CIRCLE AND LINE AS PATTERNS IN GREEK PRESOCRATIC THOUGHT

BY

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Geometry is generally referred to as the one best pattern of the Greek thought, as a purely rational view of the world. Modern sociologists and cultural anthropologists as, for instance, J. -P. Vernant are among the staunch supporters of this view. Moreover, the history of mathematics has long before furnished the general suggestion that geometry would have gained pre-eminence over arithmetics in the mathematical world itself [1]. I would raise two objections to this view.

First, it should be noticed that Greek geometry is not in the least confined to the study of regular shapes or to the setting of rules over a strictly rational space. But what is of interest is that it deals with the incommensurable as well. The discovery of irrational values, which was paradoxical, at the time came off from an attempt to see what magnitude and space as a whole should be, and A. Szabó has accurately proved that, since the Greeks found in geometry and not in arithmetics such notions as 'inexpressible' (ἄρρητον), 'incommensurable' (ἀσύμμετρον) and 'irrational' (ἄλογον), these are actually experienced as belonging to magnitudes.

The second objection is that despite the use of geometrical calculus throughout the Greek antiquity, geometry itself was only an ancillary discipline, ranking somewhat below arithmetics, which was deemed to be nobler and more important.

Number and magnitude were each intuited relatively early, and both arithmetics and geometry were to focus on them. Nicomachus of Gerasa must have been supported by tradition since he so precisely separated the number as an arithmetic element from magnitudes which were given a geometrical description. He claimed that number is a property of « discrete » objects such as « group », « choir » etc., whereas magnitudes would go with « continuous » things in nature : « animal », « tree » etc. [2] According to this typology, which was highly popular with the Greeks, geometry as a science of magnitudes not only opposed the science of

[1] As is implied, for instance, in Van der Waerden *Erwachende Wissenschaft*, pp. 204—206 (*ap.* A. Szabó, *The Beginnings of Greek Mathematics*, Budapest, 1978, p. 95). For J.-P. Vernant, see *Mythe et pensée chez les Grecs*, Paris, 1969, pp. 95—103.

[2] Nicom., *Introd. Arithm.*, ii—iii.

StCl XXIII, București, p. 13—20

numbers which was arithmetics, but also appeared as inferior to it. We can take Proclos' evidence for this fact: « It has been supported by the elder that geometry is a section (μέρος) of general mathematics (τῆς ὅλης μαθηματικῆς) and that it is second-best to arithmetics (δευτέραν τάξιν ἔχει), ...and so there is no need for extra-arguments ». And he adds that « numbers are more immaterial and pure than magnitudes » (ἀριθμοὶ τῶν μεγέθων ἀυλώτεροι καὶ καθαρώτατοι [3]).

Proclos' testimony falls short of explaining how the relation between geometry, inferior to arithmetics, and magnitudes, inferior to numbers, really worked. Yet his statement in the above quotation sounds much like the famous sentences wherein the old Pythagoreans used to praise the Number as the perfect Being. If conjoined with the former considerations on the superior nature of arithmetics as a whole, such sentences could enforce us to trace the theory of number superiority as far back as the vii[th] century. Hence less « pure » or « noble » than arithmetics, geometry appears nonetheless as a more comprehensive even though less definite domain.

With all that ancient evidence, it is all the more difficult to imagine how could geometry have endowed the Greek mind with a fundamental pattern — a fact which is beyond controversy.

The explanation I am trying to offer is that Greek geometricism preserved to some extent certain of its mythical remnants, which therefore have continually projected the eldest structures of space foreward into the growing scientific model of a rational world. It seems that this process can be traced back as far as its mythical sources, by means of two opposite notions: the straight and the circular. I have considered this opposition particularly important as it underlines Greek geometry throughout and, what is more important, it assumes in a simple and most general form a specific structure which may be found in any domain of Greek thought and experience.

In early Greek ages, space was experienced indiscriminately. In an analysis of the structural patterns of that period [4], Detienne and Vernant have described it in terms of a shifting (αἰόλος), chaotic (ἄπορος) and manifold (πολύτροπος) space, in which movements were devoid of direction. This elementary space consisted, as it were, of mere broken and curved shapes. They were unmistakenly assimilated with each other as no functional criterion had then been given. Once this criterion was formulated, it helped things towards complete geometrical order; and this process began by separating the curved from the broken, until they changed into a distinct pair of opposites: the circle and the straight lines.

The mythical cycle of Daedalus incorporates some elements of this process. Based upon Françoise Frontisi-Ducrous' re-

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[3] Procl., 48, 9 sq.; 95, 23 sq.

[4] M. Detienne — J.-P. Vernant, *Les ruses de l'intelligence*, Paris, 1974, pp. 25, 162.

search [5] one could now draw a comparative list of the tools traditionally assigned to Daedalus and Talus :

Daedalus	Talus
τύλοι, ἧλοι, γόμφοι (nails, spikes, wooden dowels)	κεραμικὸν τρόχον (potter's wheel)
κόλλη, ἰχυόκολλα (glue ; the gluing techniques)	
τρύπανον, τέρετρον (boer, wimble)	τόρνος (sort of compass : neddle and thread ?)
κάθερος (mason's balance)	
στάθμη (carpenter's rule ; string ?)	κιρίνος, διαβήτης (compasses)
πέλεκυς, σκέπαρνον (hatchelet)	
πρίων (saw)	

That these heroes were ascribed such tools so as to meet complementary skills, induces one to conclude that the battle of rivals between Daedalus and Talus relied upon a more abstract « fight » : the narrative was also suggestive of a geometric rivalry between the circular properties and the linear properties of space.

The choice of instruments indicates that this opposition was not settled as yet. Both στάθμη and κάθερος vacillate between describing straight and circular movements. The same waverings are to be found in Theognis' description of the Delphic messenger : he took three specific tools along with him, the last of which, στάθμη, can not be properly related to any geometrical form.

These confusions do not impair the evidence that two distinct models of space were already emerging. Our idea of the relation existing between this process and various periods in the Greek history can not be more precise than it was to the Greeks themselves. The Hellenistic poet Callimachus, for instance, had a half-mythical feeling about it and he expressed it in his evoking the legendary Euphorbus, of whom he said that "he was the first man who used to draw (ἔγραψε) both scalene triangles and circles" (τρίγονα σκάληνα καὶ κύκλον ἥλικα) [6]. Full mention of a regular opposition between circular and straight shapes is made in the famous Pythagorean *tabula oppositorum*, which stresses on their value as paradigms of the world. In Aristotle's record, εὐθύ and καμπύλον — that is, « straight » and « curved » (or even « circular », as the case may be) — stand among the ten Pythagorean pairs of opposites [7].

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[5] See Françoise Frontisi Ducroux, *Dédale. Mythologie de l'artisan en Grèce ancienne*, Paris, 1975, pp. 129—134 and *passim*.

[6] Callimachus, 1, 52, fr. 191 Pfeiffer (= Thales A 3 a).

[7] Arist., *Met.*, 986 a 22—26.

The man who represents at best the whole formative process of Greek geometry is Thales of Miletus. All his mathematical fragments which have come down to us speak of the feeling of tension between angular and circumferential shapes. On the one hand, Thales aimed at a clear-cut separation of the properties conveyed by either of them. We are told, for instance, that we owe to him the first attempt to demonstrate (ἀποδείξασθαι) that « the circle is divided into equal parts by its diameter » (A 20 : διχοτομεῖσθαι τὸν κύκλον ὑπὸ τῆς διαμέτρου). On the other hand, Thales was obviously in search for the lost unity between the linear and the circular rules of space, in as far as to aim at building a particular knowledge on it. It was said that « he was also the first who proved that, when related to the cycle run by the sun through heavens, the sun's breadth represented the seven hundred twentieth part, and so did the moon's when related to the same circumference » (A 1, 24).

From this moment on, the Greeks attempted to recover the primordial unity which had existed in the early age of their geometry. The scope of this tendency may be seized from two geometric definitions due to Democritus (68 b 155 sqq.), which were accurately restored by Fr. Lasserre [8]. One of them « described every circular figure as the limit of a polygon the number of whose sides is increasing to infinity ». In the second definition, which started from a controversy over the tangent, it is assumed that the sphere is « a sort of angle ». In Lasserre's own explanation, Democritus thus meant « that every circle forms with its tangent, at the point where they meet, an infinitesimal 'angle' and, separating from it by an equally infinitesimal increase of this angle, ends by closing back on itself ».

During the v[th] century the opposition between circle and line must have gained its own right and it became authoritative to the extent of being coined by philosophers in their own symbolic language. Even Heraclitus' version of the natural world proved apt to be expressed in terms of geometrical contraries. A fragment from Heraclitus states that, when simultaneously given — as in the case of the κοχλίας, a sort of screw, whose movement is both upright and circular — the circular (κύκλῳ) and the upright (ἄνω) movements are fused and this renders the above mentioned straight (εὐθεῖα) and curved (σκολιή) lines identical (B 59).

I have met the most spectacular evidence for this fashion of thought in Plato's *Phaedo*. At 72 b, Socrates, who has by now succeeded in describing the cosmic generative motion (γένεσις) in terms of an epistemological process, declares that this should necessarily involve two contraries, each of which he calls τὸ ἕτερον ('the other'). For better understanding of his statement, he concludes with a metaphorical *reductio ad absurdum* of the entire argument. The interlocutors are thus forced to admit that the opposites (τὰ ἕτερα) form the basic structure of our world, in the absence of which the whole universe would die out (παύσασθαι). Socrates' metaphor relies on the picture of what he assumes to be the two models of any conceivable human knowledge : the good one, the mo-

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Fr. Lasserre, *The Birth of Mathematics in the Age of Plato*, transl. by Helen Mortimer, 1964, pp. 19–20.

tion of which is called circular (ἐν κύκλῳ) and which matches the γένεσις; and the misleading one, which complies with the image of a linear movement, similar to the whirl of the universe towards its final destruction. The contrast here springs from the pertinent use of geometrical categories [9]. Two patterns of the contrary are thus revealed: (1) τὸ ἕτερον, which only survives inside the circular space (κύκλος). The opposite directions can in this case be simultaneously given by means of ἀνταπόδοσις; (2) τὸ καταντικρύ, which entailes the existence of a rectilinear space (εὐθύς). This can be properly described in terms of an infinite univocal growth.

Phaedo 72b is a piece of evidence for how the circle and the line as a couple of opposites may stand for the larger ethical and philosophical problem of truth opposing falsehood, of reality opposing appearance, etc. This symbolism requires further analysis of the general properties of space and of their assignment to circle and line, respectively.

The existence of a split inside the Greek geometry has been noticed even by scholars who had no particular concern for such matters. In this respect, one can recall A. Rey, who noticed that « the elements for a geometry of circumference and of circle have been found and specified later than the elements for a geometry of line and of rectangular surfaces » [10]. This functional division of the Greek space into circularity and linearity deserves a detailed inquiry as this will explain why geometry has simultaneously been a privileged symbol of the Greek rationalism and the « non-defined » space formed by irrational and indeterminate magnitudes banned continuously from the realm of numbers. This is tantamount to saying that the same opposition between a rational and an irrational element which finally succeeded in breaking geometry aloof from arithmetics and in banishing it outside mathematics was at work inside geometry as well. Neglecting this split inside the Greek geometry precludes faithful understanding.

Whatever the particular fact with which one may relate the rise of the irrationals [11], there is little doubt that the latter derived from geometry, belonging precisely to the straight-line geometry.

Incommensurability and irrationality go back to a very early date; together with Szabó, one may say that they were congeneric with Greek mathematics itself [12]. But at that time no distinction was made between the

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[9] The sentence consists of three syntagms: (a) the first deals with circular motion and uses the expression ἐν κύκλῳ in an almost technical way; (b) the second deals with rectilinear motion (εὐθεῖα) and is also technical. I took the liberty to interpret τὰ καταντικρύ as 'limits' or extremities of a given segment; (c) the third returns to circular motion (the words ἀνεκάμπτοι and ποιοῖτο ordinarily describe a spiral outline).

[10] A. Rey, *La jeunesse de la science grecque*, Paris, 1933, p. 243—244.

[11] With the relation between the diagonal and the side of the square, as Szabó suggests (*op. cit.*, p. 60), following T. L. Heat (*Mathematics in Aristotle*, Oxford, 1949, p. 2), who relies on Aristotle (*Met.*, 983 a 19 sq.; 1053 a 14 sq.); or with the ἀντυφαίρεσις described by Euclid in the opening of his ii[nd] book, which is related to the monocorde, etc.

[12] The circulation of an empirical notion of the « irrational » much before Theodorus of Cyrene, to whom it was traditionally assigned, can be proved by the xiii[th] sentence in Euclid's vii[th] book, which states the existence of a mean proportional number between two unproportional numbers; cf. Szabó, *op. cit.*, p. 53.

geometrical space described by a line and the figurative number which, according to the Pythagoreans, arose from linear approximations (if not from the line itself). If the statement is true that the irrational has been found much earlier and that it was first described as a linear magnitude, it is then likely that this discovery has helped arithmo-geometry, wherefrom the figurative number theory derived, to be fully assimilated by geometry later. In thus dealing with this transitory domain, which belonged neither to numbers nor to magnitudes, it was possible to build up a system of numbers independently of the irrational. As for the number, this was prompted to become the purest and most intelligible thing. It seems that already with the Eleatics' there was a total gap between line and number.

Owing to the original relation between the notion of incommensurability and the geometric line, the increasing difficulties of numbers — mainly related to the developing theory of the irrationals — could be diverted from arithmetics towards geometry. Thus, Greek geometry was somewhat bound to decay. Much to the contrary, the old and to some extent inertial pre-eminence of a rather mytical realm of geometrical order should be related to the symbolics of circle as the perfect shape.

However old it may be, it is to the Pythagoreans that this tradition can be ascribed. In Diogenes Laertius' record, the Pythagoreans would have said that « the sphere is the most beautiful of all solid figures and the circle is the most beautiful of all plane ones »[13]. Intelligibility was no doubt high in making this statement since the Pythagoreans believed that any circular magnitude could be numerically related to any other — a property which linear magnitudes did not share. Passing now to the Eleatics, this view set up one of the most authoritative topics in the Greek culture. Many examples could be here quoted. Plato's own claims on the circle as the perfect shape are too famous to be given here. But such contentions were very popular; so, in referring to a tradition which ascribed the fundation of five basic mechanical bodies to the circle[14], Aristotle simply quotes this, without going into futher explanatory details.

The fame of the circle also derived from the most archaic property of its centre, by virtue of which this was itself an equivalent of καιρός. Both the circle and the sphere arose from a changing and unsafe space, which gradually formed around a central point. The concept of center thus began to symbolize the human capacity to subdue the unexpected and the dangerous; it also incorporated a highly difficult movement[15]. The Greeks were overimaginative about the centre, and the history of such notions as 'Ομφαλός or 'Εστία proves it well enough.

Geometry praised the circle much the same as arithmetics exalted the virtues of the number. However, a perfect shape as it might have been, the circle exhibited nonetheless paradoxical features. The centre was believed to act over the circumference, as if all circle properties were joined by a cohesive force. Thus, this was the only shape which could

[13] D. L., *V. P.*, 1, 19.

[14] I have taken this example from J.- P. Vernant, *Mythe et pensée chez les Grecs*, Paris, 1969, p. 52.

[15] M. Detienne — J.- P. Vernant, *op. cit.*, p. 34.

reconcile the adverse by means of its centre. This appeared both as a marvellous and as an ambiguous or even damaging feature to the Greeks. Of Heraclitus, who had proclaimed that « the beginning and the end of a circle are identical » (B153), they used to think that he was an « obscure » philosopher and mock at him. But Parmenides had given much the same sentence, actually seeming only to replace « circle » by « discourse » (B 1 and B 5)[16]; and in another of his fragments he had also explained that οἱ γὰρ πάντοθεν ἴσον, ὁμῶς ἐν πείρασι κύρει (« being simultaneously at each point, everywhere within its edges, the circle is the same » B 8, 42—44). By means of its centre, the circle proved able to turn contrariety into sameness.

The circle's paradox led to the most striking intuition with the Greeks : half a unity would stand for twice the unity. This statement, which seems to anticipate the modern definition of the infinite, was half-jokingly expressed by as old and, in a certain way, unsophisticated a poet as Hesiod[17]. Its clearer mathematic form derives from the Eleatic view of the intelligible. In Aristotle's opinion, the fourth of Zeno's arguments against the movement is « one by which [Zeno] ventures to prove that half the time matches twice the time » (ἴσον εἶναι χρόνον τῷ διπλασίῳ τὸν ἥμισυν, Zeno A 28).

Plato, who made L. Robin say that « Le contraire du double est en un sens la moitié »[18], gives several passages which would require very much the same comment. One of them is at *Phaedo* 71 b. In describing the permanent swing from one contrary to the other, Socrates makes an exceptional use of the term μεταξύ ('in the middle', 'between'). Namely, he builds up a geometrical picture of the process (γένεσις) as centre, always balanced by the opposite sides — such as life and death (δύο γενέσεις). It is the only way of explaining the paradoxical equation γένεσις = δύο γενέσεις, which is implicit in Socrates' argument.

The pattern of the relation between centre and circumference can be found outside the mathematical world in a most spectacular way. This is obvious for instance in music, which furnishes a technical terminology pertinent to our assumption. I shall confine to some general examples in support of that view.

The importance attached to the central note A (μέσον) in the Pythagorean scale is well-known by now. Suffice it to recall Th. Reinach's remark that, if one could properly speak of tonic in Greek music, one should place it in the middle of the scale and not at the beginning. Thus situated, the central note — which would fairly correspond to a modern dominant, *i.e.* to the fifth step — was heard more conspicuously than a dominant but less conspicuously than a tonic. Two sensibles were accordingly created instead of one, two incomplete concomitent tonics, etc. Every tonal function appears to have been twice represented within the scale and at the

[16] For B1 (K R 342), 29, see also the commentary on Ἀληθείης εὐ κυλέος ἀτρεμὲς ἦτορ : « whenever you pick up the chain of Parmenides' reasoning, you can follow it round in a circle, passing through each of its links in turn, back to your starting-point » (G. S. Kirk — J. E. Raven, *The Presocratic Philosophers*, Cambridge, 1971, p. 268).

[17] Hesiod, *Works and Days*, v. 40 : Νήπιοι, οὐδὲ ἴσασιν ὅσῳ πλέον ἥμισυ παντός.

[18] Platon, *Phédon* (C.U.F.), Paris, 1934, p. 81.

same time divided into halves because of the circular arrangement inside this set of elements.

Music was closely related to mathematics in the Greek thought, as is today. It appears therefore even more interesting to find this same sort of binary indecision rule over wider and less artificial domains. J. -P. Vernant, M. Detienne and Zoe Petre identified a close terminological affinity of geometrical and political orders. The main argument in that demonstration is given by the notion of centre (μέσον). To debate in the Athenian βουλή meant to put the matter in the middle (ἐν μέσῳ) and to surround it by the citizens' body, in which anyone had his own share by means of an oppinion. On the other side, the most arhaïc images of sovereignity show various unsettled perceptions of the circular space [19]. Both tragedy and rhetoric arose from a similar pattern. Drama, for instance, set out opposite characters around a dynamic nucleus acting as an arbitrator whose equidistant position could neutralise all of them, while establishing each one's right.

These facts point to a two-fold evolution. Outside mathematics, the struggle between rational and irrational—which was also fought within the geometric circle — created by means of this figure a symbolism which enabled geometry to retain certain old privileges. But mathematics itself progressed in a completely different manner, which was to end up in the equation between magnitudes and the finally rejected irrationals. So the long-established idea of geometry having played a ruling part among various departments of the Greek science should not be entirely disregarded. It should rather be approached discriminately by a new image of the inner structure of geometry, which proves to be dichotomic itself. Geometry has been used as far as to provide a basic method for all Greek activities. But one should not infer from this fact that the Greeks held geometry to be a prestigious discipline; on the contrary, geometry was badly considered. And here again one should notice that this statement does not refer to geometry as a whole, but, so to speak, to its mathematical goal alone. The assumption that the circle and the line expressed a specific dichotomy within both Greek geometry and thought would account, I hope, for this sort of paradox.

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[19] A good example would be the one of king Minos: both a merechant and an eternal judge, he never parts from the golden balance. See also M. Detienne, *Les maitres de vérité dans la Grèce archaïque*, Paris, 1968, pp. 29—50. For the kinship between geometrical and political orders, see also Zoe Petre, *Géométrie et politique dans la cité classique*, An. Universităţii din Bucureşti (Istorie), 27, 1978, pp. 3—18.