or below) the fundamental plane, while θ_2 is measured along this plane from a specified point S (say) in the direction of the arrow. The coordinates (θ_1, θ_2) are related to the orientation angles (i, Ω) by:

$$i = 90^\circ - \theta_1, \quad (1)$$

$$\Omega = \theta_2 + 90^\circ, \quad (2)$$

where i is the inclination angle between the fundamental and the required planes, and Ω is the angle OSW.

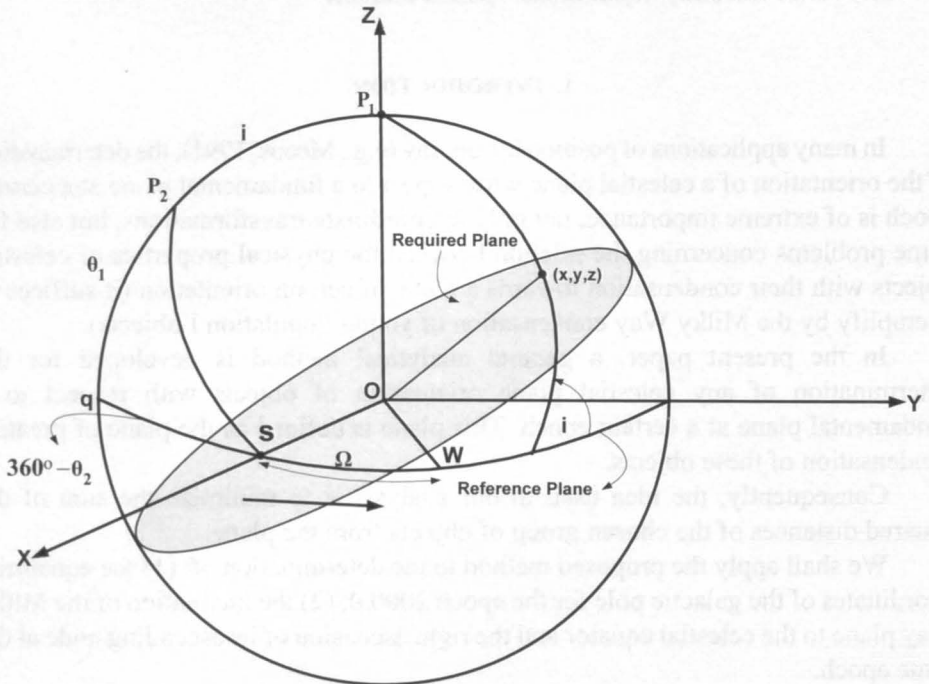


Fig. 1 – Geometry of the problem.

The position of a plane containing the origin is characterized by the scalar product $\langle \mathbf{L}, \mathbf{r} \rangle$, where $\mathbf{L} = [l, m, n]$ is the direction-cosines vector of the perpendicular to plane drawn from the origin, while $\mathbf{r} = [x, y, z]$ is the position vector. Clearly,

$$\mathbf{L}^T \mathbf{L} = 1, \quad (3)$$

where “T” denotes transposition.

From the spherical triangle P_2qS (Fig. 1) and equation (3) we get

$$\begin{aligned} l &= \cos \theta_1 \cos \theta_2, \\ m &= \cos \theta_1 \sin \theta_2, \\ n &= \sin \theta_1. \end{aligned} \quad (4)$$

Now, our problem is to get the above mentioned angular coordinates (θ_1, θ_2) or (i, Ω) , such that the required plane is that plane towards which the N (say) celestial objects (e.g., stars, clusters, etc.) of known configuration (position vectors $\mathbf{r}_i, i = \overline{1, N}$), tend to crowd. In order to determine the orientation of this plane, we shall adopt the least squares criterion, by which the sum of the squared distances, f , from the plane is minimum, where

$$f(l, m, n) = \mathbf{L}^T \mathbf{A} \mathbf{L}, \quad (5)$$

and $\mathbf{A}[a_{ij}]$ is the 3×3 symmetric matrix with the elements

$$\begin{aligned} a_{11} &= \sum_{i=1}^N x_i^2, \\ a_{12} &= \sum_{i=1}^N x_i y_i, \\ a_{13} &= \sum_{i=1}^N x_i z_i, \\ a_{22} &= \sum_{i=1}^N y_i^2, \\ a_{23} &= \sum_{i=1}^N y_i z_i, \\ a_{33} &= \sum_{i=1}^N z_i^2. \end{aligned} \quad (6)$$

By (3) and (5), the problem is finally reduced to typical constrained minimization, that is, to find the values of the set (l, m, n) that make the expression (5) a minimum subject to the imposed condition. Consequently, the objective function reduces to

$$F(l, m, n) = \mathbf{L}^T \mathbf{A} \mathbf{L} - \lambda (\mathbf{L}^T \mathbf{L} - 1), \quad (7)$$

where λ is Lagrange's multiplier to be determined.

The necessary conditions for the minimum are: $\partial F / \partial \mathbf{L} = 0$, which lead to, the eigenvalue problem

$$\mathbf{A} \mathbf{L} = \lambda \mathbf{L}. \quad (8)$$

It is well known (Ralston 1978) that the symmetric least square matrix $\mathbf{A}$ has all its eigenvalues λ_i , $i = \overline{1, 3}$, real and distinct.

Now, after solving the eigenvalue problem of equation (9) for $(\lambda_i, l_i, m_i, n_i)$, $i = \overline{1, 3}$, we select from this set the values (λ, l, m, n) that give the global minimum of F , then the required values of (θ_1, θ_2) could then be determined from

$$\begin{aligned} \theta_1 &= \sin^{-1}(n), \\ \theta_2 &= \tan^{-1}\left(\frac{m}{l}\right). \end{aligned} \quad (9)$$

3. APPLICATIONS

We shall make use of the above formulations to determine:

- (1) the equatorial coordinates of the galactic pole (α_p, δ_p) at a certain epoch 2000.0;
- (2) the inclination of the Milky Way plane to the celestial equator i_g and the right ascension of its ascending node Ω_g at the epoch 2000.0.

In order to achieve the above proposed goals, two steps are to be performed, as follows:

3.1. STEP ONE: ADAPTATION

3.1.1. Geometry

The geometry of the present application is contained in Fig. 1, in which the fundamental plane becomes the equatorial plane, and the required plane becomes the Milky Way plane, while the other transformations are listed in Table 1.

Table 1

Geometry adaptation

Fig. 1	Application
O	Sun
P_1	North celestial pole
P_2	North galactic pole
S	Vernal equinox
(θ_1, θ_2)	(δ_p, α_p)
(i, Ω)	(i_g, Ω_g)

3.1.2. The group of objects

We obviously shall expect to get the best results choosing objects which exhibit the strongest concentration towards the Milky Way. It is however, clear that, in more general investigations, other objects must be considered, too. Some useful objects for this application are the young Population I and particularly the youngest among them, as for example: galactic clusters, Wolf-Rayet stars, O-type stars, and classical Cepheids.

We shall need lists of these objects containing their right ascensions and declinations. Now it is important to realize that (α_p, δ_p) and (i, Ω_g) as deduced by using any group of these objects will refer to the same frame of reference as the coordinates of these objects.

In other words if, for instance, we used a list where the coordinates are given for the epoch 1950.0, then we shall get (α_p, δ_p) and (i_g, Ω_g) with respect to the celestial equator and equinox for the epoch 1950.0. Any determination based on coordinates for a different epoch will necessarily lead to a different result.

3.1.3. The Cartesian coordinates of the objects

Let assume that all the selected objects are at the same distance from the Sun (which distance is taken as unity). Now, let (α_i, δ_i) be the equatorial coordinates of one object. According to the above assumption, that is all objects are at unit distance from the Sun we have;

$$x_i = \cos \delta_i \cos \alpha_i, \quad y_i = \cos \delta_i \sin \alpha_i, \quad z_i = \sin \delta_i. \quad (10)$$

3.2. STEP TWO: COMPUTATIONAL DEVELOPMENTS

3.2.1. Selected objects

For the numerical studies we used four objects (actually classes of objects), their equatorial coordinates for the epoch 2000.0 are taken from Hipparcos main catalogue.

In what follows these objects are listed together with their table number at CDS (Strasbourg Astronomical Data Center), for more information about the object, we give the reference from which its data were taken

1. Wolf-Rayet stars (van der Hucht 2001): Catalogues: III/215/Table 13: *Locations of galactic WR stars* (226 rows);
2. Classical Cepheids (Tammann et al., 2003): J/A+A/404/423 Table 1: *Intrinsic colors of Galactic fundamental mode Cepheids* (324 rows);
3. O-type stars (Garmany et al., 1982): II/82/catalog: *Catalog of Galactic O-type stars* (765 rows);
4. A-type stars Clusters and Associations, Selected Data (Kluke et al., 1975): VII/5A/catalog: *The Catalogue* (1039 rows).

3.2.2. Numerical Results

In Table 2, the results $(\alpha_p, \delta_p, i_g, \Omega_g)$ at the epoch 2000.0 are listed for each object. Each result is the average of five values corresponding to galactic latitudes $b = 1^\circ, 2^\circ, 3^\circ, 4^\circ, 5^\circ$ of the object members.

Table 2
Values of $\alpha_p, \delta_p, i_g, \Omega_g$ at 2000.0 for each object

Object	α_p [°]	δ_p [°]	i_g [°]	Ω_g [°]
1	192.345	27.2984	62.7016	282.345
2	192.922	26.8941	63.1059	282.922
3	192.348	26.8466	63.1534	282.348
4	192.976	26.9445	63.0555	282.976

So, we may say that the most proper values for the epoch 2000.0 are:

$$\begin{aligned}
 \alpha_p &= 192^\circ.648, \\
 \delta_p &= 27^\circ; \\
 i_g &= 63^\circ, \\
 \Omega_g &= 282^\circ.648.
 \end{aligned}
 \tag{11}$$

In concluding the paper, we developed a general analytical method for the determination of any celestial plane orientation of objects with respect to a fundamental plane at certain epoch. This plane is defined as the plane of greatest condensation of these objects. The method is applied for the determination of:

- (a) the equatorial coordinates of the galactic pole for the epoch 2000.0;
- (b) the inclination of the Milky Way plane to the celestial equator and the right ascension of its ascending node at the same epoch.

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PRECISE DETERMINATION OF MINOR PLANET POSITIONS

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Abstract. We perform a first comparative study concerning position determination of selected minor planets observed in Bucharest. Photographic observations were made with the equatorial refractor ($d = 0.38$ m, $f = 6$ m). The plates were measured with a Carl-Zeiss Ascorecord machine and digitized with an Epson 1640XL flatbed scanner. We compare manual measurements with data processing of digitized plates for five asteroids.

Key words: astrometry – photographic observations – digitization.

1. INTRODUCTION

We perform here a first study of five comparatively good quality photographic plates from the wide-field plate archive of the Bucharest Astronomical Institute. The plates chosen for the study of the selected minor planets benefit of AGFA Gevaert emulsion, and have the size 24 cm $\times$ 24 cm. Each measured plate has two exposures, made with the Prin-Merz astrograph of the Astronomical Institute of the Romanian Academy ($d = 0.38$ m, $f = 6$ m, size of field $2^\circ \times 2^\circ$), in 1971.

To process the plates, we applied the reference system (epoch 1950.0) of the SAO Catalogue, and we considered a sample of five reference stars. The measurements were performed with the Carl-Zeiss Ascorecord measuring device. Then the plates were digitized with the Epson 1640XL flatbed scanner of the Sofia Sky Archive Data Center (SSADC, <http://www.skyarchive.org/SSADC>) and with the UMAX Alpha Vista II scanner of the Bucharest Astronomical Institute (one plate test measurements) for comparison. The calculations were done on PC's, with the same program for both cases, using a double precision accuracy, in order to compare manual measurements and data processing using the digitized plates.

2. SCANNER AND PLATE DIGITIZATION PARAMETERS

The sample of five plates from the Bucharest refractor with BUC038 designation according to the WFPDB standards (www.skyarchive.org) was scanned in Sofia using the SSADC facilities: professional flatbed scanner Epson 1640XL. We used the A3 mode (297 mm × 432 mm) to digitize the plate field with size of 241 mm × 242 mm, with the 4.5 m/sec speed of scanning. The scan resolution used was 1600 dpi – 14 bit grayscale up to 3.6 densities. With this scale 1 pixel = 15.875 microns, we had the extended field of 15184×15219 pixels. The format of the scans was FITS using the standard twain driver for Epson 1640XL and software developed by S. Mottola (Barbieri et al., 2006).

The storage requirements for one plate were about 462 MB and totally 2.3 GB for all 5 digitized plates. For the numerical data of telescope with a scale of 30"/mm we have a scan resolution 0.54" per pixel. To compute the centroid of a chosen star image we used a derivative search. The CNTRD IDL ASTROLIB program and the DAOPHOT "FIND" centroid algorithm, by locating the position where the X and Y derivatives go to zero, were applied. This is usually a more "robust" determination than a "center of mass" or fitting a 2D Gaussian if the wings in one direction are affected by the presence of a neighboring star. For the comparison with the Ascorecord measurements, the double accuracy was used.

3. MEASUREMENT AND PLATE PROCESSING RESULTS

From the coordinates α_i, δ_i of reference stars, corrected for the proper motions, and α_0, δ_0 those of the plate center, the normal coordinates X_i, Y_i are determined as follows:

$$X_i = \frac{\cos \delta_i \sin(\alpha_i - \alpha_0)}{\sin \delta_i \sin \delta_0 + \cos \delta_i \cos \delta_0 \cos(\alpha_i - \alpha_0)}, \quad (1)$$

$$Y_i = \frac{\sin \delta_i \cos \delta_0 - \cos \delta_i \sin \delta_0 \cos(\alpha_i - \alpha_0)}{\sin \delta_i \sin \delta_0 + \cos \delta_i \cos \delta_0 \cos(\alpha_i - \alpha_0)}.$$

In the first part the computations are referred to the center of gravity of the pentagon formed by reference stars:

$$\frac{\sum X_i}{5} = \frac{\sum x_i}{5};$$

$$\frac{\sum Y_i}{5} = \frac{\sum y_i}{5}, \quad (2)$$

where (x_i, y_i) are the coordinates measured on the plate. For each coordinate we have 5 equations with 2 unknown constants, one corresponding to the scale on the plate and the other to the difference in orientation of the coordinates systems (X, Y) and (x, y) . The two systems are solved using the Cracowian formulae, which are very convenient for computer programming:

$$\begin{aligned} x_i a + y_i b + c &= X_i, \\ x_i d + y_i e + f &= Y_i, \end{aligned} \quad i = \overline{1, 5}, \quad (3)$$

where (a, b, c, d, e, f) are the plate constants, and

$$\begin{aligned} [x_i x_i] p + [x_i y_i] q + [x_i 1] r &= x_p, \\ [x_i y_i] p + [y_i y_i] q + [y_i 1] r &= y_p, \quad i = \overline{1, 5}, \\ [x_i 1] p + [y_i 1] q + r &= 1, \end{aligned} \quad (4)$$

where (x_p, y_p) are the measured coordinates of the minor planet.

The (a, b, c, d, e, f) plate constants are determined from the first two systems and by replacing them in the residual values. On the basis of these ones, we determined the square mean error for α and δ , by means of the formula:

$$\varepsilon(\alpha, \delta) = \sqrt{\frac{\rho_i^2(\alpha, \delta)}{n-1}}. \quad (5)$$

By means of the third system, the values (p, q, r) are determined using the following dependences:

$$D_i = x_i p + y_i q + r. \quad (6)$$

Both Turners's (1893) and Schlesinger's (1911, 1926) constants dependences methods are used for the computation of the normal coordinates of the object. Within this method, the values of the constants are determined together with their errors.

The determination of the minor planet accurate coordinates is done by multiplying the dependences by the star coordinates;

$$\begin{aligned} X_p &= \sum_{i=1}^5 X_i D_i, \\ Y_p &= \sum_{i=1}^5 Y_i D_i. \end{aligned} \quad (7)$$

Below we present the results for a calculated position of a minor planet, in both variants: (1) when the reference stars and minor planet were measured; (2) when images of reference stars and minor planet were digitized.

For the asteroid 137 Meliboea we had a position for UT = 1971 01 04.805180, and sidereal time of exposure ST = 3^h 58^m 49^s for the exposure midtime.

3.1. ASCORECORD MEASUREMENTS

In the "classical" variant, the chosen photographic plates were measured with the Carl-Zeiss Ascorecord measuring device, the values x_i and y_i for the stars and for the asteroid are given in millimeters, whereas the plate center has the coordinates $X = 500.0000$ mm and $Y = 200.0000$ mm.

Table 1 contains the identification number of the stars in the SAO catalogue, right ascension, residuals in ascension, stars measurements in x , declination, residuals in declination, and stars measurements in y .

Table 1

Data for the chosen comparison stars

SAO	$\alpha_{1950.0}$	$\rho(\alpha)$	x_i [mm]	$\delta_{1950.0}$	$\rho(\delta)$	y_i [mm]
6853	5 ^h 07 ^m 14 ^s .347	-.028	485.1271	7° 00' 45".01	".09	209.6616
6867	5 08 00.091	.031	504.5910	6 29 04.81	.08	155.0174
7830	5 08 30.655	.019	517.8618	7 27 02.47	-.30	254.9020
7865	5 09 26.224	-.027	541.5202	6 48 02.02	-.20	187.6289
7831	5 09 34.501	.005	545.1659	7 25 05.57	.33	251.5138

$$x_{mp} = 506.8943$$

$$y_{mp} = 192.8762$$

$$\varepsilon(\alpha) = 0.026787$$

$$\varepsilon(\delta) = 0.252612$$

Results Schlesinger: 5^h 08^m 05^s.290

6° 51' 02".50

Results Turner: 5^h 08^m 05^s.290

6° 51' 02".50

Dependences: 1) 0.328269 2) 0.361689 3) 0.120620 4) 0.162884 5) 0.026539

$$(O-C)_{\alpha} = -0^s.049$$

$$(O-C)_{\delta} = -0".06.$$

3.2. PROCESSING OF THE DIGITIZED PLATES

In this case the photographic plate was digitized in Sofia with Epson 1640XL commercial flatbed scanner with a resolution of 1600 dpi, the values x_i and y_i of the stars and of the asteroid being initially expressed in pixels-mm, knowing that 1 pix = 15.875 μ and taking the upper left corner with the coordinates 380.0000 and 80.0000 as origin. Some preliminary additional test scans were made also in Bucharest using the UMAX Alfa Vista II scanner, with a resolution of 2400 $\times$ 1200 dpi.

In Fig. 1 the scheme of the photographic plate dimensions with pixels and millimeters equivalent is presented.

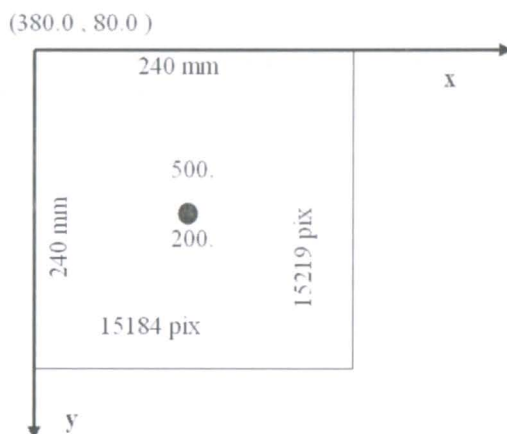


Fig. 1 – Scheme of the photographic plate dimensions with pixels and millimeters equivalent.

For example, the transformations of the pixels in mm are:

$$x = 380.0000 + 6506.4757 \times 15.875 = 483.2903,$$

$$y = 80.0000 + 8021.4108 \times 15.875 = 207.3399.$$

Table 2

Positions measured with Ascorecord and digitized with the Epson scanner

X [pix]	x [mm]	Y [pix]	y [mm]
506.4754	483.2903	8021.4108	207.3399
7736.0187	502.8093	4560.9133	152.4045
8573.2471	516.1003	10878.3110	252.6932
10066.9540	539.8129	6619.9558	185.0918
10296.1130	543.4508	10660.2203	249.2310
7880.7368	505.1067	6955.9872	190.4263

In Table 2 an example of the position measured is presented. The values of the digitized with the Epson scanner and their transformations into millimeters in comparison with measurements using the Carl-Zeiss Ascorecord device are shown. There are small differences between data from the measured and scanned plates due to the fact that the plate centers in both cases do not coincide.

Table 3 contains the identification number of the comparisons stars from the SAO catalogue, right ascension, residues in ascension, x_i scanning of stars, declination, residues in declination and y_i digitization of stars.

Table 3

Data of the comparison stars from the SAO catalogue
and their residuals in right ascension and declination calculated after digitization

SAO	$\alpha_{1950.0}$	$\rho(\alpha)$	x_i [mm]	$\delta_{1950.0}$	$\rho(\delta)$	y_i [mm]
6853	5 ^h 07 ^m 14 ^s .326	-.038	483.2903	7° 00' 45".18	".32	207.3399
6867	5 08 00.096	.034	502.8093	6 29 04.37	-.20	152.4045
7830	5 08 30.670	.036	516.1003	7 27 01.89	-.41	252.6932
7865	5 09 26.210	-.025	539.8129	6 48 02.27	.08	185.0918
7831	5 09 34.523	-.007	543.4508	7 25 05.45	.22	249.2310

$$x_{mp} = 505.1067$$

$$y_{mp} = 190.4263$$

$$\epsilon(\alpha) = 0.033617$$

$$\epsilon(\delta) = 0.300724$$

Results Schlesinger: 5^h 08^m 05^s.281 6° 51' 02".42

Results Turner: 5^h 08^m 05^s.281 6° 51' 02".42

Dependences: 1) 0.328326 2) 0.361725 3) 0.120599 4) 0.162829 5) 0.026521

$$(O-C)_\alpha = -0^s.058$$

$$(O-C)_\delta = -0".14$$

We present below two tables of right ascension and declination for five minor planets, each one with two positions, and the results being obtained through the measuring or digitization of the photographic plates in order to comparatively outline the processing quality for a greater number of cases.

Table 4 contains columns referring to the right ascension: the minor planet name, coordinates for the plate positions measured or digitized respectively at the epoch 1950.0, values $(O-C)_\alpha$ and the square average errors $\epsilon(\alpha)$.

Table 4

Comparison for the five minor planets:
 α coordinate measured and calculated from the plate digitized

Minor planet	Measured positions			Scanned positions Epson		
	$\alpha_{1950.0}$	$(O-C)_\alpha$	$\epsilon(\alpha)$	$\alpha_{1950.0}$	$(O-C)_\alpha$	$\epsilon(\alpha)$
129 Antigone	15 29 04.805	-.006	.00827	15 29 04.801	-.010	.01998
"	15 29 04.212	-.006	.00558	15 29 04.242	+.024	.01678
137 Meliboea	5 08 05.290	-.049	.02678	5 08 05.281	-.058	.03361
"	5 08 04.707	-.050	.02964	5 08 04.699	-.058	.03559
148 Gallia	16 47 40.603	-.012	.03870	16 47 40.354	-.016	.05211
"	16 47 40.035	-.013	.04528	16 47 39.782	-.017	.06097
161 Athor	11 21 57.997	+.009	.01378	11 21 57.987	-.001	.02182
"	11 21 57.441	+.009	.00736	11 21 57.432	.000	.01608
498 Tokio	2 56 42.050	+.008	.05093	2 56 42.056	+.014	.03926
"	2 56 41.761	+.007	.02910	2 56 41.765	+.011	.02736

Table 5 contains columns referring to the declination: the minor planet name, and data for the positions measured or digitized, respectively, at the epoch 1950.0, the values $(O-C)_\delta$ and the square average errors $\varepsilon(\delta)$.

Table 5

Comparison for the five minor planets:
 δ coordinate measured and calculated from the plate digitized

Minor planet	Measured positions			Scanned positions Eposn		
	$\delta_{1950.0}$	$(O-C)_\delta$	$\varepsilon(\delta)$	$\delta_{1950.0}$	$(O-C)_\delta$	$\varepsilon(\delta)$
129 Antigone	3 25 16.08	-.05	.19515	3 25 16.04	-.09	.21562
"	3 25 18.25	-.06	.08014	3 25 18.19	-.12	.15593
137 Meliboea	6 51 02.50	-.06	.25261	6 51 02.42	-.14	.30072
"	6 51 03.18	-.06	.11562	6 51 03.10	-.14	.17622
148 Gallia	11 35 25.02	-.65	.63903	11 35 24.95	-.72	.68937
"	11 35 21.81	-.66	.29201	11 35 21.81	-.66	.29201
161 Athor	14 23 30.04	+.17	.14943	14 23 30.10	+.23	.14299
"	14 23 33.86	+.17	.12571	14 23 33.81	+.12	.20431
498 Tokio	7 59 03.82	+.58	.34531	7 59 03.90	+.66	.07791
"	7 59 07.23	+.57	.14202	7 59 07.23	+.55	.09659

4. CONCLUSIONS

We have analyzed and compared the square average errors $\varepsilon(\alpha, \delta)$ and the values $(O-C)_{\alpha, \delta}$, in both cases: Ascorecord and digitized plate processing. We showed that the results are good and even in very good consideration. They depend on the scanning resolution and determination of the precise positions of the minor planets, as well as on the number of reference stars. The results obtained with a resolution of 1200–1600 dpi have the square average error comparable with the measuring machine. An essential factor for the precise scanning consists of folding the plate stability in the scanner and providing a stable condition of the table. In the future the manufacturers should complete the scanner and provide plastic frames for plates of different dimensions (30 cm $\times$ 30 cm, 24 cm $\times$ 24 cm, 16 cm $\times$ 16 cm, 13 cm $\times$ 18 cm, 9 cm $\times$ 19 cm) often used in photographic astronomical observations, and shorten the time for digitization, if possible.

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NEW LAWS OF PLANETARY DISTANCES

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Abstract. The present paper gives a detailed analysis of the new formulation of Titius-Bode law by Poveda and Lara (2008) and of the hypothesis that this law may exist in extra-solar planetary systems. A thorough study of the correspondences between the predicted distances and the observed ones in the Solar System and in 55 Cancri is given. A less known formula of planetary distances – Butusov’s (1973) law – is discussed. New predicted distances in 55 Cancri based on Butusov’s law are proposed.

Key words: Titius-Bode law – planetary systems.

1. INTRODUCTION

The very interesting article published by Arcadio Poveda and Patricia Lara (Poveda and Lara 2008) is very important indeed, as the authors try to make a major step towards proving the physical reality of the famous Titius-Bode law (TBL). Namely, the authors suppose that TBL (rewritten in an exponential form) is governing not only planetary distances in the Solar System, but also in 55 Cancri and predict (on the basis of TBL) the existence of two not yet observed planets in that system. If this prediction is confirmed by further observation, then TBL will acquire the status of a correct phenomenological astronomical law – like Kepler’s laws before Sir Isaac Newton.

However, this paper has some flaws that may diminish its scientific value. In the present paper I will try to discuss these problems.

It is important to mention that TBL still attracts the attention of astronomers and mathematicians wishing to find a theoretical basis for this still unexplained formula (see, for example, Graner and Dubrulle 1994, and Dubrulle and Graner 1994) and/or to find a mathematical formulation of this law that would ensure a better fit of the calculated distances with the observed ones. A detailed overview of these attempts can be found in the works of Nieto (1972) and Badolati (1980, 1982). An essay to include all “major” bodies of the Solar System (i.e., planets and minor planets) into a Titius-Bode-like formula is outlined by Ortiz et al. (2007). Neslušan (2004) performed a statistical analysis of the Titius-Bode distribution and showed that it is not due to a

chance. At last, it is necessary to list the important publication of Alcobé and Cubarsi (2007), who applied a Titius-Bode-like formula to the distribution of stellar populations.

2. TITIUS-BODE FORMULA

Poveda and Lara (2008) wrote down the formula for TBL as

$$a = 0.1912 \exp(0.5994n), \quad (1)$$

where a = distance from the Sun (in AU), and n = orbital number (number of the planet counting from the Sun).

As the authors admit themselves, their formula does not predict distances for Mercury and Pluto. Mercury is excluded from the formula because its orbital number n in the traditional presentation of the formula of TBL

$$a = 0.4 + 0.3 \times 2^n \quad (2)$$

is $-\infty$, which is “devoid of any physical meaning”.

Pluto is excluded because its orbit underwent many changes since the formation of the Solar System.

Obviously, Eris is not included in this formula either.

Unfortunately, this approach does not seem correct to me:

First of all, it is extremely fallacious to believe that n in formula (2) represents the orbital number (while n in the formula (1) is the orbital number indeed). Actually, it would be more correct to write down the traditional formula for TBL as follows:

$$a = 0.4 + 0.3 \times 2^N, \quad (3)$$

where

$$N = F(n), \quad (4)$$

that is, N is a function of the orbital number, but not the orbital number itself. The authors should have realized this, as the orbital number 0 (for Venus in the traditional TBL formula) is also “devoid of any physical meaning” – therefore, it is not the orbital number at all.

I would propose the following explicit presentation of $N = F(n)$:

$$N = \frac{n-2}{\text{sign}(n-1)}. \quad (5)$$

In this case the orbital number for Mercury is 1 – as it is expected to be, but $N = -\infty$, as it is prescribed by TBL. Interestingly enough, while formula (5) is absolutely obvious, to the best of my knowledge it was not proposed by anybody.

Pluto is not the only planet with modified orbit – according to recent studies, many planets happened to change their orbits since the formation of the Solar System. Does it mean that all these planets should have been excluded from formula (1)? Of course, it does not – but then Pluto should have been included in formula (1).

Real distances differ dramatically from predicted ones, not only for Mercury and Pluto, but also for other planets:

Table 1

Comparison of real and predicted distances (formula (1))

<i>n</i>	Planet	a_R (real, in AU)	a_P (predicted by formula (1), in AU)	$\Delta = (a_P - a_R)/a_R$ [%]
1	<i>Mercury</i>	0.387	0.335	-13.56
2	<i>Venus</i>	0.723	0.585	-19.05
3	<i>Earth</i>	1.000	1.024	2.40
4	<i>Mars</i>	1.524	1.792	17.57
5	<i>Ceres</i>	2.766	3.135	13.33
6	<i>Jupiter</i>	5.203	5.485	5.41
7	<i>Saturn</i>	9.537	9.596	0.62
8	<i>Uranus</i>	19.191	16.790	-12.51
9	<i>Neptune</i>	28.263	29.376	3.94
10	<i>Pluto</i>	39.482	51.396	30.18
11	<i>Eris</i>	67.668	89.924	32.89

The discrepancies are huge for Venus, Mars, Ceres and Uranus. Unfortunately, authors did not provide us with any good explanation of these differences between the formula and the reality. They do not even mention them.

It is not necessary to highlight that, if we accept such big discrepancies, then we are allowed to use virtually *any* formula.

In Section 4 of the present paper I will try to demonstrate that Eris should be included in the formula of planetary distances.

Last, but not least: why do we have right to exclude any planet from the formula that is expected to describe the distribution of planetary distances? It seems to me that there is a logical contradiction – either the formula is valid for all planet or it does not describe the structure of the Solar System. I am afraid that in this case the latter hypothesis is true.

In my opinion, these facts substantially undermine the value of the formula (1).

3. REAL AND PREDICTED DISTANCES IN 55 CANCRI

Unfortunately, the same discrepancies between predicted and observed distances exist in 55 Cancri, too:

Table 2

Comparison of real and predicted distances in 55 Cancri

n	A_R (real, in AU)	A_P (predicted by formula (1), in AU)	$D = (A_P - A_R)/A_R$ [%]
1	0.038	0.039	2.63
2	0.115	0.104	-9.57
3	0.24	0.283	17.92
4	0.781	0.768	-1.66
5	?	2.08	?
6	5.77	5.643	-2.20
7	?	15.3	?

As we can see, D is high for $n = 2$ and $n = 3$, which may be indicative of a problem with the formula for TBL for 55 Cancri.

The authors did not provide any explanation for these differences either. Neither did they indicate them.

4. BUTUSOV'S LAW

In 1973 Kirill Butusov, an astronomer from the USSR, proposed a very interesting structural law of the Solar System (Butusov, 1973):

$$a_6^2 = a_{5+i} \times a_{5-i}, \quad (6)$$

where a_6 = distance between Jupiter and the Sun (in AU), while $i \in \{1, 2, 3, 4, 5\}$ (see the bodies considered in Table 3 below).

Table 3

Butusov's law

Planet	n	i	a (AU)	$a_{6+i}a_{6-i}$	d [%]
Mercury	1	5	0.387	26.188	-3.264
Venus	2	4	0.723	28.545	5.446
Earth	3	3	1.000	28.263	4.402
Mars	4	2	1.524	29.247	8.038
Ceres	5	1	2.766	26.379	-2.556
Jupiter	6	—	5.203	27.071	0.00
Saturn	7	1	9.537	26.379	-2.556
Uranus	8	2	19.191	29.247	8.038
Neptune	9	3	28.263	28.263	4.402
Pluto	10	4	39.482	28.545	5.446
Eris	11	5	67.668	26.188	-3.264

Butusov's law simply says that there is a Jovian symmetry of planetary distances in the Solar system.

Butusov's results are summed up in Table 3, in which the quantity d in the last column is given by the formula

$$d = \frac{a_{6+i} \times a_{6-i} - a_6^2}{a_6^2} \times 100\%. \quad (7)$$

Indeed, Butusov's law is not absolutely precise – but the difference between its predictions and the observed distances does not exceed 8%, which is substantially better than the predictions provided either by TBL or by the Poveda-Lara's formula (1). Interestingly enough, Butusov's law is also valid for Eris, which was discovered long after Butusov published his paper. In my opinion, Butusov's law gives a very good description of our planetary system, and, as it includes Eris, *any* other structural formula should also be valid for Eris – which, unfortunately, is not the case for Poveda-Lara's formula.

Butusov's law – like TBL – is purely phenomenological, that is, it has no recognized theoretical basis. Butusov tried in his later works to provide such a basis, but his results were not accepted by the scholarly community, as they were based on non-traditional approaches towards the Solar System.

A more traditional hypothesis would be that Jupiter, as the most massive planet, determines the precise structure of the Solar system – while the Sun is responsible for the formation of this system. However, this hypothesis – if Butusov's law is applicable not only to our system, but also to extra-solar ones – should be rejected, which will be shown in Section 5 of the present paper.

The general form of Butusov's law would be:

$$r_Z^2 = r_{Z+k} \times r_{Z-k}, \quad (8)$$

where r = distance of the planet from the star (in AU), Z = number of the planet that is the center of symmetry in the given planetary system, $k = \overline{1, Z-1}$.

Another form of this law is:

$$r_{Z+j} \times r_{Z-j} = \text{const.}, \quad j = \overline{0, Z-1}. \quad (9)$$

5. NEW DISTANCE PREDICTIONS FOR 55 CANCRI

Let us suppose Butusov's law is also valid for 55 Cancri. This hypothesis can be accepted if we suppose, in full accordance with Poveda and Lara (2008), that there still are two planets to be discovered, which will be assigned orbital numbers 5 and 7. The fifth planet known to us is therefore actually the sixth. The parameter Z is then equal to 4 (see Table 4).

Table 4

Butusov's law in 55 Cancri

n	$j \in \{0, 1, 2, 3\}$	r_n [AU]	$r_{4+j}r_{4-j}$	d [%]
1	3	0.038	?	?
2	2	0.115	0.6634	8.79
3	1	0.24	?	?
4	0	0.781	0.61	0.00
5	1	?	?	?
6	2	5.77	0.6634	8.79
7	3	?	?	?

We can therefore try to predict possible planetary distances for the hypothetical planets 5 and 7:

$$r_5 = \frac{r_4^2}{r_3} = \frac{0.61}{0.24} = 2.541, \quad (10)$$

$$r_7 = \frac{r_4^2}{r_1} = \frac{0.61}{0.038} = 16.052. \quad (11)$$

These distances differ from the ones predicted by Poveda and Lara (but, interestingly enough, the difference is not high), and only real observations will make us able to choose between these results.

6. CONCLUSION

These structural laws (Titius-Bode and Butusov) may seem to be just number games, but if any one of them is confirmed, not only for the Solar System, but also for other planetary systems, it will clearly show that we are far from the understanding of the mechanisms that are responsible for the formation of these systems.

The hypothesis that the most massive planet is responsible for the precise structure of the planetary system – which we adopted above – could not be accepted, as in 55 Cancri the center of symmetry (planet 4) is not the most massive one. So we have to find an alternative explanation.

And, of course, it is extremely important to find out if there are any planets in 55 Cancri that correspond to the orbital numbers 5 and 7. It is also important to carry out Titius-Bode and Butusov calculations for other extra-solar planetary systems.

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FROM SYNCHRONIZATION OF THE ATOM TO ORGANIZATION OF THE UNIVERSE VIA GRAVITATION

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Abstract. This paper is a contribution to develop a unified theory for the basic physical forces of nature. A model in the sense of a scientific hypothesis is designed, in which the gravitational force is explained as an electromagnetic effect. We take the following approach: the force, which is called gravitation, is effective between any masses, therefore between atoms, too. This leads us to the idea to study the interaction of atoms in order to find causes for the gravitational force, i.e., an attractive force between them. Since electromagnetic forces are effective on that scale, mass is equivalent to energy, and general experience in physics recommends to try to explain phenomena with as few causes as possible, we analyze the interaction of atoms under electromagnetic aspects. Based on the hypothesis of a certain kind of “synchronization” of the atom and taking into account the electron sheath as a rotating system of charges, we explain the attractive force between atoms as an electromagnetic effect. The explanation of gravitation, which we present here, together with basic cosmic laws presented in a previous paper (Helmecke and Herkenrath 2003), leads to important consequences for the understanding of dark matter, black holes and the evolution of the Universe.

Key words: gravitation – unified theory – synchronization of the atom – dark matter – black holes – evolution of the Universe.

1. INTRODUCTION: AIM AND SCOPE OF THE PAPER

Our aim is to design a model in the sense of a scientific hypothesis in which the gravitational force is explained as an electromagnetic effect. Since mass is equivalent to energy, it seems to be natural that gravitation does not play an exceptional role in the series of basic physical forces or interactions, but is integrated into the electromagnetic character of the other physical forces. Thus a further search for gravitons or any new particles which transmit the gravitational force is superfluous.

We think that general experience in physics recommends to try to explain phenomena with as few causes as possible. Moreover, the search for new particles which could transmit the gravitational force was long and unsuccessful.

Another remarkable effect is that the law of gravitation has the same structure as Coulomb's law for the interaction between electric charges at rest: the force of attraction respectively repulsion (according to the signs of the charges) is proportional to the product of the magnitude of the charges and inversely proportional to the square of the distance between them (see, e.g., Purcell 1965, p. 7). Moreover, down to distances less than 10^{-12} m, no quantum modifications of the electromagnetic forces have to be taken into account (according to Purcell 1965, p. 3). Considering the size of an atom on the scale of 10^{-10} m and the fact that "electromagnetic forces are the only essential forces which act on electrons" (according to Wichmann 1989, pp. 34 and 45), the impact and importance of electromagnetic forces on atoms and any agglomeration of matter and its conditions are clear.

Of course, the explanation or model presented here is far from being complete or even detailed. It is up to physicists who study elementary particles to justify the model by suitable measurements.

A consequence of this model is the view that the basic or original subject of nature is electromagnetic energy and not matter; matter is conceived as materialized energy.

2. THE ELECTROMAGNETIC STRUCTURE OF THE ATOMS: THE ORIGIN OF GRAVITATION

The key to our explanation is that an atom is not completely neutral in the sense of electric charges. This effect is caused

(1) by a certain kind of synchronization of the atom;
and

(2) since the atom sheath is a rotating system of charges like, e.g., a ball-lightning.

These two principles rule the interplay between positive and negative charges of an atom in itself (1), and of different neighbouring atoms (2).

We refer to the wave-character of matter with respect to de Broglie, Schrödinger, and others (see, e.g., Gerthsen and Kneser 1966; Crawford 1968; Hey and Walters 1998).

In that context an electron can be modelled as a standing wave formed by a wave packet on some elliptical orbit around the nucleus. It forms a cloud of charge along its orbit. The spatial density of the cloud of charge is given by the intensity of the standing wave, which is defined as the square of the absolute value of the wave function ψ . The intensity, as a real-valued function with the unit interval $[0, 1]$ as

range, represents, respectively can be interpreted, as the density of the sojourn probability of the electron on its orbit. That means:

- the probability of the electron being in a neighbourhood of a point, where the intensity function has a value near 0, is very low;
- the probability of the electron being in a neighbourhood of a point, where the intensity function has a value near to 1, is very high.

In order to illustrate a standing wave pattern, we present Fig. 1, taken from Vlasak's (1999) book.

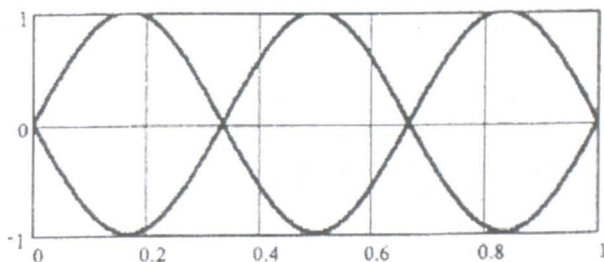


Fig. 1 – Standing wave pattern.

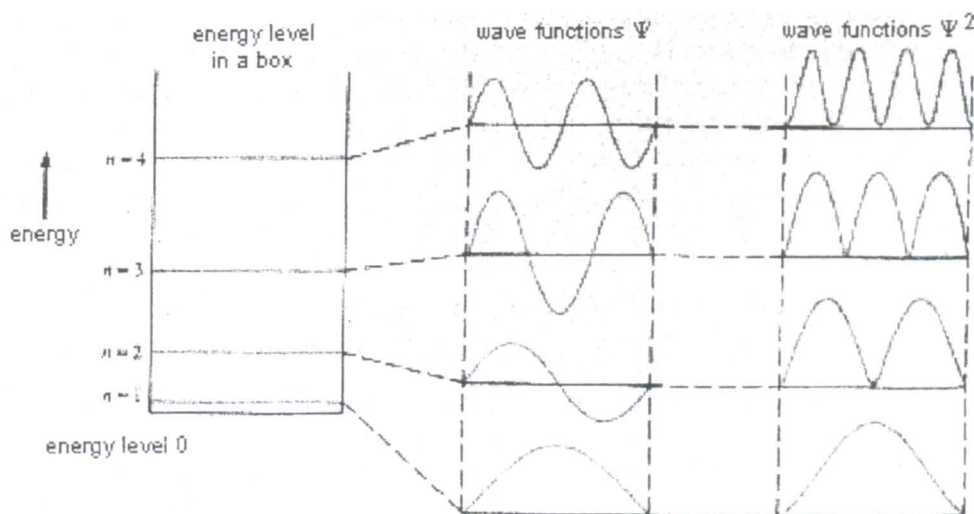


Fig. 2 – Wave functions and their intensities.

Fig. 2, taken from Hey and Walters (1998), shows, for an electron “in a box” for different levels of energy (left-hand part of the figure), the corresponding wave functions ψ (in the middle) and, next, the corresponding intensity functions ψ^2 (right-hand part).

Our hypothesis of the synchronization of the atom means that the distribution of protons in the nucleus corresponds to the electron sheath in such a way that positive radiation of the nucleus, i.e., the protons, overcomes the electron sheath, since the position of protons relates to those points in the electron sheath where the intensity function is near to zero.

Therefore, an atom is a “synchronized” system, as the interplay between protons and electrons is organized in such a way that the collection of all electrons does not interfere with the collection of all protons so as to neutralize the positive charge of the nucleus as a whole.

The size or the power of the excess of positive radiation of the nucleus depends on:

- the number of protons, neutrons and electrons;
- the distribution of the electrons over the different orbits of electrons;
- and, of course, the conditions of the neighbourhood of the atom under consideration.

3. THE ELECTROMAGNETIC INTERACTION BETWEEN ATOMS

As for the interaction between two neighbouring atoms, A and B , we have to consider: first, the excess of positive radiation of both atoms as two synchronized systems; next, the negative charges of both electron sheaths. What is the result of that interplay of different charges?

We have to take into account:

- first, there are two attractive forces, one $F_{nA,eB}^{\text{attr}}$ between the nucleus of A and the electron sheath of B , and, by symmetry, an attractive force $F_{nB,eA}^{\text{attr}} = F_{nA,eB}^{\text{attr}}$ between the nucleus of B and the electron sheath of A ;
- next, there are two repulsive forces, one $F_{eA,eB}^{\text{rep}}$ between the electron sheaths of A and B , and one $F_{nA,nB}^{\text{rep}}$ between the nuclei of both atoms.

The repulsive force $F_{nA,nB}^{\text{rep}}$ between the nuclei is extremely weak, since part of their positive charges is shielded by the both electron sheaths, and because of the “large distance” between the two nuclei in terms of Coulomb’s law.

The repulsive force $F_{eA,eB}^{\text{rep}}$ between the both electron sheaths is not strong, as the hulls are near to each other, both on a “mean level of intensity” and the charges of electrons are distributed along the whole orbit. Moreover, we point out the fact that the electron sheath is a rotating system of electric charges. The comparison of this system with a ball-lightning indicates that the repulsive force of the two electron sheaths is

very weak, even more, a ball-lightning, e.g., may attract more negative electric charge up to a certain level. Such an effect generally may be explained by the law of nature that rotation of any physical system creates a certain self-dynamics of the system, thus leading to a stabilization of the system.

Because of the synchronization of the atom, the dominant forces are the attractive ones: $F_{nA, eB}^{\text{attr}}$ and $F_{nB, eA}^{\text{attr}}$. The positive radiation of the protons is focused on certain “windows” within or along the orbits corresponding to sections of the orbits where the intensity functions of the electrons have values near to zero. This focused radiation attracts the electron sheath of the neighbouring atom.

The balance of these forces is:

$$2F_{nA, eB}^{\text{attr}} > F_{eA, eB}^{\text{rep}} + F_{nA, nB}^{\text{rep}}, \quad (1)$$

namely, the acting attractive forces are stronger than the sum of the both repulsive ones.

The remaining difference in the above inequality is the gravitational force $F_{g; A, B}$ between the two atoms. In other words, plugging $F_{g; A, B}$ in (1), the inequality turns to an equality:

$$2F_{nA, eB}^{\text{attr}} = F_{g; A, B} + F_{eA, eB}^{\text{rep}} + F_{nA, nB}^{\text{rep}}, \quad (2)$$

As a trivial consequence of (2), the attractive gravitational force between the atoms A and B is given by

$$F_{g; A, B} = 2F_{nA, eB}^{\text{attr}} - F_{eA, eB}^{\text{rep}} - F_{nA, nB}^{\text{rep}}. \quad (3)$$

By introducing a suitable constant g_a (the subscript “a” referring to the atomic scale), the force $F_{g; A, B}$ may be conceived as an attractive force between two point charges q_1 and q_2 at rest, of opposite sign, each point charge resting in the nucleus of its own atom. Therefore, $F_{g; A, B}$ may be represented as a force inversely proportional to the squared distance r^2 of the both centers of the nuclei, according to Coulomb’s law:

$$F_{g; A, B} = g_a \frac{q_1 q_2}{r^2}. \quad (4)$$

More detailed calculations for the very special case of the hydrogen atom can be found in Vlasak’s (1999) book.

There even remains an excess of positive radiation of the two neighbouring atoms, which is directed towards the electron sheaths of other neighbouring atoms. For

a physical body consisting of N atoms, an electromagnetic field of positive charges results. Newton's law of gravitation means:

For two physical bodies with masses m_1, m_2 , whose barycenters are situated at a distance r each other, the gravitational force between them is

$$F_G = g \frac{m_1 m_2}{r^2}, \quad (5)$$

with the gravitational constant g . In some sense, Newton's law is a consequence of Coulomb's law, which becomes active on the atomic scale because of our principles (1) and (2) (see Section 2).

4. CONSEQUENCES

Our model, respectively hypothesis, is based on an essential asymmetry of the interplay between positive and negative charges, depending on belonging to the same atom or not. Due to the "synchronization" of the atom and the rotation of electrons, the electron sheath is something like a semi-permeable diaphragm for positive charges: it allows the escape of an essential portion of radiation of its own nucleus, but shields its own nucleus against the positive radiation of neighbouring atoms. By such an interplay of positive and negative radiation, atoms may agglomerate to a body with an excess of positive radiation, which is active also outside the body, thus explaining gravitation.

In a previous paper (see Helmecke and Herkenrath, 2003), we dealt with and explained basic cosmic laws concerning dimensions, (evolutionary) causality, and stability/instability. Those cosmic laws, in combination with our present hypothesis on gravitation, lead to important consequences for dark matter, black holes, and the structure, respectively evolution, of the Universe.

Since the matter of a black hole lost its atomic structure, black holes only attract and absorb other matter. They have an effect on other matter, but do not interact with that. The condensation of matter in a black hole does not only destroy the space-structure of the atoms, but, even more, the space-time structure.

That implies for cosmic evolution. Since the Big-Bang, the Universe expands into all directions. Within the galaxies, black holes arise due to excessive mass-concentrations, according to the cosmic law of critical stability (see Helmecke and Herkenrath, 2003). Those black holes destroy the surrounding matter and thereby the space-time structure. Since the natural dimensionality disappears, the distances in space of the different black holes disappear, too. There arises a singularity, in which the entire energy of the Universe is concentrated. As the principle of evolutionary causality remains valid (see Helmecke and Herkenrath 2003), and the stability of that state reaches a critical value, a new Big-Bang arises, according to the cosmic law of

critical stability. A slowing down of the expansion of matter up to its reversion is not necessary to explain origin and end of the Universe.

Finally, if the gravitational effects can be explained as electromagnetic interactions, no concept of dark matter is needed, i.e., a corresponding matter needs not to exist, but only electromagnetic energy.

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THE INTERNATIONAL CONFERENCE “EXPLORING THE SOLAR SYSTEM AND THE UNIVERSE” Bucharest, Romania, 8–12 April 2008

The place and role of Romanian astronomy in the international context are well known, but a permanent image improvement is required, especially due to the integration of national research into the European programs.

The interest in astronomy at the international level was decreasing in the last years, but now it knows a new upsurge in the context of the 7th Framework Programme of the European Commission. The activities due to the International Heliophysical Year and the upcoming International Year of Astronomy contributed consistently to this.

The visibility of the Romanian astronomical researches (both past, but especially present ones) at international level has considerably increased after 1990. The decisive contribution to this was brought by the Astronomical Institute of Romanian Academy. Theoretical and observational results, as well as resounding discoveries, have become better known. Key factors in this development have been the ever more extensive international cooperation and ever more significant participation of the Romanian astronomers in prestigious scientific meetings, as well as in working or doctoral stages. It is also worth mentioning that today Romanian astronomers are part of the management boards of international astronomical organizations.

At national level, the interest in Romanian astronomy continues to be rather low. In spite of an intense outreach, as well as of the growing cooperation between the Astronomical Institute and other research and high-education centers, the decision factors' lack of interest in astronomy impedes the natural development of this research field.

This was the international and national context at the time of the centenary of the Astronomical Institute of Romanian Academy. The Astronomical Institute's centenary celebration (1st of April 2008) was the best opportunity for the organization of an ample anniversary international conference, aimed at providing a forum for a detailed presentation of the scientific results obtained in this long period of time, as well as a large exchange of scientific ideas with researchers of the European and international astronomical community.

In this context, the Astronomical Institute organized the International Conference “Exploring the Solar System and the Universe”. It was held from April 8 to 12, 2008, at the Romanian Academy, in Bucharest. This event was the most important astronomical scientific meeting ever organized in Romania. This could be appreciated by looking at: fields approached by Romanian astronomy (classical and present), part of modern astronomical research; general number and scientific prestige of participants; participation with contributions of all researchers of the Astronomical Institute; participation of numerous specialists in astronomy from all over Romania; the number and quality of the contributions presented.

Besides the celebration of the Institute's centenary, another main goal of the conference was the promotion of the European and international dimension of Romanian astronomy. The response from the

astronomical community was enthusiastic, with more than 100 participants (from 22 countries: Argentina, Australia, Belgium, Bulgaria, Canada, Czech Republic, Denmark, France, Germany, Greece, Israel, Italy, Japan, Mexico, Poland, Romania, Russia, Serbia, Spain, UK, Ukraine, USA) and 109 contributions presented. The conference consisted of four one-day symposia, each one structured on invited talks, oral presentations, and posters.

There were many invited speakers from: Argentina (Lilia Bassino), Belgium (Stefaan Poedts), Canada (Jaymie Mark Matthews and Cristina Stoica), Czech Republic (Petr Heinzel, Jan Palouš, and Jan Vondrák), Denmark (Hans Kjeldsen and Birgitta Nordstrom), France (Annie Baglin, Chantal Balkowski, Ziad Mouradian, and Brigitte Schmieder), Greece (Eleni Rovithis-Livaniou), Italy (Mauro Messerotti), Japan (Umin Lee and Hideyuki Saio), Mexico (Ernesto Pérez-Chavela), Poland (Wojciech Dziembowski), Romania (Cristiana Dumitrache, Vasile Mioc, Magda Stavinschi, and Marian Doru Şuran), Spain (José Luis Ballester), UK (Cristina Popescu), USA (Terry G. Forbes, Donald Gene Saari, and Zhihong Jeff Xia).

Symposium 1, *History of Romanian Astronomy, and Education by Astronomy in the European Framework*, held on 8 April, tackled problems and subjects as: moments and figures from the history of Romanian astronomy, in general, and from the history of the Bucharest Observatory of the Astronomical Institute, in special; Romanian cooperation in astronomy along centuries; old astronomical institutions in Romania; Romanian contributions to the International Heliophysical Year, to the International Year of Astronomy, and so forth (especially from the education standpoint); role of the education via astronomy in the context of the needs and requirements of the space era. There were 15 oral presentations and 8 poster presentations, as follows:

Oral presentations:

- M. Stavinschi, V. Mioc: *100 years and more of Romanian astronomy*
- L. Bennaceur: *The family of Nicolae Coculescu, the first Director of the Bucharest Observatory*
- Z. Mouradian: *Discovering astronomy 50 years ago*
- S. Débarbat: *Astronomy at the Observatoire de Paris at the time of Notara's visit*
- P. Rovithis, E. Rovithis-Livaniou: *Educational actions of some Greek scholars in Romania*
- M. Tsvetkov: *European Virtual Observatory – Bulgarian and Romanian cooperation: a historical overview*
- M. S. Dimitrijević, M. Stavinschi: *Romanian-Serbian collaboration in astronomy*
- H. Minți: *Beginnings of the modern astrophysics in Bucharest Observatory*
- M. D. Şuran: *Cosmology in the Bucharest Observatory*
- G. Mariş: *Romanian solar physics research within the framework of international cooperation (1955-2005)*
- F. Szenkovits: *The first astronomical observatory in Cluj-Napoca*
- C. Balkowski: *Astronomy education in France*
- C. Dumitrache, N. A. Popescu, V. Mioc: *Romanian contributions to the International Heliophysical Year*
- C. Mosoia, E. A. Pică, M. Stavinschi: *Romania before the International Year of Astronomy*
- M. V. Rusu, M. Stavinschi: *Moments of astronomy education in Bucharest*

Posters:

- G. Pinigin, M. Stavinschi: *Bucharest-Nikolaev astronomical observatories – collaboration in astronomy*
- E. Theodossiou, V.N. Manimanis, M. S. Dimitrijević, E. Danezis: *Rigas Velestinlis and astronomy in his "Anthology of Physics"*
- L. Farkas: *Astronomy-connected scientific works in early Transylvania and Banat*
- N. Pazmany, M. V. Rusu: *Astronomy in fortress of Oradea*

- T. Oproiu, E. A. Pică: *Decorative elements with astronomical subjects on medieval buildings in Transylvania*
 E. Botez: *Remember on some among those who passed/crossed the Bucharest Astronomical Observatory*
 S. Cojocaru, M. Păvăloiu: *Astronomy in the Romanian naval education*
 O. Bădescu: *Astrometry knowledge in geodetic education*

Symposium 2, *Celestial Mechanics and Astrometry Today*, held on 9 April, was oriented mainly to theoretical problems: Saari's celebrated conjecture and its implications in intra- and intergalactic dynamics; Hénon-Heiles model in intragalactic dynamics; chaos and stability in problems of dynamical astronomy; dynamics in post-Newtonian fields. But many practical problems of today's astrometry were equally discussed: mutual phenomena in the Solar System; campaigns of astrometric observations from ground and from space; astrometric catalogues; databases and virtual observatories. There were 22 oral presentations and 12 poster presentations, as follows:

Oral presentations:

- D. G. Saari: *Virial theorem, dark-matter, and N-body problems*
 F. Diacu, T. Fujiwara, E. Pérez-Chavela, M. Santoprete: *Saari's conjecture in celestial mechanics*
 C. Stoica: *Constant inertia trajectories: on Saari's conjecture and more*
 V. Mioc, D. Pașca, C. Stoica: *A double pitchfork bifurcation in a generalized Hénon-Heiles problem*
 Z. J. Xia: *Symmetries in N-body problems*
 A. V. Kyuldjiev, V. S. Gerdjikov, V. Mioc: *On the local equivalence between Manev and Kepler problems*
 K. Tsvetkova, V. Mioc: *Manev's field problem in contemporary science*
 T. Kalvouridis: *3-D motions in the field of two rotating oblate magnetic dipoles*
 J. Vondrák, V. Štefka: *Combination of space- and ground-based astrometric observations to create astrometric catalogs*
 J.-E. Arlot, M. Birlan, V. Robert, V. Lainey, D. Pascu, L. Winter, J.-P. de Cuyper, G. Bocşa, L. Șerbănescu, E. Khrutskaya: *Exploration of the Solar System: projection of the past into the future*
 J. Souchay, A. H. Andrei, C. Barache, S. Bouquillon, A.-M. Gontier, S. B. Lambert, C. Le Poncin Lafitte, F. Taris, E. F. Arias, D. Suchet, M. Baudin: *The LQAC compilation of quasars catalogues*
 M. Birlan, A. Nedelcu: *Astrometry in the Uranian system of satellites*
 T. Bonev: *Dust particles of comet Schwassmann-Wachmann 3 during its return in 2006*
 A. V. Ivantsov, L. A. Hudkova, Z. Aslan, R. Gumerov, I. Khamitov, G. Pinigin: *Positions observations of NEAs at the RTT-150*
 L. A. Hudkova, A. V. Ivantsov: *Error statistics for position observations of numbered asteroids in six observatories of the world*
 D. A. Nedelcu, M. Birlan: *Ground-based science of ESA's Rosetta mission targets: (21) Lutetia and (2867) Steins*
 P. Popescu, M. Assafin, O. Bădescu, D. A. Nedelcu, P. Paraschiv, L. Iliev, A. Antov: *A pilot astrometric survey of 59 Northern ICRF radio-sources*
 P. Paraschiv, O. Bădescu, D. A. Nedelcu, P. Popescu, L. Iliev: *Narrow-field astrometry with different-quality CCD cameras*
 N. Maigurova, G. Pinigin: *Results of CCD observations of ecliptic zone in Nikolaev Observatory*
 K. Tsvetkova, P. Popescu: *Astrometric plates in the Bucharest and Cluj observatories archives*
 G. Bocşa, P. Popescu: *Qualitative study of the plates observed with the photographic equatorial of Bucharest Observatory since 1930*
 Y. Chapanov, J. Vondrák, C. Ron: *Decadal oscillations of the Earth rotation*

Posters:

- M.-C. Anisiu: *Spatial families of orbits in 2D conservative fields*
 F. Szenkovits: *On the elliptic restricted three-body problem*
 M. Ciobanu: *Euler cone aperture in a special Euler-Poinsot case*
 Z. Makó: *Chaotic variation of the capture effect around the weak stability boundary*
 R. Roman, T. Oproiu: *The dumb-bell's restricted, photogravitational, circular three-body problem*
 G. Bocşa, M. Tsvetkov: *Measuring and scanning methods in astrometric processing of the photographic plates*
 O. Bădescu, M. Ploeanu, D. A. Nedelcu, P. Popescu, P. Paraschiv: *Geodetic astronomy and modern astrometry developments*
 Y. Chapanov: *Earth rotation response to ENSO events*
 Y. Chapanov, D. Gambis: *Influence of AAM and OAM on the Universal Time variations*
 L. Şerbănescu: *Recovering information from photographic plates*
 G. Bocşa: *The study of the dynamics of HD 180642 area from the CoRoT programme*
 A. Drăguşan, S. Lupu, S. Cojocaru: *Astronomical principles of satellite positioning*

Symposium 3, *Dynamics of Solar Atmosphere and Heliosphere*, held on 11 April, approached very actual problems of solar physics: solar coronal structures; acoustic observations of solar flares; study of solar atmosphere via observations from the space missions; Sun-Earth effects of the solar radio activity; mathematical modeling and simulation of the phenomena in the solar atmosphere. There were 20 oral presentations and 1 poster presentation, as follows:

Oral presentations:

- Z. Mouradian: *A new method for measuring solar activity rotation*
 S. Ustyugov: *Realistic simulation of solar local supergranulation*
 P. Heinzel: *Understanding the solar chromosphere*
 T. G. Forbes: *Predicting the onset of solar eruptions*
 D. Beşliu-Ionescu, A.-C. Donea, P. Cally: *A comparison of the acoustic hardness of acoustically active and non-active solar flares*
 A. R. Osokin, M. Livshits: *Solar flares of different types and their influence on the formation of interplanetary medium disturbances*
 L. Mircea: *Effects of large solar events on atmospheric drag of Earth artificial satellites*
 B. Schmieder, T. Török, G. Aulanier: *Magnetic field changes preceding filament eruptions and coronal mass ejections*
 I. A. Bilenko: *Conditions for the formation of CMEs associated with filament eruptions*
 D. R. Constantin, C. Dumitrache, C. Oprea, M. Mierla: *Polar filament evolution*
 I. Chifu, O. Chiricuţă, D. R. Constantin, C. Dumitrache: *Multiwavelength study of active region 09778*
 M. Messerotti: *Observing, modeling and predicting the effects of solar radio bursts on radio communications*
 J. L. Ballester: *Oscillations in coronal structures: recent progress*
 S. Poedts, A. Soenen, F. P. Zuccarello, C. Jacobs, B. van der Holst: *Magnetic flux emergence and shearing motions as CME trigger mechanisms*
 C. Dumitrache: *Contribution to the Sun-heliosphere studies using SOHO and Ulysses data*
 A. S. Popescu, M. Mierla: *Mass estimates and wave propagation for the eruptive event of 8th of January 2002*
 A. Marcu, G. Mocanu, B. Orza: *Standing waves in solar periodic structures*
 M. Lazar, R. Schlickeiser: *On the origin of turbulent fields in interplanetary plasmas*

A. Oncica, N. A. Popescu, C. Dumitrache: *A CME-ICME sequence investigation, August 14–15, 2001*

E. Popescu, N. A. Popescu: *Models for heavy tailed data and applications*

Poster:

O. Chiricuță, D. R. Constantin, C. V. Oprea, I. Chifu, A. Oncica, C. Dumitrache: *The study of AR09767 and its neighbourhood*

Symposium 4, *Stellar Astrophysics, Extragalactic Astronomy, and Cosmology*, held on 12 April, tackled many fascinating domains in today's research: pulsations of magnetic and/or rotating stars; new methods in stellar variability research; asteroseismology as main tool for investigating stellar interior; exoplanets; investigation of pairs, groups, and clusters of galaxies, and of their dynamics; observation campaigns from ground and from space. There were 15 oral presentations and 16 poster presentations, as follows:

Oral presentations:

H. Saio: *Modelling pulsations of magnetic Ap stars*

U. Lee: *Pulsational stability of rotating stars*

W. A. Dziembowski: *Looking into B star interiors by means of asteroseismology*

E. Rovithis-Livanou: *Exoplanets: an overview*

A. Baglin: *Ultra-high photometry from space – first results*

J. M. Matthews: *Stellar and exoplanetary astrophysics in the era of space-based ultraprecise photometry*

E. Michel, A. Baglin, and CoRoT team: *CoRoT, a new insight on the field of stellar oscillations*

H. Kjeldsen: *Measurements of stellar structure through asteroseismology*

B. Nordström: *Our galactic neighbourhood – a melting pot of migration*

L. P. Bassino: *Extragalactic globular clusters: tracers of galaxy evolution*

C. Balkowski: *Environmental effects on galaxies in clusters*

C. C. Popescu, R. J. Tuffs: *Dust in galaxies*

J. Palouš, R. Wünsch: *Star formation and evolution of galaxies*

M. D. Şuran: *Seismology of the Universe – the inverse method in cosmology*

N. A. Popescu: *Galaxy pairs and groups in the AGNs' environments*

Posters:

R. Konstantinova-Antova, M. Aurière, E. Alecian, P. Petit, J.-F. Donati, G. Wade, S. Boeva, L. Iliev, A. Kostov: *Magnetic field study in single late giants*

M. S. Dimitrijević, M. Christova, Z. Simić, S. Sahal-Bréchet: *Electron-impact broadening of Ar I 737.212 nm spectral line for stellar atmospheres research*

M. S. Dimitrijević, A. Kovačević, Z. Simić, M. Dačić: *Stark broadening of O V 1371 Å line in stellar atmospheres*

A. Dumitrescu, M. D. Şuran, Ş. G. Sorescu-Surdu: *Exo-planets transits: orbit and size estimations*

A. Dumitrescu, M. D. Şuran, Ş. G. Sorescu-Surdu: *First ground-based observations of the eclipsing binary V449 Aur made at Bucharest Observatory*

L. Farkas: *The Beta Cephei type variable star BW Vulpeculae*

D. Ficuț-Vicaș: *LITTLE THINGS survey – a quest to understand dwarf galaxies*

V. S. Gerdjikov, V. Atanasov, V. Mioc: *Constructing exact solutions for special cases of Einstein equations in vacuum*

L. Iliev, A. Dumitrescu, M. D. Şuran, Ş. G. Sorescu-Surdu: *Recent spectral observations of V449 Aur*

D. Moldovan: *PHEMU 2003 campaign – CCD observations made at Cluj-Napoca Astronomical Observatory*

A. Pop, R. Roman: *RT Andromedae: period variation revised*

A. Pop, C. Vamoş: *Detection and diagnosis of low-level stellar variability in non-evenly sampled data*

A. Pop, V. Turcu, A. Marcu: *γ Leonis reloaded*

A. S. Popescu: *Knot-based large-scale structure code*

M. D. Şuran, D. Pricopi: *About the star ϵ Oph*

V. Turcu, A. Pop, D. Moldovan: *GP Andromedae survey byproducts*

All the 37 contributions presented as poster papers were exposed all along the conference. Such a rich “exhibition” constituted a good opportunity for new proposals of international cooperation.

We have to remark that all the researchers of the Astronomical Institute (from Bucharest, Cluj-Napoca and Timişoara observatories) participated in the conference with presentations. We also remark the contributions of many professors and teachers with preoccupations and results in astronomy (from Bucharest, Cluj-Napoca, Constanţa, Iaşi, Oradea, Sibiu, Târgu Mureş).

An important contribution to the visibility of the Romanian astronomy was brought by the presence of Romanian specialists who live and work abroad today, in Australia, Canada, France, Germany, Israel, UK, USA. Both their presence and their contributions constitute a supplementary evidence of the Astronomical Institute’s efforts aimed at increasing the prestige of the national astronomy and promoting its international dimension.

Many presentations resulted from a fruitful cooperation between Romanian and foreign astronomers. Among such cooperation results we mention the papers signed by co-authors from: Romania-Australia, Romania-Belgium-France-Russia-USA, Romania-Bulgaria, Romania-Canada, Romania-France, Romania-Serbia, Romania-Ukraine.

Lastly, this outstanding event constituted an excellent opportunity for the presentation of many other contributions elaborated within international cooperation, such as: Belgium-Brazil-France, Bulgaria-France, Bulgaria-Czech Republic, Canada-Japan-Mexico, France-Serbia, Greece-Serbia, Russia-Turkey-Ukraine. This is another argument in favour of the international prestige of the conference.

On the same occasion, three booklets were published by the Astronomical Institute and were distributed at the conference: M. Stavinschi, C. Dumitrache, V. Mioc, N. Popescu, M. D. Şuran, *Astronomical Institute of the Romanian Academy, Bucharest Observatory Centenary, 1908-2008*, Griffon & Swans, Bucharest; M. Stavinschi, *Histoire céleste entre la Roumanie et la France*, Griffon & Swans, Bucharest; V. Mioc, C. Dumitrache, N. Popescu (eds), *Exploring the Solar System and the Universe*, Matrix Rom, Bucharest.

The main aim of the conference was twofold. On the one hand, to provide a realistic image of the Romanian astronomy to the international astronomical community. On the other hand, to provide a scenario to Romanian advanced students and researchers on the actual topics of astronomy, in order to keep them in contact with work going on in other parts of the world and, at the same time, to motivate and support the young and mid-career researchers from Romania. To achieve this goal, we invited as lecturers the most distinguished experts in the subjects of the conference.

The contributions presented at the conference are published by the prestigious Editing House of the American Institute of Physics; see V. Mioc, C. Dumitrache, N. A. Popescu (eds), *Exploring the Solar System and the Universe*, American Institute of Physics Conference Proceedings Series, Vol. 1043, Melville, New York, 2008. Unfortunately, not all the contributions are presented in this volume. There were unavoidable last-minute cancellations, there also were problems of bureaucracy that made the sending of papers too late.

The conference could not have been achieved without the financial support of the Romanian Ministry of Education, Research, and Youth (via the Contract No. 243/2006), and, obviously, of the Romanian Academy. The support of the Embassy of France, French Institute in Bucharest, for a Romanian-French evening meeting is duly acknowledged.

Last, but not least, Academician Ionel Haiduc (President of Romanian Academy) and Academician Romulus Cristescu (President of the Section of Mathematical Sciences of Romanian Academy), supported consistently the Astronomical Institute in this international endeavour.

We conclude that the centenary conference was a brilliant opportunity to increase the international visibility of the Romanian astronomical community.

Vasile Mioc, Cristiana Dumitrache, Nedelia Antonia Popescu

NOTICE TO AUTHORS

ROMANIAN ASTRONOMICAL JOURNAL appears twice a year and is open to original contributions in Astronomy and related disciplines. The contributions – in English or French – can be accepted only if they were neither published before nor destined to another publication.

Manuscripts should, preferably, be submitted as e-mail attachments to roaj@aira.astro.ro, to the Editor in Chief (vmioc@aira.astro.ro), or to one of the members of the Editorial Board. They must be written in a well-known editor, preferably a recent version of Word. Authors who cannot submit an electronic version of their manuscript should send it to the Editorial Board. In this case two copies of the manuscript (along with the text of the article on floppy disk) are required. The first page should contain: article's title (brief and informative), author's name and affiliation, followed by an *Abstract* in English and *Key words*. The text should be clear and concise (it is recommended not to exceed 10 pages). The *Abstract* will present clearly the principal conclusions on the work, in no more than 10–15 lines.

Chapters and Paragraphs. Papers, except short notes, should be divided into chapters, numbered by Arabic numerals. Chapters may be divided into paragraphs denoted by the number of the chapter and the number of the paragraph; each chapter and each paragraph should have a short descriptive title (e.g. "3.2. Results").

Formulae have to be centered and numbered consecutively in Arabic numerals, too, but included in brackets on the right-hand side of the manuscript.

Tables should be numbered consecutively in Arabic numerals; they should be introduced in the text at their appropriate place.

Figures and Illustrations should be submitted separately, in such a form as to permit reproduction without retouching. Any lettering should be large enough to be legible after the figure has been reduced in size for printing. Captions should be introduced in the text at their appropriate place. All figures should be numbered consecutively in Arabic numerals and referred to in the text, e.g. Fig. 2 or Figs. 2–5. Photographs should be given only if essential and should be enlarged enough to permit clear reproduction.

References are indicated in the text by the author's name and year of publication. They should be listed in alphabetic and chronologic order at the end of the paper, as follows: name and initials of the author(s), year of publication, suitable abbreviation of the journal (or title of the book and editing house), its volume and page.

The *fonts* to be used are: 11 for the text proper, 13 for the paper title, 9 for author's name and affiliation, abstract, key words, titles of chapters and paragraphs, figure captions, tables, running titles, and references.

Detailed instructions for the preparation of manuscripts can be found at <http://www.astro.ro/~roaj>.

Please pay attention to these recommendations; the manuscripts that do not observe them will be returned to the authors.

ROMANIAN ASTRONOMICAL JOURNAL

Vol. 18, No. 1, 2008

CONTENTS

Magda STAVINSCHI, Vasile MIOC, The Bucharest Observatory of the Astronomical Institute at Centenary: 100 Years and More of Romanian Astronomy	3
Ponsamy SRIRAMACHANDRAN, Ramachandran SHANMUGAVEL, Narayanan RAJAMANICKAM, Somashekar Pundalikarao BAGARE, Evaluation of Astrophysically Useful Parameters for LaO Molecule	21
Ovidiu C. FURDUI, On the Stability of Planetary Orbits with a Modified Hermite Integrator.....	29
Vasile MIOC, Rodica ROMAN, Libration Points in Fock's Restricted Three-Body Problem	41
Mohamed Adel SHARAF, Bielliptic Transfer with Plane Changes	51
Mohamed Adel SHARAF, Determination of a Celestial Plane Orientation: Application to Milky Way	61
Gheorghe BOCŞA, Milcho TSVETKOV, Precise Determination of Minor Planet Positions	69
Ivan KOTLIAROV, New Laws of Planetary Distances	77
Gerd HELMECKE, Ulrich HERKENRATH, From Synchronization of the Atom to Organization of the Universe via Gravitation	85

MISCELLANEA

The International Conference "Exploring the Solar System and the Universe", Bucharest, Romania, 8–12 April 2008 (Vasile MIOC, Cristiana Dumitrache, Nedelia Antonia Popescu)	93
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