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Romania. Phone/Fax: 004 0332 408922. Email: symposion.journal@yahoo.com. www.symposion.acadiasi.ro.

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RESEARCH ARTICLES

What Can Synesthesia Teach Us About Higher Order Theories of Consciousness?¹

Fred Adams and Charlotte Shreve

Abstract: In this article, we will describe higher order thought theories (so-called H.O.T. theories) of consciousness. Then we will describe some examples from synesthesia. Finally, we will explain why the latter may be relevant to the former.

Keywords: synesthesia, Ramachandran, Higher-Order Thought (H.O.T.) theories

1. Introduction

H.O.T. theories of consciousness maintain that what makes an experience conscious is a higher order thought that takes that experience as its content. So, for example, one may be exerting pressure on the seat of the chair upon which one is sitting, but not be consciously experiencing that pressure. However, as soon as one's attention turns to that pressure, it will be consciously experienced. On this view, what turns a non-conscious experience into a conscious one is the higher order thought that takes the non-conscious experience as its content.

While experiences themselves may be concept-free, H.O.T.s of their nature involve concepts because thoughts, unlike experiences, involve concepts. The thought that it is raining involves the concept of rain. The thought that Obama is president involves the concept of the presidency. The non-conscious experience of the pressure you are exerting upon your chair does not involve a concept. Not being conscious, there is no concept applied to it, nor is your experience applying a concept to the chair or to pressure. Your non-conscious experience may be responding to pressure or sensory input, but unlike a thought, it is not categorizing or conceptualizing that input. A thought however, by its nature categorizes and conceptualizes.

H.O.T. theories also apply to thoughts themselves. They maintain that what makes a thought conscious is that there is a second higher-order thought that takes the lower-order thought as its content. For example, one may non-consciously think (fear) that one is becoming an alcoholic. Upon reading a brochure about the symptoms of alcoholism, one may form the higher order thought that one has for some time now feared becoming an alcoholic. Thus, this makes conscious the formerly non-conscious thought (fear). However, here we will confine our attention to conscious experiences only.

¹ We are grateful to Andre Galois and Rocco Gennaro for very useful conversations.

We will limit our discussion here to only the ‘thought’ versions of H.O.T. theories. That is, there are several different versions of higher order theories. Some appeal to thoughts as the key to making lower order states conscious. Others appeal to higher order experiences or non-thought forms of representation to do this. Still others discuss more global forms of representation (Gennaro 2004) as the mechanism to turn a non-conscious state into a conscious one.²

Synesthesia derives from the Greek root ‘syn’ meaning together and ‘aesthesia’ or perception, and is a condition where otherwise normal people experience the blending of two or more senses (Ramachandran and Hubbard 2003).

Synesthesia is a condition in which stimulation of one sensory modality causes unusual experiences in a second unstimulated modality (Ramachandran and Hubbard 2005).

Examples of experiences reported by synesthetes would include seeing colors when seeing numerals, seeing red when hearing C-flat or chicken tasting pointy (Ramachandran and Hubbard 2003). One subject experienced a bitter taste when shaping hamburger patties. As a phenomenon, synesthesia has been known at least since the work of Francis Galton (1880). The different types of synesthesia number over 100.³

There are different attempts to explain synesthesia, but knowing it runs in families, there is likely an evolutionary explanation. Perhaps the best-known explanation is that there is a ‘cross-wiring’ of sensory modalities due to proximity of brain regions and then some selectional advantage for this new capacity (Ramachandran and Hubbard 2003, Ramachandran 2011).

2. Why some examples from synesthesia may present a problem for H.O.T.s

One of the most important reasons why we think synesthesia is relevant to theories of consciousness is that the phenomenon seems not to be conceptually driven or triggered. In support of this, we turn to the research of Ramachandran (2011).

When Ramachandran first interacted with some students who reported having synesthesia, he wondered whether their seeing colors when seeing numerals was due to a conceptual association or whether it was truly a perceptual phenomenon. His first thought was to see whether a student who saw red when looking at the number 7 would also see red when looking at the Roman

² We agree with Rosenthal (1990) that the ‘thought-versions’ of H.O.T. are the best versions of the higher-order theories, and that is why we will limit our remarks here to them.

³ Indeed, there are so many varieties that there has been discovery of a kind associated with ideas—so called ‘ideasthesia.’ However, we will not be addressing this variety in this paper. Here we will be concerned only with varieties that are purely perceptual varieties. We explain below how this is determined.

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numeral VII. She did not. This was some evidence that seeing red was not due to a conceptual association, but Ramachandran did not stop there.

He next tested showing the subject uncolored drawings of fruits that would be colored and asking what the subject saw when looking at the drawings—one of which was of a tomato. She reported seeing no colors. She acknowledged that carrots are orange but denied seeing orange when looking at the drawing of carrot. But when observing the numeral 7, she reported that "it keeps screaming red at me."

Next, with the subject's eyes closed, he drew a 7 on her hand. She denied seeing red when feeling the drawn 7, but then reported a tinge of red when she started visualizing a 7. Ramachandran said the word 'seven' to her and again nothing happened until she started to visualize the 7.

Next he drew a green 7. She reported: "I certainly don't mix the real color with the mental color. I see both colors simultaneously, but it looks hideous."

On a subject with a different form of synesthesia, he introduced a galvanic skin response (GSR) paradigm to test the same question about this being a perceptual phenomenon. For this subject, rather than seeing colors when observing numerals, the person felt emotions when touching fabrics or textures. The same question applies: is the synesthesia a conceptual or perceptual phenomenon?

In normal subjects touching mundane textures caused no GSR response. But in one subject who had experiences of fear, anxiety and disgust when touching certain textures, there was a strong GSR response when touching the relevant items. Ramachandran concluded that this is a perceptual phenomenon that he observed. GSR's cannot be faked. The point of giving the GSR test is to determine whether the phenomenon is top-down or purely perceptual (sometimes called 'bottom up').

Ramachandran was thoroughly investigating as many ways as he and his researchers could think of to test whether this phenomenon was conceptual (or 'top-down') vs. perceptual (not driven by conceptual association or deployment). Clearly GSRs are not driven by conceptual deployment.

In a 'pop-out' experiment, Ramachandran (2011) produced a grid of 5s and 2s that were mirror images of one another. The grid was presented for about one half second. To a non-synesthete, looking at the grid produced only the experience of random figures. The subjects had to press one of two buttons on a computer depending upon whether they saw a triangle or a circle. The instruction was to find a general shape. Twenty 'normal' subjects scored about 50% on whether the shapes were circles or triangles. Hence, they were at chance for success.⁴

⁴ While it is true that pop-out is not experienced by all subjects with synesthesia, there is a significant population in which it is experienced and it is those subjects who we think present difficulty for H.O.T. theory (Ramachandran and Hubbard 2005). A distinct possibility is that

However, when subjects with synesthesia looked at the grid, the colors that they saw on the numbers caused the shapes to pop out. That is, the 2s were arranged either in a circular pattern or a triangular pattern among the 5s (which were randomly placed). The colors seen when observing the 2s and their shapes in circular or triangular pattern were apparent to them at a hit rate of 80-90%. For example, a pattern of 2s that was triangular jumped out as a red triangle. Or a pattern of 2s shaped as a circle popped out as a circular shape. The subjects with synesthesia experienced something the subjects without synesthesia did not.

The reason this is interesting in regard to H.O.T. theories, is that the 'pop-out' phenomena is a bottom-up visual experience. The subjects did not first see the shape (triangle or circle) and then have the higher-order thought ('triangle' or 'circle') causing the experience of the shape to become conscious. Rather, the perceptual pop-out produced the conscious visual experience of the shape prior to the having of the thought about the shape experienced.

According to H.O.T. theory, the experience should be non-conscious before a higher-order thought about it raises it to consciousness. So, an H.O.T. theorist would need to say that when the circular or triangular shape pops out, first the subject is having a non-conscious experience until the H.O.T. is applied. But this seems to have it backwards. The subject has no idea of which shape to look for or whether there will actually be one. The visual pop-out is immediate and vivid in its color presentation. It first looks red and circular or red and triangular and only then has the subject the time to apply the relevant concept ('circle' or 'triangle').

Contrast this pop-out phenomenon with the sort of example mentioned earlier. When you turn your attention to the pressure you are exerting on the seat beneath you, you suddenly make conscious the experience of exerting pressure on the seat. But it takes an act of conscious will or attention to make this happen. First comes the thought about the experience of pressure and then comes the conscious experience.⁵

In the pop-out case, you have no idea which shape you may be about to behold. You have no act of will or attention that is directed towards the circle or triangle.⁶ The suddenness of the pop-out experience comes as complete surprise,

there are distinct neural mechanisms for different varieties of synesthesia. Some synesthetes have color experiences upon hearing spoken words or names.

⁵ We are not saying that every H.O.T. itself must be conscious. However, when one directs attention to a state, attention is a conscious state.

⁶ Again, there are individual differences among some subjects, but in some only the minimal attention needed to look at the display is needed to induce the pop-out phenomenon. What is more, attention itself may require a 'hybrid' model to explain the differences being discovered (Ramachandran and Hubbard 2005). In addition, both Andre Galois and Rocco Gennaro suggested to us that unconscious H.O.T.s may be able to explain both pop-out phenomena and the example of becoming conscious of the pressure you exert on the chair. But if so, why are you not conscious of the pressure on the chair even prior to one's calling your attention to it?

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even to the synesthetes, who are not expecting it (and who viewed it for only a half-second).

In the pop-out case it is highly implausible to say you are having the experience of the circle or the triangle non-consciously prior to your H.O.T. about the experience. Why? For one, the pop-out phenomena is so sudden and unexpected that the experience and the consciousness are simultaneous. With only half second presentation, there was little time for the visual system to scan the pattern of the 2s prior to the visual pop-out. So we know they don't first see the pattern of 2s, apply the concept 'circle' (or 'triangle') and then have the visual experience of colored circle. There simply is not time.

Second, color qualia are not non-conscious. For the H.O.T. explanation to apply, the subject's non-conscious H.O.T. would have to be applied to the non-conscious color experience, thereby making the experience conscious. If the non-conscious experience lacks the red qualia, how does the subject know which color concept to apply prior to the pop-out? And which colored shape pops out, circle or triangle, should not come as a surprise to the subject, if one has already applied the concept triangle or circle. After all, one would already have the *thought it was a circle* (or a triangle) prior to what phenomenally pops out. This leads us to believe the conscious visual experience happens prior to the H.O.T.

Of course, to our first point, an H.O.T. theorist can always reply: "How long does a thought take?" Not that long. So they may find this example unpersuasive. But we think this example is different than others in the literature. Not just because it doesn't take long. Not just because of Ramachandran's excellent discussion of the difference between perceptually driven phenomena and conceptually driven phenomena. But because the H.O.T. explanation just seems to get things the wrong way around to say that the concepts are applied first in the pop-out cases of synesthesia.

The seeing of color when observing a numeral is not under conscious control. The explanation of why a synesthete sees red when looking at a 2 has nothing to do with having the concept of red or the concept of the number 2. What brings on the color has nothing to do with the application of an H.O.T.. So when the entire shape among the 2s pops out, this too is a low-level sensory phenomenon.⁷

Attention surely plays a role in that case and attention surely is not non-conscious. In the pop-out case one is consciously scanning the array, looking for a shape. Something brings one's attention to hidden shapes revealed by their color. It is only when attention is focused on the shapes that H.O.T.s would be applied to the shapes. So we don't see how unconscious H.O.T.s can be doing the work needed to explain the phenomenon.

⁷ Ramachandran and Hubbard (2005) and Ramachandran (2011) suggest an extensive theory of the evolutionary origins of synesthesia. Nothing in this explanation involves the role of H.O.T.s. The explanation of its origin is that of a purely mechanical low-level phenomena and is attributed to the spatial proximity of the relevant brain regions (for example the exact location of V4 and the number processing areas of the brain). They suggest that crossactivation may occur between adjacent brain regions of the fusiform gyrus involved in letter recognition and

Interestingly, Gennaro (2012) (an H.O.T. theorist) discusses synesthesia in regard to another matter and argues that his account of higher-order theory is compatible with this phenomena. He is not discussing the issue we raise here. Instead, he is discussing the 'binding problem,' of what binds together certain features of conscious experience. He argues:

"Indeed, I have elsewhere argued at length that feedback loops and top-down integration of brain activity are necessary for having any kind of conscious state (Gennaro 2006; 2012). For example, the brain structures involved in feedback loops seem to resemble the structure of at least some form of higher-order theory of consciousness whereby lower-order and higher-order states combine to produce conscious states. On my view, there is essential and mutual interaction between the relevant neuronal levels." (Gennaro 2012, 77)

Gennaro believes this is compatible with higher-order accounts. However, we think this explanation would make higher-order theories rely crucially on sub-personal states. If they do, this removes the 'higher' from the higher-order theories and resorts to replacing higher-order thoughts with the lower level information processing in the brain that is well below what can be accessed even in principle by the person. These sub-personal feedback processes seem too low-level to be compatible with higher-order thoughts.

Other examples of conscious visual phenomena that may be explainable by sub-personal cognitive states might include illusory contours (Lee and Nguyen 2001) or even the Ishihara test for color vision (Dain 2004).

3. Conclusion

In this paper we have suggested that the phenomena of conscious color experience of subjects with synesthesia is problematic for higher order theories of consciousness (H.O.T.s). The phenomena of 'pop-out' for synesthetes who experience color when viewing certain shapes is a specific example of the problem. The phenomena seems to be perceptually driven, not conceptually driven. So the explanation of the pop-out phenomena seem not to be caused by the application of a higher-order thought. There may be higher order thoughts associated with the phenomena but they may happen after the conscious experience not be the cause of the conscious experience. Herein lies the tension between the conscious experience of synesthesia and the higher order thought theory of consciousness.

color processing. Higher cognitive forms of synesthesia would involve other brain regions such as parietal cortex, and particularly in the region of the angular gyrus, the ventral intraparietal area and the lateral intraparietal area. Their lower level cross-activation model is also useful in explaining phenomena such as 'phantom limb.'

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A Methodology for Teaching Logic-Based Skills to Mathematics Students

Arnold Cusmariu

Abstract: Mathematics textbooks teach logical reasoning by example, a practice started by Euclid; while logic textbooks treat logic as a subject in its own right without practical application to mathematics. Stuck in the middle are students seeking mathematical proficiency and educators seeking to provide it. To assist them, the article explains in practical detail how to teach logic-based skills such as: making mathematical reasoning fully explicit; moving from step to step in a mathematical proof in logically correct ways; and checking to make sure inferences are logically correct. The methodology can easily be extended beyond the four examples analyzed.

Keywords: Inference chain, justifying and checking inferences, matching logical form, mathematical proof, syntactic and semantic validity.

Introduction

Logical reasoning is an absolute requirement of mathematical proficiency and has been since ancient times. The most famous textbook in the history of mathematics, the *Elements* of Euclid,¹ showed by example after example that mathematical propositions^{2,3} are to be justified by a non-empirical method: logical argumentation.⁴ Thus, measuring the interior angles of a triangle is not how mathematics justifies the proposition that those angles add up to 180

¹ According to scholars, Pythagoras was probably the source for most of Books I and II of the *Elements*; Hippocrates of Chios for Book III; and Eudoxus of Cnidus for Book V, while books IV, VI, XI, and XII probably came from other ancient Greek mathematicians. See Ball 1960 [1908], 44. See also Kneale and Kneale (1962) and Gabbay and Woods (2004).

² The author is aware of philosophical concerns over the ontology of logic, such as whether arguments are composed of sentences (type or token) as opposed to propositions, which are abstract entities expressed by sentences in a language. The sentence ontology is easier for students to understand and will be used here.

³ In modern terms, mathematical propositions are analytic, a priori, and necessary; while the propositions of science are synthetic, a posteriori and contingent. Though it is beyond the purview of this article to address philosophical controversies stemming from this distinction, they are mentioned here because mathematics students should be encouraged to ask questions that are not, strictly speaking, mathematical.

⁴ Logical argumentation in mathematics predated Euclid by several centuries. Thales of Miletus is usually credited with its application in geometry.

degrees.⁵ While Euclid makes extensive use of drawings, it is not being suggested that geometry is about figures on paper (or papyrus, in Euclid's case). The drawings are a heuristic device to facilitate comprehension.⁶

Euclid developed a wealth of mathematics using what came to be known as the axiomatic method: First, state some propositions assumed without argument, along with definitions of key terms⁷; and then derive everything else by logical argumentation. Once established, results can be used to derive more results the same way. Euclid made logical argumentation the standard method for deriving mathematical results.

What this method itself is, the *Elements* does not explain. Perhaps it should have. Euclid was trained by students of Plato and as such was probably aware of the Platonist distinction between *F* itself and instances of it; and between a list of instances of *F* itself and a definition of it. Moreover, the *Meno* argues in effect that conceptual analysis is a pre-condition of pedagogy. Given this background, Euclid should have provided an explanation of: (1) the method his examples of logical argumentation instantiated; (2) why he believed it was applied correctly in all of them; and (3) what 'correctly' meant.⁸ Generations of students were left to fend for themselves.

(1), (2) and (3) were not really new problems.⁹ In his *Prior Analytics*, Aristotle, who slightly preceded Euclid, proposed what he probably thought was a sufficiently general analysis of logical argumentation.¹⁰ Stated in modern terms, his answer was insightful in principle: Logical argumentation means the

⁵ Such measurements could be used to convince students that mathematical applications to 'the real world' always involve approximation; and that there is a fundamental difference between mathematics and science in how results are justified. See Cusmariu 2012.

⁶ Descartes would later dispense with drawings by introducing algebraic methods into geometry.

⁷ Book I of the *Elements* assumes ten propositions without argument: Five "Postulates" and five "Common Notions." Teachers should explain the difference between them as well as the role of definitions.

⁸ It is unknown how Euclid proceeded in classroom settings. Berlinski (2013, 17) claims that Euclid "had no interest" in what made his arguments valid and that he "was not a mathematician disposed to step back to catch himself in the act of stepping back." What an odd characterization of Aristotle, and of Frege!

⁹ Ball states: "It would appear that he [Euclid] was well acquainted with the Platonic geometry, but he does not seem to have read Aristotle's works." (Ball 1960, 43). Even so, Euclid's fellow mathematicians would have been aware of what Aristotle accomplished.

¹⁰ An *Encyclopedia of Mathematics* article states: "At the time [of Euclid] the problem of the description of the logical tools employed to derive the consequences of an axiom had not yet been posed," evidently unaware that Aristotle had in fact 'posed' this problem and suggested a solution; and that the need for analysis of important concepts is a key theme in Plato. See Novikov citation in References.

application of rules of inference, of which Aristotle supplied 15 in syllogistic form.¹¹

Unfortunately, Aristotle seems not to have tested his analysis against logical argumentation as practiced in mathematics; why not is unknown.¹² Had he done so, he would have realized that mathematical propositions were not in general reducible to the four types in his syllogisms;¹³ and that his 15 rules were insufficient to capture all logically correct mathematical reasoning.¹⁴ Revision would have followed.

Greek mathematics after Euclid showed no interest in the problems that Aristotle tried to solve. Archimedes, Apollonius, Diophantus, Pappus, Eratosthenes and their contemporaries continued to use logical argumentation to derive results (what else?), also without an analysis of logical argumentation itself.¹⁵ This was true many centuries later also of Descartes, Newton, Euler, Gauss, Cauchy and their contemporaries. We can only wonder what might have been if Euclid had taken a Fregean turn – or even Descartes, who was a philosopher as well as a mathematician. As we shall see, logic today is still a ‘silent partner’ in mathematics and its instruction, despite Frege’s insights.¹⁶

¹¹ Aristotle’s analysis yielded 256 syllogisms in standard form, of which 15 are logically correct and effectively can function as rules of inference. Briefly, a syllogism is composed of two premises and a conclusion in subject-predicate form, designated by the letters **A**, **E**, **I** and **O**. **A** is of the form “All S are P,” **E** is of the form “No S are P,” **I** is of the form “Some S are P,” and **O** is of the form “Some S are not P.” The syllogism also contains at most three terms, which can occur as subject, predicate, and middle terms. Kant was reporting a historical fact when he stated in the preface to the second edition of his *Critique of Pure Reason* that “logic ... is thus to all appearances a closed and completed body of doctrine,” (Kant 1929 [1787], 17, Bviii) meaning Aristotelian logic. Gottlob Frege’s analysis in the *Begriffsschrift* (1879) showed otherwise. Some elementary logic textbooks, e.g., the highly popular Copi, Cohen and McMahon (2010), still cover Aristotelian logic.

¹² As noted above, many of the results in Euclid’s *Elements* predate Euclid and would have been known to students of Plato’s Academy, which Aristotle attended, including the use of logical argumentation to derive mathematical results.

¹³ Sentences stacking quantifiers are counterexamples. “For any x there is a y such that $f(x, y)$ ” cannot be analyzed using Aristotelian propositions as, for example, a conjunction of the **A** proposition “for any x , $f(x)$ ” and the **I** proposition “there is a y , $f(y)$.” See Cusumariu 1979A for an explanation why Aristotle’s solution to the problem of universals is wrong.

¹⁴ Logical analysis of the Pythagorean proof that $\sqrt{2}$ is not a rational number requires the machinery of first-order logic with the equality symbol. Other examples of mathematical proofs requiring sophisticated logical machinery for a full analysis are in Muller 1981.

¹⁵ Of the five major ancient Greek mathematicians just cited, Kneale and Kneale (1962, 62) only mentions Diophantus and it is a passing reference to algebraic notation he introduced.

¹⁶ Berlinski (2013, 2) claims that in the view of most mathematicians, mathematical logic is not part of mathematics. This attitude toward logic is common among mathematics teachers as well, as the author can testify from his own experience. While in college, the author expressed interest in studying the concept of mathematical proof as a subject in its own right, to which he got the response “that’s not mathematics.”

From the fact that mathematics has been able to progress without an analysis of logical argumentation it should not be inferred that the problems Aristotle tried to solve are pedagogically unimportant; far from it. Yet, mathematics textbooks continue to assume that students will ‘get’ logical reasoning on their own just by working through examples of mathematical reasoning.¹⁷ Standard logic textbooks, on the other hand, fail to explain in practical terms why studying the methods of logic can help students learn mathematics and do a better job of solving problems.¹⁸

Educational systems cannot realistically expect the student population to figure out the principles of logical reasoning and their application in mathematics without the special training necessary to acquire a skill that is absolutely essential to learning and doing mathematics. It is imperative that schools add logic to mathematics curricula. The best way to do that is by means of a sound, practical and clearly laid out methodology. The following must be explained to mathematics students: how to apply rules of logic in ways that are explicitly linked to mathematical contexts; how they can make sure that applications are carried out correctly; and how they can correct errors if they are not. That is the purpose of this article: To provide step-by-step guidance for teaching logic-based skills to mathematics students, suitable even at elementary levels.

The methodology presented here, illustrated in detail with examples from several mathematical disciplines, will enable students to:

- Distinguish assumed from inferred statements in arguments.
- Build a logical sequence of steps from assumptions to conclusion.
- Identify logical links justifying inferences from one step to another.
- Check to make sure inferences are logically correct.
- State proofs in a way that makes the reasoning logically explicit.¹⁹

Basic Concepts

For purposes of this article, an argument is a finite sequence of sentences such that some, the premises (assumptions), are claimed to logically imply another sentence, the conclusion. The argument is valid when this claim is correct.²⁰

¹⁷ A classic text by Edmund Landau is typical. He writes: “I will ask of you [students] only the ability to read English and to think logically.” (Landau 1951 [1929]), v). Landau does not explain what it means to think logically. Like many mathematics teachers, he leaves it to students to ‘pick it up’ on their own. Example 1 below presents a detailed analysis of one of Landau’s proofs using the methodology of this article.

¹⁸ Mathematical logic textbooks, e.g., Kleene 1952, Church 1956, Mendelson 1964 and Schoenfield 1967, are too difficult for most high school mathematics students.

¹⁹ Other benefits of the methodology will be explained along the way in the main text and in footnotes as appropriate.

²⁰ It is disconcerting to find a professional mathematician writing: “The conclusion of a valid argument is entrained by its premises.” (Berlinski 2013, 16). Teachers should avoid using

However, we note right away that validity is an ambiguous concept in logic, something not generally recognized. Validity has a *semantic* and a *syntactic* meaning.²¹

In a semantically valid argument, if premises are true, the conclusion must be true. An argument can be semantically valid even if (a) premises are false and the conclusion is true; (b) premises and conclusion are false; but not if (c) premises are true and the conclusion is false. On the other hand, (d) an argument can be semantically invalid even though premises and conclusion are all true.

Point (a) might seem unintuitive but is nevertheless correct: “All roses have thorns” follows logically from “all roses are purple flowers” and “all purple flowers have thorns” even though both premises are false and the conclusion is true.²² A semantically valid argument is easy to construct using only falsehoods, as students can verify. To verify point (d), students should be asked to construct a logically incorrect argument using true premises and a true conclusion – also easy to do.

The mathematically relevant and pedagogically useful concept of validity is syntactic, just as Aristotle thought. Syntactic validity means correct application of rules of logic, which involves matching logical form as explained in detail below.²³

Logical Symbolism

The following symbols, called ‘logical connectives,’ will be used below to state mathematical arguments.

- ~ Negation, meaning ‘it is not the case that.’
- & Conjunction, meaning ‘and’ and its cognates.
- ∨ Disjunction, meaning ‘or’ and its cognates.
- Material implication, meaning ‘if __, then __.’
- ≡ Material equivalence, meaning ‘if and only if.’

informal language to explain the logical concept of validity. For example, Berlinski would not dare use informal language to teach basic concepts of calculus such as ‘continuity’ and ‘integration.’

²¹ The two concepts are related. Students should be encouraged to find out how.

²² Students should internalize as early as possible the difference between validity, which is a property of arguments, and truth, which is a property of argument components. Logic, mathematics and science often use common words in a technical sense, which must be applied as defined in those fields.

²³ An elementary treatment of the subject is Cusmariu 2016. Texts suitable for high-school mathematics courses are Velleman 1994 and Wohlgemuth 1990. Advanced texts are Takeuti 1987 and Kunen 2012.

$(\forall x)\Phi x^{24}$, meaning “the predicate Φx holds for all x objects.”²⁵

Example: *All even numbers are sums of primes.*

$(\exists x)\Phi x^{26}$, meaning “the predicate Φx holds for some x objects.”

Example: *Some numbers are expressible as ratios.*

Mathematics textbooks use ordinary words and phrases to express logical connectives, though it is not always easy for students to match words and phrases with logical connectives and their symbols. For mathematical purposes and depending on context, the following can be taken to mean ‘and’ and are symbolized as **&**: ‘also,’ ‘however,’ ‘though,’ ‘too,’ ‘but,’ ‘besides,’ ‘what’s more,’ ‘in addition,’ ‘nonetheless,’ ‘moreover,’ ‘yet.’ The following can be taken to mean ‘or’ and are symbolized as **v**: ‘unless,’ ‘otherwise,’ ‘except,’ ‘else.’ It would be useful for students to put together, and share with each other, a vocabulary listing the various ways that logical connectives can be expressed in words. Spotting words and phrases for logical connective and interpreting them correctly is an important skill.

The standard way of defining logical connectives is by means of truth tables.²⁷

p	q	$\sim p$	$p \& q$	$p \vee q$	$p \rightarrow q$	$p \equiv q$
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

Rules of Logic: Preliminaries

Because syntactic validity means correct application of rules of logic to yield new mathematical knowledge,²⁸ it will be useful to have a list of such rules up front.

²⁴ The notation $(\forall x)$ denotes the universal quantifier.

²⁵ The terms ‘predicate’ and ‘holds for’ are used neutrally here without taking a stand on issues associated with the philosophical problem of universals. A concise statement of this problem can be found in Cusmariu 1979A and Cusmariu 2016A. Universal and existential quantifiers along with predicates and variables belong to the predicate calculus of logic, first developed by Frege.

²⁶ The notation $(\exists x)$ denotes the existential quantifier.

²⁷ It does not matter that connectives as defined in the truth tables are not in complete agreement with common usage. For example, the first row of the truth table for $p \vee q$ shows that ‘or’ is defined in the inclusive sense as ‘one or the other or both.’ The truth table also shows a weaker sense of ‘if __, then __’ than is used in non-mathematical contexts.

Below are five rules of the propositional calculus and two rules of the predicate calculus that will be applied to the mathematical arguments studied in this article.²⁹ However, these rules are not sufficient to capture all logically correct mathematical arguments. The object here is to get students used to operating with the concept of syntactic validity, starting with relatively simple examples. More rules can be added after students have become proficient at operating with the ones presented here.

Note that in Rules 1-3 premise components need not occur immediately above or below one other. Thus, *Modus Ponens* has been applied correctly even if $p \rightarrow q$ occurs on line 3 of a proof while p occurs on line 10, and vice versa.

Seven Rules of Logic³⁰

Rule 1: *Modus Ponens* (MP)³¹

$p \rightarrow q$

p

$\therefore q$

Students encountering MP for the first time may find the rule unhelpful if they see the conclusion, q , as ‘part of the premise,’ $p \rightarrow q$. They may take this to mean that the rule is circular or redundant because it seems to assume what is to be proved. To clear up this misunderstanding, it should be pointed out that $p \rightarrow q$ is in conditional form and as such does not assert q ; only that IF p is the case, THEN q is the case. The expression ‘part of’ has a specific, defined meaning in $p \rightarrow q$, as shown in the truth table. Logic can help mathematics students learn to operate with concepts as defined. Exercises should be devised to show students correct as well as incorrect ways of matching the form of MP or any other rule. Form-matching exercises will also get students used to thinking in abstract terms, which is another critical skill in mathematics.

Asserting the Consequent is a popular but fallacious argument form that closely resembles MP:

²⁸ The author is aware of philosophical problems associated with the idea that ‘new knowledge’ can be generated from ‘old knowledge’ by means of ‘pure reason.’ See Cusmariu 2012 and Cusmariu 2016A.

²⁹ A standard logic text that can be consulted for more rules of logic is Copi, Cohen and McMahon 2010.

³⁰ There is significant evidence in the developmental psychology literature that students are able to master some rules of inference remarkably early. See Stylianides and Stylianides 2008.

³¹ Students should be informed that sentence letters in propositional calculus rules can be replaced by sentences of any logical complexity whatever. Thus, an inference from $(p \vee r) \rightarrow (q \ \& \ s)$ and $(p \vee r)$ to $(q \ \& \ s)$ is also an MP inference. This fact is part of the formal nature of rules and should be accepted as early as possible. The formal nature of rules of logic will help students get used to abstraction in mathematics.

$$\begin{array}{l} p \rightarrow q \\ q \\ \therefore p \end{array}$$

p does NOT follow logically from $p \rightarrow q$ and q . Counterexamples are easy to devise and may usefully be posed as homework. Logic can help mathematics students learn to pay careful attention to formal details.

Rule 2: *Modus Tollens* (MT)³²

$$\begin{array}{l} p \rightarrow q \\ \sim q \\ \therefore \sim p \end{array}$$

Students encountering MT for the first time may object that the pairs of statement forms $(p, \sim p)$ and $(q, \sim q)$ cannot be part of the rule because they are contradictory but rules of logic cannot contain contradictions. To clear up this misunderstanding, teachers can note that MT neither asserts nor implies $p \ \& \ \sim p$ and $q \ \& \ \sim q$, which are contradictions. The rule says, in words, “given $p \rightarrow q$ as well as $\sim q$, it is logically correct to infer $\sim p$.” MT is another opportunity for students to learn careful attention to formal details.

A popular misapplication of MT is the fallacy of *Denying the Antecedent*:

$$\begin{array}{l} p \rightarrow q \\ \sim p \\ \therefore \sim q \end{array}$$

$\sim q$ does NOT follow logically from $p \rightarrow q$ and $\sim p$.

Proofs by *reductio ad absurdum* rely on MT and MP.³³ We show A by deriving a contradiction (inconsistent sentence) C from the negation of A , $\sim A$, from which A follows because contradictions are false. In outline, the argument looks like this:

1. $\sim A \rightarrow C$
2. $\sim C$
- $\therefore \sim \sim A$, by MT
3. $\sim \sim A \rightarrow A$

³² MT refutes the popular misconception that “you can’t prove a negative.” Mathematics proves negatives routinely. Thus, Pythagoras proved that $\sqrt{2}$ is not a rational number and Bertrand Russell proved that there is no set of just those sets not members of themselves.

³³ The mathematician G.H. Hardy regarded the *reductio* proof as “one of a mathematician’s finest weapons.” See Hardy, G.H. (1940, 94). On the other hand, mathematician Jordan Ellenberg (2014, 133) describes the *reductio* proof as “a weird trick, but it works.” There is nothing ‘weird’ about *reductio* proofs. One hopes Ellenberg does not say such things in class.

$\therefore A$, by MP

Many proofs in Euclid are in *reductio ad absurdum* form.³⁴

Rule 3: Hypothetical Syllogism (HS)

$$p \rightarrow q$$

$$q \rightarrow r$$

$$\therefore p \rightarrow r$$

Replacing the arrow in HS with the equality symbol yields Euclid's Common Notion I.1 (see below), which, however, is not equivalent to HS. The arrow is a truth-functional symbol; the equality symbol in mathematics usually designates identity.³⁵

Rule 4: De Morgan's Theorem (De M.)

$$\sim(p \& q) \equiv \sim p \vee \sim q$$

that is,

$$(\sim(p \& q) \rightarrow (\sim p \vee \sim q)) \& ((\sim p \vee \sim q) \rightarrow \sim(p \& q)).$$

Rule 5: Material Implication (Imp.)

$$p \rightarrow q \equiv \sim p \vee q$$

that is,

$$((p \rightarrow q) \rightarrow (\sim p \vee q)) \& ((\sim p \vee q) \rightarrow (p \rightarrow q))$$

De M. and Imp. are rules of replacement rather than rules of inference, meaning that expressions flanking the equivalence symbol can be replaced for one other without affecting the validity of an argument.

³⁴ *Reductio* proofs can also be found in modern physics, e.g., quantum theory. See Jauch 1968, 115, where Proposition 1 – *Every dispersion-free state is pure* – is proved by *reductio* reasoning. An extensive analysis of Jauch's argument and its implications for quantum mechanics is Cusmariu 2016B.

³⁵ 'Usually' because the equality symbol sometimes is used in mathematics as shorthand for the definition symbol '=df.' The definition symbol does not mean identity and is used rather to introduce terms, specifying under what conditions they hold and how they are to be used. A defined term may be used to define other terms. Thus, Euclid's Definition 11 in Book I of *Elements*, "an *obtuse* angle is an angle greater than a right angle," assumes the definition of 'right angle' provided in Definition 10.

Rule 6: Universal Generalization (UG)³⁶

Φy	read, "the predicate Φ holds for arbitrarily selected individual y ."
$\therefore (x)(\Phi x)$	read, " Φ holds for any x ."

Mathematics often proves that Φ holds for all objects of kind K (geometric figures, numbers, etc.) by selecting an arbitrary instance of K and proving that Φ holds for it. Then it is inferred that Φ must hold for all objects of kind K . According to UG, this inference is correct. The proof from Euclid studied in detail below applies UG to derive its main conclusion, the last line of the argument.

Rule 7: Universal Instantiation (UI)³⁷

$(x)(\Phi x)$	read, " Φ holds for any x ."
$\therefore \Phi v$	read, "the predicate Φ holds for individual v ."

UI is used in mathematics more often than realized. Unfortunately, it is common to see inferences 'from the general to the particular' without any hint that such inferences are based on, and therefore justified by, a rule of logic. Applying a definition to a specific case means applying UI, as does assigning values to variables in a formula. Thus, when students encounter the expression "let x be such and such," UI has been applied.

Comments on Proofs

- Knowing that syntactic validity means matching the form of rules of logic can simplify the process of argument building and offer useful hints how to proceed. Students familiar with these rules will know what assumptions must be marshaled to match the relevant forms. Thus, applying MP and MP requires conditional premises; while HS requires all sentences be in conditional form. Mathematical arguments frequently omit conditional premises, even though they are necessary for arguments to go through as we shall see below.
- Definitions in mathematics are often key steps in arguments. The symbol =df is often used to write a definition, $A =df B$, where A is the concept being defined and B the concept(s) used to define A . Because definitions record equivalence, it is helpful to express $A =df B$ as $A \equiv B$. Because $A \equiv B$ is expanded as $(A \rightarrow B) \& (B \rightarrow A)$, an argument can use part of the definition, $A \rightarrow B$, as one of its steps. The A component of a conditional is called the 'antecedent' and B the 'consequent.'

³⁶ UG and UI are rules of the predicate calculus. The careful student will ask, for example, whether propositional calculus rules also apply to the predicate calculus. They do indeed and work the same way.

³⁷ Students should be informed that UG and UI are correct for Φ of any logical complexity whatever. Moreover, despite appearances to the contrary, UI is not restricted to sentences with a single quantifier.

- Mathematics often requires proving conditionals. There are several strategies to accomplish this. Strategy 1 is to prove that the negation of $p \rightarrow q$, a sentence of the form $p \ \& \ \sim q$, implies something that is false, thus $\sim(p \ \& \ \sim q)$ follows by MT and $p \rightarrow q$ follows by De M. and Imp. Strategy 2 is to infer $p \rightarrow q$ from $p \rightarrow r$ and $r \rightarrow q$ using HS. Strategy 3 is to prove q and then infer $(p \rightarrow q)$ from q and the tautology $q \rightarrow (p \rightarrow q)$ by MP. Euclid uses Strategy 1 to prove his Proposition III.6 – see below. Strategy 3 is a modern development.
- Proofs in mathematics frequently use results that cut across disciplines. This clarifies further the sense in which rules of logic are formal. Thus, MP has the same meaning in all of mathematics, so that p and q can be replaced with formulas of different disciplines and still yield a syntactically valid argument.
- Proofs in mathematical textbooks follow Euclid in presenting what might be called ‘proof sketches.’ As we shall see, they do not list all the steps necessary and sufficient to derive the final conclusion, or indicate which rules of logic have been applied to justify moving the argument from one step to the next.
- Students should be encouraged to ask probing questions about mathematics and its methods. For example, as they work through proofs to identify assumptions driving a result, students will come to realize, as Euclid did, that mathematics must make some assumptions without argument. It is an interesting and important question how such assumptions are to be justified and in what sense of ‘justified.’

Seven Logic Lessons

The 17th century French mathematician Pierre de Fermat famously stated that “*la qualité essentielle d’une démonstration est de forcer à croire*” (“the essential attribute of a proof is that it compels belief” (Fermat 1891-1912, Vol. II, 483). However, a line of reasoning can “compel belief” only if proof elements and their logical links are readily apparent. This is not always the case in mathematics, as Fermat’s own “Last Theorem” showed. Students encountering proof narratives may well have difficulty ‘tracking’ the reasoning from beginning to end because mathematical arguments often omit assumed as well as inferred steps deemed ‘obvious’; and there is near universal absence of the rules of logic used to derive steps. It is assumed that the student will ‘see’ the logic without instruction. Keeping logic a ‘silent partner’ in mathematics instruction is pedagogically unwise to say the least. The dreaded ‘fear of math’ can be traced in part to the fact that the logic of mathematical reasoning is not transparent, leaving students confused and discouraged if they fail to ‘get it.’ Learning can be stifled by negative emotional reactions to the subject matter.

It is preferable to teach students how to build logically explicit arguments in stages. We believe the seven lessons explained and applied to mathematical examples below represent a practical methodology.

Lesson I: Distinguishing assumed from inferred steps.

Lesson II: Identifying assumed steps needed in the argument.

Lesson III: Justifying assumed steps.

Lesson IV: Displaying the entire inference chain of the argument.

Lesson V: Identifying rules of logic applied.

Lesson VI: Checking that rules of logic have been applied correctly.

Lesson VII: Building a fully explicit argument.

Lesson I will teach students that every step in an argument is either assumed or inferred from one or more assumptions. It is critical that students develop an ability to work with both types of argument components. Information that is of neither type should be discarded when constructing proofs, as it is logically irrelevant.

Lesson II will teach students to make sure they have compiled a complete list of assumptions before going forward with an argument. Missing assumptions can easily wreck an argument, sow confusion, and slow down the inference process.

Lesson III will teach students that assumptions may also need justification and they should be prepared to provide it. It is impossible to justify all assumptions, of course. In mathematics we can take for granted axioms, definitions, and previously established results. Justifying a step on grounds that it is a bad idea, certainly in the beginning stages.

Lesson IV will teach students how to lay out argument components sequentially so that the chain of inferences can be checked easily. Inferential chains contain many steps on the way to the final one; how many such steps will be needed is not predictable. Mathematical arguments seldom prove a result in single inference.

Lesson V will teach students how to be explicit about the rules of logic applied to derive inferred steps and how rules work to move from step to step in an argument.

Lesson VI will teach students how to check that rules of logic have been applied correctly to every inferred step.

Lesson VII will teach students how to build mathematical arguments that are fully explicit in all relevant respects: assumptions, inferred steps, and rules of logic. This is a stronger concept of rigor than is customary in mathematics.³⁸ The stronger concept has many pedagogical advantages, as the following four examples show.

³⁸ Proofs in Hilbert's axiomatization of geometry (Hilbert 1902) do not cite the rules of logic but they are cited in proofs in his mathematical logic book (Hilbert and Ackermann 1950 [1938]). This may create the (false) impression – not intended by Hilbert – that ‘proof’ does not mean the same thing in both subjects; that rules of logic are not what determines the validity of mathematical proofs; or that mathematical logic applies different rules of logic than mathematics itself.

EXAMPLE 1

Consider the following theorem and its proof in Edmund Landau's classic textbook on analysis (1951[1929], 9):

Theorem 12: If $x < y$, then $y > x$.

Proof: Each of these [$x < y$ and $y > x$] means that $y = x + v$ for some suitable v .

Lesson I: Distinguishing Assumed From Inferred Steps

The Theorem-Proof format, which students encounter routinely in mathematics, illustrates the sense in which logic is a 'silent partner' in mathematics. This format is actually shorthand for an argument, claiming that a sentence, labeled Theorem, follows logically from sentences listed in the Proof. The student is challenged to discover the inferential chain from sentences in the Proof to the conclusion, the Theorem.

A good deal of confusion and misunderstanding can be avoided by telling students right away that the relationship between Theorem and Proof is purely logical, which may not be obvious to all of them. Students should also be told that the proof component may well contain additional arguments. That is, the proof component may well contain other theorems, even though these are not always labeled as such and justification for them is not always included. Mathematical tradition might have evolved differently had Euclid stated explicitly that the relationship between his Propositions and the sentences in the narrative below was purely logical; that justification in mathematics does not mean taking measurements of any kind.

In the example at hand, Landau asserts that his Theorem 12 follows logically from the sentence listed in the proof. In other words, he is claiming that

(a) Each of $x < y$ and $y > x$ means that $y = x + v$ for some suitable v ,

logically implies

(b) If $x < y$, then $y > x$.

Students are likely to find this claim mystifying, for several reasons.

To begin with, it is not apparent that sentence (a) is a conjunction of sentences, which it is. Let us label all components, Theorem and Proof, and arrange them vertically:

(a1) $x < y$ means that $y = x + v$.

(a2) $y > x$ means that $y = x + v$.

(b) If $x < y$, then $y > x$.

Landau is claiming that (b), Theorem, follows logically from the conjunction of (a1) and (a2), in Proof. The familiar three dot symbol \therefore is a useful way to distinguish inferred from assumed steps and we shall do so henceforth.

The sequence of steps and their logical relationship in Landau's argument as stated is this:

(a1) $x < y$ means that $y = x + v$.

(a2) $y > x$ means that $y = x + v$.

\therefore (b) If $x < y$, then $y > x$.

Lesson II: Identifying Assumed Steps

The second reason students may find Landau's argument hard to fathom is this. Intuitively, if A means B and C means B , it follows that A and B mean the same thing, from which it follows that $x < y$ and $y > x$ mean the same thing. This is not literally true. However, Landau is not suggesting that $x < y$ and $y > x$ mean the same thing just because he says in the Proof component that "each means that $y = x + v$." He is not deliberately sowing confusion.

Rather, (a1) is intended to suggest

(1) If $x < y$ then $y = x + v$,

while (a2) is intended to suggest

(2) If $y = x + v$ then $y > x$.

It is (1) and (2) that are the 'real' premises of Landau's argument. This fact is by no means obvious. Landau evidently expects students to know already how think through information and piece it together into a logically correct argument. He assumes, as do mathematics texts in general, that students already understand logical argumentation, in theory as well as in practice. This assumption is by no means obvious.

Students can realize that (1) and (2) must replace (a1) and (a2) by focusing on the logical form of the conclusion sentence, which is always a good place to start, even though it means starting at the end of an argument. Having noticed that Theorem 12 is in conditional form, students should next consider which rules of inference are relevant to its derivation. This means determining which rules have a conditional sentence in the conclusion; namely, the component in the rule prefixed by the \therefore symbol. In this case, it is HS, which requires conditional premises. This is a kind of 'working backwards' from rules of inference to the argument structure but it is helpful and it works.

Lesson III: Justifying Assumed Steps

Landau provides justification for (1) and (2) in the form of definitions stated a few lines above Theorem 12.

Definition 2: If $x = y + u$ then $x > y$.

Definition 3: If $y = x + v$ then $x < y$.

Here students may well wonder what concepts Landau is trying to define. It is clear that he is not trying to define $=$ nor $+$, which leaves $>$ and $<$. In general, it is preferable to be explicit about the concepts (terms) being defined.

A second, related criticism is that Landau fails to make clear that definitions in mathematics are not conditionals but rather biconditionals, i.e., conjunctions of conditionals. Students should be warned that definitions in mathematics are not always stated in proper form as biconditionals. Landau is the norm rather than the exception. Even if a mathematics text shows a definition only in conditional form, students should assume that a biconditional is intended. Stating the above definitions in biconditional form in a way that makes it clear they define the symbols $>$ and $<$ yields:

Definition 2a: $x > y \equiv x = y + u$.

Definition 3a: $x < y \equiv y = x + v$.

Because (3a) is a conjunction of conditionals, we are entitled to use half of it:³⁹

(3b) If $x < y$ then $y = x + v$.

(3b) is premise (1) above, which completes the justification for premise (1).

Because (2a) is a conjunction of conditionals as well, we are also entitled to use half of it:

(2b) If $x = y + u$ then $x > y$.

However, there are some discrepancies between (2b) and premise (2)

(2) If $y = x + v$ then $y > x$

that make it less easy to see a definition as the justification for this premise.

First, the variables x and y are reversed. Second, (2b) has u where (2) has v . These discrepancies can safely be ignored here. Landau could have made life simpler by writing his definitions consistently. By way of general warning, students should be prepared for less than complete logical rigor in mathematics textbooks, even those of celebrated teachers like Landau. In this case no harm is done because the discrepancies are easily resolved. However, the author urges teachers to avoid 'looseness' and informality and strive for complete clarity as much as possible.

Lesson IV: Displaying the Entire Inference Chain

Here is the inference chain from assumptions to the conclusion, Theorem 12:

(1) If $x < y$ then $y = x + v$.

³⁹ Some students will ask if a rule of logic is being applied when we use only one component of a conjunction p & q . The answer is in Example 4, Lesson III below.

(2) If $y = x + v$ then $y > x$.

\therefore (b) If $x < y$ then $y > x$.

Lesson V: Identifying Rules of Logic Applied

Once the elements of an argument have been identified, it should be evident which rules apply – provided, of course, students know rules of inference and how they work. Only one rule of logic was applied to infer (b) from (1) and (2), HS.

A comment is in order about the justification of (1) and (2), which were based on the two definitions provided. Students should realize that when a mathematics text justifies a sentence using the words ‘from definition so-and-so’ a rule of logic is in fact being applied, though this rule is seldom if ever identified. The rule is UI. Here is why.

Definitions in mathematics stipulate the meaning of a term and usually contain no limitation on the range of values of their variables.⁴⁰ Thus, the definition of a circle holds for any circle whatever, meaning that definitions are actually universally quantified biconditionals, something of the form ‘ $(x)(Fx \equiv Gx)$,’ where ‘ Fx ’ abbreviates the term being defined and ‘ Gx ’ its necessary and sufficient defining conditions. This is also not always apparent in mathematics, as the two examples from Landau illustrate. When a mathematics text claims that a sentence about a specific circle ‘follows from’ the definition of a circle, the explanation for this inference – yes, it is an inference – is UI. Students should be alerted to this fact.

Lesson VI: Checking Logical Justification

First, we display an argument structure side by side with the rule of inference applied to derive the conclusion of the argument, Theorem 12, inserting the arrow symbol \rightarrow for the ‘if __then__’ material conditional:

<u>HS Form</u>	<u>Inference Chain to Step (b)</u>
$p \rightarrow q$	$x < y \rightarrow y = x + v$
$q \rightarrow r$	$y = x + v \rightarrow y > x$
$\therefore p \rightarrow r$	$\therefore x < y \rightarrow y > x$

Inspection shows that sentence letters in HS have been correctly and consistently replaced by the sentences of the argument. Thus, p has been replaced by $x < y$; q by $y = x + v$; and r by $y > x$. The argument matches the form of HS and, accordingly, is syntactically valid.

⁴⁰ Definitions in mathematics are often stipulative, setting down the meaning of a term rather than making a common term (or concept) precise through analysis. A useful text on the subject is Robinson 1954.

Lesson VII: Building a Fully Explicit Argument

Here we display the argument structure side by side with the justification for all steps, assumed as well as inferred. Logic is no longer a 'silent partner':

(1) $x < y \rightarrow y = x + v$From definition component (3a), UI.

(2) $y = x + v \rightarrow y > x$From definition component (2a), UI.

\therefore (b) $x < y \rightarrow y > x$From (1), (2), HS.

EXAMPLE 2

Addition and multiplication are commutative operations, meaning that the order in which they are carried out does not affect the result. Both of these equations are correct:

$$a + b = b + a$$

$$a \cdot b = b \cdot a$$

What about division? Is it true that $a \div b = b \div a$? In different notation, is it true that $a/b = b/a$?

Here is how a standard mathematics textbook (Edwards, Gold and Mamary 2001, 35) argues that division is not commutative, using the numbers 12 and 3 as examples and displaying argument components on the page this way:

$$12/3 = 4$$

$$3/12 = 1/4$$

$$4 \neq 1/4$$

\neq means unequal.

Therefore, $12/3 \neq 3/12$. Division is not commutative.

This is another proof sketch of the sort typically found in mathematics textbooks. The term 'therefore' signals an inference from information presented, so we know which formula is the conclusion. It is left to the student to figure out what information belongs in the premises and how the conclusion is derived from them.

Lesson I: Distinguishing Assumed From Inferred Steps

We begin once again by labeling all sentences provided and arranging them in vertical order.

(a) $12/3 = 4$

(b) $3/12 = 1/4$

(c) $4 \neq 1/4$

(d) $12/3 \neq 3/12$

(e) Division is not commutative.

It is a relatively simple matter to see that (d) and (e) are inferred steps, while (a), (b) and (c) are assumed steps.

(a) $12/3 = 4$

(b) $3/12 = 1/4$

(c) $4 \neq 1/4$

\therefore (d) $12/3 \neq 3/12$

\therefore (e) Division is not commutative.

Figuring out which steps should be prefixed with the three-dot symbol is not always this easy. Mathematics textbooks present proofs in narrative form. Words and phrases commonly used to distinguish assumed from inferred steps are not always present. Inferred-step indicators include: 'therefore,' 'consequently,' 'it follows that,' 'as a result,' 'so,' 'thus,' 'hence,' 'we conclude,' 'we infer,' and 'accordingly.' Words and phrases used to indicate assumptions are: 'assuming,' 'because,' 'since,' 'as,' 'supposing,' 'for the reason that,' 'given that,' and 'seeing that.'⁴¹

Lesson II: Identifying Assumed Steps

Creativity is required once again to compile the list of assumptions necessary and sufficient to derive a result. In a simple example such as this, the effort is minimal but this is very much the exception. A useful exercise is to provide students with one rule of logic and have them build an inference chain that will apply only that one rule.

A moment's thought will show that premises (a), (b) and (c) are insufficient to derive the two inferred steps (d) and (e). In this case, it is a fairly simple matter to supply missing premises by bearing in mind that rules of logic made available in this article feature conditionals. With a bit of work we get the following:

(1) Division is a commutative operation $\rightarrow a/b = b/a$.

(2) $a/b = b/a \rightarrow 12/3 = 3/12$.

(3) $12/3 = 3/12 \rightarrow 4 = 1/4$.

(c) $4 \neq 1/4$.

Lesson III: Justifying Assumed Steps

Once a complete list of assumed steps has been compiled, we need to explain the basis for each one. Assumed steps are usually axioms, definitions, or theorems.

We get Step (1),

(1) Division is a commutative operation $\rightarrow a/b = b/a$.

⁴¹ These lists are by no means complete. Students should assemble a vocabulary of premise and conclusion words and phrases and share them with each other.

from

(DC*) R is a commutative operation $\rightarrow R(a,b) = R(b,a)$,

which is half the definition of commutation

(DC) R is a commutative operation $\equiv R(a,b) = R(b,a)$,

by applying UI and letting R be division in the antecedent of (DC*) and replacing R with the division symbol $/$ in the consequent on (DC*).

We get Step (2),

(2) $a/b = b/a \rightarrow 12/3 = 3/12$

also by applying UI and assigning arbitrary values to a and b , letting $a = 12$ and $b = 3$. As an exercise, students should work out the application of UI to derive steps (1) and (2).

We get Step (3),

(3) $12/3 = 3/12 \rightarrow 4 = 1/4$

by carrying out division on the numbers in the antecedent of (3).

Step (c),

(c) $4 \neq 1/4$

is an arithmetical fact that can be assumed here without argument.

Lesson IV: Displaying the Entire Inference Chain

Listing argument components vertically once again makes it easy to see how the inference chain proceeds from link to link.

(1) Division is a commutative operation $\rightarrow a/b = b/a$.

(2) $a/b = b/a \rightarrow 12/3 = 3/12$

(3) $12/3 = 3/12 \rightarrow 4 = 1/4$

(c) $4 \neq 1/4$

\therefore (d) $12/3 \neq 3/12$

\therefore (2*) $a/b \neq b/a$

\therefore (e) Division is not a commutative operation.

Lesson V: Identifying Rules of Logic Applied

Inferred steps (d), (2*) and (e) were derived using the same rule, MT. UI was used to justify (1) and (2).

Lesson VI: Checking Logical Justification

Inference chains to inferred steps (d), (2*) and (e) match the form of MT. The application of UI to justify derive steps (2) and (3) was explained above.

<u>MT Form</u>	<u>Inference Chain to Step (d)</u>
$p \rightarrow q$	$12/3 = 3/12 \rightarrow 4 = 1/4$
$\sim q$	$4 \neq 1/4$
$\therefore \sim p$	$\therefore 12/3 \neq 3/12$
<u>MT Form</u>	<u>Inference Chain to Step (2*)</u>
$p \rightarrow q$	$a/b = b/a \rightarrow 12/3 = 3/12$
$\sim q$	$12/3 \neq 3/12$
$\therefore \sim p$	$\therefore a/b \neq b/a$
<u>MT Form</u>	<u>Inference Chain to Step (e)</u>
$p \rightarrow q$	Division is a commutative operation $\rightarrow a/b = b/a$
$\sim q$	$a/b \neq b/a$
$\therefore \sim p$	\therefore Division is not a commutative operation.

Students should be assigned exercises that will help them see the form of rules of logic being matched by different content. Perception of logical form will lead to more advanced abstraction skills essential in mathematics and will make it easier to understand complex mathematical structures at a glance. Students need to get to the point where they can operate with symbols in mathematics without having to ask for specific examples.

The fact that the same sentence letter in a rule of logic can be replaced by different mathematical formulas shows the sense in which rules of logic are formal. MT is a correct rule of logic and has been matched correctly in all three cases, so the argument to show that division is not commutative is syntactically valid.

Lesson VII: Building a Fully Explicit Argument

- (1) Division is a commutative operation $\rightarrow a/b = b/a$Definition of commutation, **UI**
- (2) $a/b = b/a \rightarrow 12/3 = 3/12$Value assignments to a and b , **UI**
- (3) $12/3 = 3/12 \rightarrow 4 = 1/4$Application of division
- (c) $4 \neq 1/4$Arithmetic fact
- \therefore (d) $12/3 \neq 3/12$(3), (c), MT
- \therefore (2*) $a/b \neq b/a$(2), (d), MT
- \therefore (e) Division is not a commutative operation.....(1), (2*), MT

EXAMPLE 3

Algebra is an opportunity to show students that logical reasoning occurs also in mathematical contexts not associated with proving a result.

Suppose a student is asked to 'solve for x ' in a simple equation such as

$$(1) x - 7 = 6.$$

Textbooks usually state that the procedure to follow is to add 7 to both sides of the equation, resulting in 13 as the answer (Edwards, Gold & Mamary 2001, 130).

$$(1) x - 7 = 6$$

$$(2) x - 7 + 7 = 6 + 7$$

$$(3) x = 13$$

The word 'therefore' does not occur anywhere here to help students realize that argumentation is taking place. Students are merely instructed to carry out an operation according to a routine to be memorized. Before showing that computing a solution does in fact rely on logical argumentation, we consider issues in algebra that logic can clarify and will help students get used to the algebra environment.

Let us explain what 'x' means in the argument. Some algebra textbooks state that 'x' designates an 'unknown' and equations are described as containing several 'unknowns.' This creates the impression that algebra is about subjective, mysterious things called 'knowns' and 'unknowns.' Algebra – mathematics in general – is not about subjective or mysterious things of any kind. The helpful answer is that x is variable – as compared to a constant, such as *a*, also called in logic an 'individual symbol.' As such, x is ambiguous between 'for some x' and 'for all x.'⁴²

To resolve this ambiguity for the equation " $x - 7 = 6$," notice that "for all positive integers x, $x - 7 = 6$ " is false, so that x in $x - 7 = 6$ must mean "for some positive integer x ..." Teachers should make clear that x in an algebraic equation means either 'for any x' or 'for some x.' There is no third alternative, e.g., 'for many x' or 'for most x.' When equations contain more than one variable, we must be clear which quantifier is intended. Equations can combine quantifiers, e.g., 'for some x, for any y, $\Phi(x, y)$.'

Lesson I: Distinguishing Assumed From Inferred Steps

In light of the above comments, we need to start by inserting quantifiers in addition to distinguishing assumed from inferred steps. (2) and (3) are inferred steps, while step (1) is assumed:

$$(1) (\exists x)(x - 7 = 6)$$

⁴² Algebra is also an opportunity to draw a basic distinction between bound and free variables. In the example under study where we wish to find the value of a variable, the variable is bound by a quantifier, in this case an existential quantifier. Some mathematics texts refer to bound variables as 'apparent variables' and to free variables as 'real variables.' It is not always obvious which variables in mathematical formulas are free and which are bound.

$$\therefore (2) (\exists x)(x - 7 + 7 = 6 + 7)$$

$$\therefore (3) (\exists x)(x = 13)$$

It is possible to simplify matters by eliminating quantifiers, provided we do it carefully. Thus, the existential quantifier $(\exists x)$ can be replaced with an individual symbol such as a but only if we have made sure that a has not occurred earlier in the argument. This requirement is necessary because using a again to get Ga from $(\exists x)(Gx)$ after we have proved Fa would have the strange consequence that we 'proved' $Fa \& Ga$! At an elementary level, it may be better to eliminate quantifiers, provided we do not make such mistakes, we can replace $(\exists x)$ with an individual symbol and add $(\exists x)$ after we have proved Ga to get $(\exists x)(Gx)$.⁴³ In the example under discussion, we may safely remove the existential quantifier and replace it with an individual symbol. Students will find it easier to work with the quantifier-free version in the beginning stages.

$$(1^*) a - 7 = 6$$

$$\therefore (2^*) a - 7 + 7 = 6 + 7$$

$$\therefore (3^*) a = 13$$

Lesson II: Identifying Assumed Steps

The inference to step (2^*) is not apparent. To make the inference explicit, we need once again a step that links steps (1^*) and (2^*) by means of material implication:

$$(2^{**}) a - 7 = 6 \rightarrow a - 7 + 7 = 6 + 7$$

We also need a step in conditional form that carries out the computation in the consequent of (2^{**}) .

$$(3^{**}) a - 7 + 7 = 6 + 7 \rightarrow a = 13$$

Lesson III: Justifying Assumed Steps

Step (1^*)

$$(1^*) a - 7 = 6$$

is stipulated as part of the exercise.

The new step (2^{**})

$$(2^{**}) a - 7 = 6 \rightarrow a - 7 + 7 = 6 + 7,$$

follows by UI from an addition property of equality:

⁴³ The inference from Ga to $(\exists x)(Gx)$ applies the rule of *Existential Generalization* (EG), not included in the list above. The fourth predicate logic rule is *Existential Instantiation* (EI). We leave it to teachers to research the matter. It is beyond the scope of this article to explain how the rules of predicate logic apply to sentences containing multiple quantifiers.

$$(AP) a = b \rightarrow a + c = b + c$$

For the time being, we can take it for granted that (AP) is true. No doubt (AP) will seem 'obvious' at a glance. This does not mean that a proof of (AP) is easy or that it is not necessary.

The new step (3**)

$$(3**) a - 7 + 7 = 6 + 7 \rightarrow a = 13$$

is a conditional that carries out the computation in the consequent of (2**).

Lesson IV: Displaying the Entire Inference Chain

Here is the sequence of steps making explicit the inference to the final step (3*):

$$(1*) a - 7 = 6$$

$$(2**) a - 7 = 6 \rightarrow a - 7 + 7 = 6 + 7$$

$$\therefore (2*) a - 7 + 7 = 6 + 7$$

$$(3**) a - 7 + 7 = 6 + 7 \rightarrow a = 13$$

$$(2*) a - 7 + 7 = 6 + 7$$

$$\therefore (3*) a = 13$$

Lesson V: Identifying Rules of Logic Applied

MP justifies steps (2*) and (3*) and UI justifies step (2**).

Lesson VI: Checking Logical Justification

Inspection shows that inference chains match MP.

<u>MP Form</u>	<u>Inference Chain to Step (2*)</u>
$p \rightarrow q$	$a - 7 = 6 \rightarrow a - 7 + 7 = 6 + 7$
p	$a - 7 = 6$
$\therefore q$	$\therefore a - 7 + 7 = 6 + 7$

<u>MP Form</u>	<u>Inference Chain to Step (3*)</u>
$p \rightarrow q$	$a - 7 + 7 = 6 + 7 \rightarrow a = 13$
p	$a - 7 + 7 = 6 + 7$
$\therefore q$	$\therefore a = 13$

Sentence letters in MP (or any other rule) can be matched by formulas even though formulas are not sentences in the grammatical sense of containing a subject and a predicate like "snow is white." It was one of Frege's discoveries that subject-predicate form, which is assumed in Aristotelian logic, is not sufficiently general for mathematical purposes and had to be replaced by a more powerful analysis.

Lesson VII: Building a Fully Explicit Argument

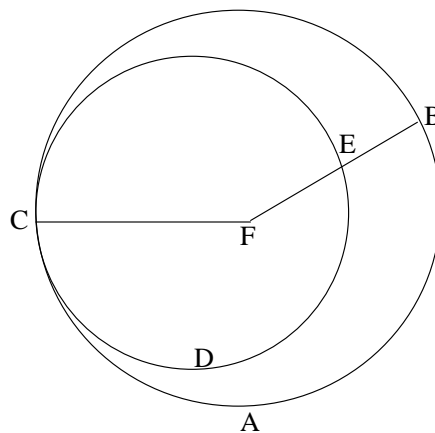
Arnold Cusmariu

- (1*) $a - 7 = 6$ Assumption
 (2**) $a - 7 = 6 \rightarrow a - 7 + 7 = 6 + 7$ Addition property of equality by UI
 \therefore (2*) $a - 7 + 7 = 6 + 7$ (2**), (1*), MP
 (3**) $a - 7 + 7 = 6 + 7 \rightarrow a = 13$ Arithmetic addition on (2*)
 \therefore (3*) $a = 13$ (3**), (2*), MP

EXAMPLE 4

This example from Euclid is much more challenging to analyze, despite the appearance of simplicity. Students will find that mathematicians often strive for simplicity or elegance of presentation at the expense of logical rigor, leaving out information they consider ‘obvious’ to avoid cluttering the text – including, as we have already seen, rules of logic used to derive results at various stages of an argument.

Proposition III.6 of the *Elements* states that if two circles touch one another, they will not have the same center. Below is the language of Euclid’s proof (2013, 55) along with the drawing he used in presenting the proof.



“For let the two circles ABC , CDE touch one another at the point C ; I say they will not have the same center. For, if possible, let it be F ; let FC be joined, and let FEB be drawn through at random. Then, since the point F is the center of the circle ABC , FC is equal to FB . Again, since the point is the center of the circle CDE , FC is equal to FE . But FC was proved equal to FB ; therefore, FE is also equal to FB , the less to the greater: which is impossible. Therefore F is not the center of the circles ABC , CDE . Therefore etc., Q.E.D.”

Lesson I: Distinguishing Assumed From Inferred Steps

Extracting argument components from Euclid's language and translating them into symbolic notation using truth-functional connectives requires interpretation.

Euclid: "Since the point F is the center of the circle ABC , FC is equal to FB ."

(1) If F is the center of circle ABC , then $FC = FB$.

Euclid: "Since the point $[F]$ is the center of the circle CDE , FC is equal to FE ."

(2) If F is the center of circle CDE , then $FC = FE$.

Usually, the term 'since' is a premise indicator but in (1) and (2) it is correct to interpret Euclid as asserting conditionals.

Euclid: " FC was proved equal to FB ."

(3) $FC = FB$.

Euclid: "Therefore, FB is also equal to FE ."

\therefore (4) $FB = FE$.

Euclid: "[Therefore], the less [is equal] to the greater."

\therefore (5) $FE = FE + EB$.

Euclid: "which is impossible."

(6) $FE \neq FB$.

Euclid: "Therefore, F is not the center of the circles ABC , CDE ."

\therefore (7) F is not the center of the circle ABC and the circle CDE .

Euclid: "Therefore, etc. Q.E.D."

\therefore (III.6) If two circles are tangent, then they do not have the same center.

Lesson II: Identifying Assumed Steps

Euclid argues for III.6 by *reductio at absurdum*. That is, he derives a contradiction from the negation of III.6

(III.6*) Two circles are tangent and have the same center,

from which he concludes that (III.6*) is false, therefore its negation, (III.6) is true.

Euclid does not explain why *reductio* arguments work. Specifically, he does not explain why deriving a contradiction from a sentence p proves that p false and thus its negation, $\sim p$, is true. If only he had! Here is the explanation.

The preferred term in logic for 'contradiction' is 'inconsistent sentence.' An inconsistent sentence is a sentence of the form:

(IS1) $p \ \& \ \sim p$.

This is the form Euclid applies in his argument. (IS1), however, is not the only inconsistent sentence form; so is this form:

$$(IS2) p \equiv \sim p.^{44}$$

Truth-tables introduced earlier to define logical connectives explain what's wrong with (IS1) and (IS2). We will leave it to the student to apply them to (IS1) and (IS2), which will show that only the truth value False occurs in columns under & in (IS1) and under \equiv in (IS2). What's wrong with contradictions, then, is that they are false no matter what truth values we assign to p in (IS1) and (IS2).

There is also the general question why contradictions cannot be allowed in mathematics. The short answer is that any sentence whatever can be proved to follow logically from an inconsistent sentence, meaning that every (well-formed) sentence is a theorem so that the entire system is inconsistent. We leave it to students to carry out the proof. Contradiction is 'check-mate' in mathematics.

Continuing with the analysis of Euclid's argument, let us start by translating its components into symbolic notation to identify logical connectives involved. Let us use Euclid's notation for referring to triangles, points, and segments – a notation that in fact 'hides' an inference to be explained later. Thus, Euclid's (III.6*)

(III.6*) Two circles are tangent and have the same center

is symbolized as,

(a) Circles ABC & CDE are tangent & F is the center of both circles.

Repeating this procedure the rest of Euclid's argument yields:

(b) $((ABC \text{ \& } CDE \text{ are tangent}) \text{ \& } (F \text{ is the center of both circles})) \rightarrow FC=FB \text{ \& } FC=FE.$

(c) $FC=FB \text{ \& } FC=FE \rightarrow FB=FE.$

(e) $FB=FE + EB \text{ \& } EB>0 \rightarrow FB > FE.$

(f) $FB=FE + EB \text{ \& } EB>0.$

(h) $FB > FE \rightarrow FB \neq FE.$

(k) $\sim((ABC \text{ \& } CDE \text{ are tangent}) \text{ \& } (F \text{ is the center of both circles})) \rightarrow (\sim(ABC \text{ \& } CDE \text{ are tangent}) \vee \sim(F \text{ is the center of both circles})).$

(l) $(\sim(ABC \text{ \& } CDE \text{ are tangent}) \vee \sim(F \text{ is the center of both circles})) \rightarrow ((ABC \text{ \& } CDE \text{ are tangent} \rightarrow \sim(F \text{ is the center of both circles})).$

This example should give students a better idea than the previous three examples how much work is required in mathematics to compile even a

⁴⁴ In the paradox that bears his name, Bertrand Russell uses (IS2). As an exercise, students should show that (IS2) implies (IS1).

reasonably complete list of assumed steps. Mathematical proof sketches contain gaps that must be filled to produce a syntactically valid argument. Practice is the only way to acquire this skill.

Lesson III: Justifying Assumed Steps

The justification for assumed steps needed to make the argument explicit is this:

- Step (a) is an arbitrary instance of III.6*.
- Step (b) combines Euclid's Steps (1) and (2).
- Step (c) applies Euclid's Common Notion I.1 (2013, 2): *Things which are equal to the same thing are also equal to one another.*
- Step (e) applies Euclid's Common Notion I.5 (2013, 2): *The whole is greater than the part.*
- Step (f) is apparent from Euclid's diagram.
- Step (h) follows from the definition of 'greater than,' assumed without statement in Common Notion I.5.
- Step (k) applies half of De M., bringing the negation sign inside brackets.
- Step (l) applies half of Imp., replacing $\sim_v_$ with $_\rightarrow_$.

Regarding steps (k) and (l), students should recall that $p \equiv q$ is equivalent with $(p \rightarrow q) \& (q \rightarrow p)$ and that, accordingly, 'applying half' of De M. and Imp. to these steps is in fact an inference from a sentence of the form $p \& q$ to p , which is justified by a rule of logic called 'Simplification.' Steps (k) and (l) are deliberately not stated as inferences to help teachers determine which students are paying attention to details. The author believes this is good pedagogy in general.

Lesson IV: Displaying the Entire Inference Chain

- (a) Let circles ABC & CDE be tangent & F be the center of both circles.
- (b) $((ABC \& CDE \text{ are tangent}) \& (F \text{ is the center of both circles})) \rightarrow FC=FB \& FC=FE.$
- (c) $FC=FB \& FC=FE \rightarrow FB=FE.$
- \therefore (d) $((ABC \& CDE \text{ are tangent}) \& (F \text{ is the center of both circles})) \rightarrow FB=FE.$
- (e) $FB=FE + EB \& EB>0 \rightarrow FB > FE.$
- (f) $FB=FE + EB \& EB>0.$
- \therefore (g) $FB > FE.$
- (h) $FB > FE \rightarrow FB \neq FE.$
- \therefore (i) $FB \neq FE.$
- \therefore (j) $\sim((ABC \& CDE \text{ are tangent}) \& (F \text{ is the center of both circles})).$
- (k) $\sim((ABC \& CDE \text{ are tangent}) \& (F \text{ is the center of both circles})) \rightarrow (\sim(ABC \& CDE \text{ are tangent}) \vee \sim(F \text{ is the center of both circles})).$

(l) $(\sim(ABC \ \& \ CDE \text{ are tangent}) \vee \sim(F \text{ is the center of both circles})) \rightarrow ((ABC \ \& \ CDE \text{ are tangent} \rightarrow \sim(F \text{ is the center of both circles.}))$

$\therefore (m) \sim((ABC \ \& \ CDE \text{ are tangent}) \ \& \ (F \text{ is the center of both circles})) \rightarrow ((ABC \ \& \ CDE \text{ are tangent} \rightarrow \sim(F \text{ is the center of both circles})).$

$\therefore (n) ABC \ \& \ CDE \text{ are tangent} \rightarrow \sim(F \text{ is the center of both circles.})$

$\therefore (III.6) \text{ Two circles are tangent} \rightarrow \text{they do not have the same center.}$

Lesson V: Identifying Rules of Logic Applied

The logical justification for inferred steps is as follows:⁴⁵

- Step (d) follows from steps (b) and (c) by HS.
- Step (g) follows from steps (e) and (f) by MP.
- Step (i) follows from steps (g) and (h) by MP.
- Step (j) follows from steps (d) and (i) by MT.
- Step (k) follows step (j) by De M.
- Step (l) follows from step (k) by Imp.
- Step (m) follows from steps (k) and (l) by HS.
- Step (n) follows from steps (j) and (m) by MP.
- Step (III.6) follows from step (n) by UG.

Lesson VI: Checking Logical Justification

All inference chains match the form of the corresponding rule.

<u>HS Form</u>	<u>Inference Chain to Step (d)</u>
$p \rightarrow q$	$(ABC \ \& \ CDE \text{ are tangent} \ \& \ F \text{ is the center of both circles})$ $\rightarrow (FC=FB \ \& \ FC=FE)$
$q \rightarrow r$	$(FC=FB \ \& \ FC=FE) \rightarrow FB=FE$
$\therefore p \rightarrow r$	$\therefore (ABC \ \& \ CDE \text{ are tangent} \ \& \ F \text{ is the center of both circles}) \rightarrow FB=FE$
<u>MP Form</u>	<u>Inference Chain to Step (g)</u>
$p \rightarrow q$	$FB=FE + EB \ \& \ EB>0 \rightarrow FB > FE$
p	$FB=FE + EB \ \& \ EB>0$
$\therefore q$	$\therefore FB > FE$
<u>MP Form</u>	<u>Inference Chain to Step (i)</u>
$p \rightarrow q$	$FB > FE \rightarrow FB \neq FE.$
p	$FB > FE.$
$\therefore q$	$\therefore FB \neq FE.$

⁴⁵ Steps (c) and (e) are also inferred steps because they apply UI. We leave this as an exercise to the student.

<u>MT Form</u>	<u>Inference Chain to Step (j)</u>
$p \rightarrow q$	$((ABC \ \& \ CDE \text{ are tangent}) \ \& \ (F \text{ is the center of both circles})) \rightarrow FB=FE.$
$\sim q$	$FB \neq FE.$
$\therefore \sim p$	$\therefore \sim(ABC \ \& \ CDE \text{ are tangent} \ \& \ F \text{ is the center of both circles}).$
<u>De M. Form</u>	<u>Inference Chain to Step (k)</u>
$\sim(p \ \& \ q) \rightarrow (\sim p \vee \sim q)$	$\sim(ABC \ \& \ CDE \text{ are tangent} \ \& \ F \text{ is the center of both circles})$ $\rightarrow (\sim(ABC \ \& \ CDE \text{ are tangent}) \vee \sim(F \text{ is the center of both circles}))$
<u>Imp Form</u>	<u>Inference Chain to Step (l)</u>
$(\sim p \vee \sim q) \rightarrow (p \rightarrow \sim q)$	$(\sim(ABC \ \& \ CDE \text{ are tangent}) \vee \sim(F \text{ is the center of both circles})) \rightarrow ((ABC \ \& \ CDE \text{ are tangent} \rightarrow \sim(F \text{ is the center of both circles})).$
<u>HS Form</u>	<u>Inference Chain to Step (m)</u>
$p \rightarrow q$	$\sim((ABC \ \& \ CDE \text{ are tangent}) \ \& \ (F \text{ is the center of both circles})) \rightarrow (\sim(ABC \ \& \ CDE \text{ are tangent}) \vee \sim(F \text{ is the center of both circles}))$
$q \rightarrow r$	$(\sim(ABC \ \& \ CDE \text{ are tangent}) \vee \sim(F \text{ is the center of both circles})) \rightarrow ((ABC \ \& \ CDE \text{ are tangent} \rightarrow \sim(F \text{ is the center of both circles})).$
$\therefore p \rightarrow r$	$\therefore \sim((ABC \ \& \ CDE \text{ are tangent}) \ \& \ (F \text{ is the center of both circles})) \rightarrow ((ABC \ \& \ CDE \text{ are tangent} \rightarrow \sim(F \text{ is the center of both circles})).$
<u>MP Form</u>	<u>Inference Chain to Step (n)</u>
$p \rightarrow q$	$\sim((ABC \ \& \ CDE \text{ are tangent}) \ \& \ (F \text{ is the center of both circles})) \rightarrow (ABC \ \& \ CDE \text{ are tangent} \rightarrow \sim(F \text{ is the center of both circles})).$
p	$\sim((ABC \ \& \ CDE \text{ are tangent}) \ \& \ (F \text{ is the center of both circles})).$
$\therefore q$	$\therefore ABC \ \& \ CDE \text{ are tangent} \rightarrow \sim(F \text{ is the center of both circles}).$
<u>UG Form</u>	<u>Inference Chain to Step (III.6)</u>
$\Phi(a,b) \rightarrow \sim(\Psi(c,a) \ \& \ \Psi(c,b))$	$ABC \text{ and } CDE \text{ are tangent} \rightarrow F \text{ is not the center of both } ABC \text{ and } CDE.$
$\therefore (\exists x)(\forall z)(\Phi(xy) \rightarrow \sim(\Psi(zx) \ \& \ \Psi(zy)))$	\therefore If two circles are tangent, they do not have the same center.

where Φ is 'tangent with' and Ψ is 'center of.'

Euclid selected circles a and b and center c arbitrarily, therefore, UG is satisfied and the inference from (n) to what he set out to prove, (III.6), goes through.

Lesson VII: Building a Fully Explicit Argument

(b) ((Circles ABC and CDE are tangent) &
 $(F$ is the center of both circles)) \rightarrow
 $FC=FB$ & $FC=FE$Assumption for *reductio*
(c) $FC=FB$ & $FC=FE \rightarrow FB=FE$Common Notion I.1, UI
 \therefore (d) (Circles ABC and CDE are tangent) &
 $(F$ is the center of both circles) $\rightarrow FB=FE$(b), (c), HS
(e) $FB=FE + EB$ & $EB>0 \rightarrow FB > FE$Common Notion I.5, UI
(f) $FB=FE + EB$ & $EB>0$Assumption
 \therefore (g) $FB > FE$(e), (f), MP
(h) $FB > FE \rightarrow FB \neq FE$Assumption
 \therefore (i) $FB \neq FE$(h), (g), MP
 \therefore (j) $\sim((ABC$ and CDE are tangent) &
 $(F$ is the center of both circles.))(d), (i), MT
 \therefore (k) $\sim((ABC$ and CDE are tangent) &
 $(F$ is the center of both circles)) \rightarrow
 $(\sim(ABC$ and CDE are tangent) \vee
 $\sim(F$ is the center of both circles)).....(j), De M.
 \therefore (l) $(\sim(ABC$ & CDE are tangent)
 $\vee \sim(F$ is the center of both circles)) $\rightarrow ((ABC$ & CDE
are tangent $\rightarrow \sim(F$ is the center of both circles)).....(k), Imp.
 \therefore (m) $(\sim(ABC$ & CDE are tangent) &
 $(F$ is the center of both circles)) $\rightarrow ((ABC$ & CDE
are tangent $\rightarrow \sim(F$ is the center of both circles)).....(k), (l), HS
 \therefore (n) ABC and CDE are tangent \rightarrow
 $\sim(F$ is the center of both circles).(m), (j), MP
 \therefore (III.6) Two circles are tangent \rightarrow
they do not have the same center.(n), UG

Logic courses focusing on making Euclid's proofs logically explicit as explained above would be more useful to mathematics students than standard logic courses.⁴⁶

Concluding Remarks

We consider briefly comments by professors of mathematics Robin Hartshorne, Kenneth Kunen, Reuben Hersh, and David Berlinski.

Hartshorne writes:

Euclid's proof [of Proposition I.1] depends only on the definitions, postulates, and common notions set out at the beginning of Book I. (Hartshorne 2002, 20)

Left unstated (here and elsewhere in the book) is the fact that transitions from one step to the next in a mathematical proof are justified by ('depend on') rules of logic. We have here a 21st century mathematics textbook that still treats logic as a 'silent partner' and presents proofs as they were in Euclid. Hartshorne, however, is hardly alone.

There is also this comment:

Among experienced mathematicians, there would be little disagreement about what constituted a valid proof, once it was found. (Hartshorne 2002, 11)

It is not made clear how students are to resolve such disagreements; or what 'valid proof' means. In any case, logical argumentation as explained here is sufficient for the purpose of determining whether a mathematical proof is (syntactically) valid.

Kunen writes:

The justification for the axioms (why they are interesting, or true in some sense, or worth studying) is part of the motivation, or physics, or philosophy; not part of the mathematics. The mathematics itself consists of logical deductions from the axioms. (Kunen 2012, 3-4)

Indeed, the reason theorems are justified (derivation from axioms) does not apply to axioms themselves. This, however, does not mean that the epistemic status of axioms must be extra-mathematical; that students are to look for it elsewhere – not that there is anything wrong with looking in philosophy! For example, an axioms system is justified to the extent that it leads to the derivation of useful results. Sometimes axioms must be rejected because they lead to contradiction, as did Frege's Law V; or because one or more are redundant; these are legitimate, and mathematical, reasons for deciding the epistemic status of axioms. In any case, students should not believe that axioms are 'arbitrary' or 'mere conventions' and thus not objectively true; or true only in some special sense of 'true.' Mathematics must assume that axioms are true (under an

⁴⁶ More advanced proof sketches such as those presented in Aigner and Ziegler 2000 would require a great deal more work to make logically explicit. See also Mueller 1981.

interpretation) and that it is this sense of 'true' that logical argumentation moves from axioms to theorems. The truth of a mathematical result does not appear by magic once it is proved!

Hersh writes:

I say the 3-cube or the 4-cube – any mathematical object you like – exists at the social-cultural-historic level, in the shared consciousness of people (including retrievable stored consciousness in writing). In an oversimplified formulation, "mathematical objects are a kind of shared thought or idea." (Hersh 1997, 19)

Students told that mathematical objects exist in a 'shared consciousness' may well conclude that the objectivity of mathematics is in doubt. They might be led to infer, as Hersh seems to suggest, that sentences about mathematical objects are true only in a 'social-cultural-historic' sense. Teachers will sow confusion and inhibit learning if they tell students that the interpretation under which the axioms of mathematics are true also depends on 'social-cultural-historic' factors; or that rules of logic and the concept of syntactic validity depend on such factors as well. Mathematics teachers should avoid suggesting that sociology has anything to do with mathematics.

Berlinski writes:

If the theorems of an axiomatic system follow from its axioms, it is reasonable to ask what *following from* might mean. What *does* it mean? The image is physical, as when a bruise follows a blow, but the connection is metaphorical. The connection between the axioms and the theorems of an axiomatic system is, when metaphors are discarded, remarkably recondite, invisible for this reason to all of the ancient civilizations but the Greek. (Berlinski 2013, 14, original italics)

The logical sense of 'follows from' is not the same as the causal sense Berlinski describes as a 'physical image,' a common (and elementary) misunderstanding. As to the relation between axioms and theorems, Aristotle made it clear in principle long ago, to which modern logic added technically correct details: Derivations of inferred steps are syntactically valid if and only if they match the form of rules of logic.⁴⁷ There is nothing 'recondite,' 'metaphorical' or 'invisible' going on here. The seven lessons presented in this article should enable mathematics teachers to make logic transparent and useful to students, who are much more likely to become mathematically proficient as a result.

⁴⁷ Berlinski writes (2013, 16): "Good arguments are good by virtue of their form" and credits Aristotle with this insight; yet, curiously, he seems not to grasp that he has in effect described the syntactic nature of mathematical proof as something not the least bit 'recondite.' Incidentally, Berlinski's claim that only the ancient Greeks understood the formal nature of logical reasoning is debatable. See Gabbay and Woods 2004, Vol. 1, *Greek, Indian and Arabic Logic*.

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Multiculturalism, or the Vile Logic of Late Secularism. The Case of Anders Breivik

Ignaas Devisch

Abstract: More than four years ago, Anders Breivik launched his apocalyptic raid in Norway. His killing raid was not an action standing on its own but a statement to invite people to read his manifesto called *2083. A European Declaration of Independence*. The highly despicable and disgusting mission of Anders Breivik addresses us whether we like it or not. Maybe there are good reasons to read and analyze Breivik's 'oration?' He confronts us with many questions we cannot simply run away from: What about the Islamization? How could this happen in secular Norway? What about the role of religion in European societies? In this article, I will argue that Breivik's plea can only happen from within a secular society in which the homogeneity already has been lost, which allows him to deal with religion and politics on a very specific basis. In no way whatsoever, the context of our secular society forced Breivik to do what he did. However Breivik could only construct his actions and ideas within the (Christian) democratic context he lives in. I will analyze this with the writings of Hannah Arendt on political theology and the complex relationship between politics and religion and a late secular society.

Keywords: secular, Breivik, Arendt, violence, political theology

'Whoever sheds human blood, by humans shall their blood be shed; for in the image of God has God made mankind.' (Genesis, 9: 6)

1. Introduction

More than four years ago, Anders Breivik launched his apocalyptic raid in Norway that began when he set off a car bomb near government buildings in Oslo killing eight people, and ended with the massacre of tens of participants in a Labour Party youth camp on the nearby island of Utøya. Not only was he after the destruction of many young lives, he also aimed to destroy the hope and innocence of a whole generation of 'leftists' because, he kept on repeating, he had a mission to tell the world in general and Europe in particular, namely: we have to stop 'Islamization,' the great danger from which (he believed) Europe suffers.

His killing raid was not an action standing on its own but a statement to invite people to read his manifesto called *2083. A European Declaration of Independence* (Breivik 2011). This 1500 page document is stuffed with declarations, statements, and fulminations against the world, as it is, Europe in particular. Page by page, Breivik develops his crusade against 'Islam imperialism' supported with fragmented quotes from books, articles, lectures, and newspapers. He presents us with his final product, an intellectual bricolage,

through which he hopes to convince us all of the great danger of Islamization and at the same time directing us about how to stop it.

The highly despicable and disgusting mission of Anders Breivik addresses us whether we like it or not. We can always ignore him and not read the manifesto for good reasons of which moral deprecation of Breivik's actions is only one, but an important one. Many people wonder, why should we pay attention to the manifesto of a mass murderer? Although these moral objections are reasonable, from a philosophical point of view, they are insignificant. Naturally, we disapprove of Breivik – what else could we do? – Nevertheless, does his case stop here? Maybe there are good reasons to read and analyze Breivik's 'oration;' he confronts us with so many questions we cannot simply run away from: What about the Islamization? How could this happen in secular Norway? (Mogensen 2013), What about the role of religion in European societies? And so forth.

For some among us, it may seem disgraceful to spend time on Breivik's manifesto. However, our disrespect of his highly excessive and morbid actions and their background ideology must not dispel us from thinking about these frictions and the precarious role religion plays in it. Despite his childish fantasy about some mythical European past, the way(s) Breivik writes about the relationship between politics and religion challenges us to think over our understanding of Europe as a secular continent and religion as a matter of individual, private choices. What about the (political) role of religion in contemporary Europe? (Goldstone 2007).

In this article, I will argue that Breivik's plea can only happen from within a secular society in which the homogeneity already has been lost, which allows him to deal with religion and politics on a very specific basis. In no way whatsoever, the context of our secular society forced Breivik to do what he did. However, it is my argument that Breivik could only construct his actions and ideas within the (Christian) democratic context he lives in. I will analyze this with the writings of Hannah Arendt on political theology and the complex relationship between politics and religion and a late secular society.

2. 2083

From its very origin, philosophy has never ceased to handle the most aporetic questions we humans are confronted with, albeit their moral connotation. Therefore, I will argue we should do more than declare our moral deprecation of Breivik's actions. We will not understand anything if we only condemn Breivik on moral grounds, just as we do not legitimate his actions because we want to understand them. As Jean-Luc Nancy makes it clear in *The Forgetting of Philosophy*: "It is one thing to denounce the ignominy of slavery; it is quite another to think sovereignty, which is not simply the cessation or the opposite of slavery. And which brings another essence – or another meaning – into play" (Nancy 1997, 20).

It is (too) easy to distance ourselves from Breivik and describe him as a lunatic or a 'sick mind,' and absolve ourselves from any attempt to try to understand what is at work here. Nor should we reduce Breivik to the contemporary context of Islamization and religious tensions by calling him a mere representation of today's democratic politics – Breivik is no more a representation of democracy than Bin Laden was of Islam. Of course, given today's political setting and the increasing anti-Islam tendency, it is perhaps not a coincidence that Breivik relates his actions to anti-Islam ideas and not, for instance, to alchemy or astrology. However, the attempt to understand how Breivik used society's setting as it is does not make of that setting as a whole a criminal given. To give another example: if a 'deep ecologist' would kill the CEO of Unilever, we need to investigate if and how his actions are related to his ideology, we have to analyze if the ideology as such tends towards violence, but if even this were the case, that does not make any ecologist idea suspect or violent.

Therefore, I call the act of Breivik a major dilemma of our time: how do we understand the ideological context behind his unimaginable act? I am not interested in the psychological analysis of this man, all the more I want to understand his ideas. Breivik is most and foremost an idealist in the strict sense of the word: a man who did and does everything to realize his ideas, no matter how excessive they are. Remarkably, although there is no comparison to whichever movement in Europe in what Breivik did and does, for more than a decade now, his background ideas and ideology are common sense in liberal, right or extreme right wing parties all over Europe for who 'leftist' ideas – Marxism, multiculturalism, the 'nanny-state,' etc. – are responsible for all what goes wrong in today's society. Think, for instance, about the success of the work of psychiatrist Theodor Dalrymple and his critique on the 'sentimental society.'¹ This kind of critique is a 'sign of the times' and a symptom of a paradigm shift, such as the work of Herbert Marcuse was crucial for the revolts of May '68 or Woodstock for the sixties.

For everyone reading his compendium, it is obvious that Breivik had the ambition to present himself as the avant-garde spokesman of this anti-left tendency and to declare war against the "the ongoing Islamisation of Europe which has resulted in the ongoing Islamic colonization of Europe through demographic warfare" (Breivik 2011). Breivik is very clear in his highly stated ambitions:

Time is of the essence. We have only a few decades to consolidate a sufficient level of resistance before our major cities are completely demographically overwhelmed by Muslims. Ensuring the successful distribution of this compendium to as many Europeans as humanly, possible will significantly contribute to our success. It may be the only way to avoid our present and future dhimmitude (enslavement) under Islamic majority rule in our own countries (Breivik 2011).

¹ <http://www.telegraph.co.uk/news/uknews/crime/7894907/Sentimentality-is-poisoning-our-society.html>

Breivik called his own actions 'spectacular', but what can be said about the quality of his writing? Starting from a very clear but high ambition, he writes "This compendium presents the solutions and explains exactly what is required of each and every one of us in the coming decades. Everyone can and should contribute in one way or the other; it's just a matter of will" (Breivik 2011). As we read this manifesto, we are consistently confronted with an ongoing copy cat-bricolage of quotes, (re)writings, statements, and references to all sorts of articles and books about what goes wrong on the 'old continent,' in particular on the supposed increasing influence of Islam in Western world. Breivik's thesis is born out of this: he is obsessed with what he calls 'the dangers of Islam' and how to restore Europe into a Christian bastion from which all 'leftist' sympathy with Islam is banned.

2083 refers to a 'prophetic year' in which Europe would defeat Islam, thereby reminding us of former crusades and battles between Christianity and Islam. The subtitle of the manifesto is twofold. First of all, "A European Declaration of Independence" Van Gerven makes clear in his "Anders Breivik: on copying the obscure" this subtitle is copied from a blog post by Peter Are Nøstvold Jensen operating under the pseudonym Fjordman, integrally inserted into Breivik's manuscript. The second subtitle is called "De Laude Novae Militiae Pauperes Commilitones Christi Templique Solomonici," to be translated as "In Praise of the New Knighthood, the poor fellow soldiers of Christ and of the Temple Solomon" (Breivik 2011). The first part is taken from a title of a text written by Bernard de Clairvaux between 1128 and 1146, entitled *Liber ad milites Templi: De laude novae militiae* (A Book for the Knights Templar: In Praise of the New Knighthood), the second part *Pauperes Commilitones Christi Templique Solomonici*, also abbreviated as PCCTS, was according to Breivik the official name of a Christian military order founded in 1119 also known as the Knights Templar.

As Van Gerven sorted out, the two Latin parts do not match grammatically:

In his manifesto, Breivik refers to himself as Justiciar Knight Commander for Knights Templar Europe and one of the several leaders of the National and pan-European Patriotic Resistance Movement". It is unclear to what extent the "Knights Templar Europe" organization actually exists; in chapter 3, "A Declaration of Pre-emptive War," Breivik refers to the "PCCTS, Knights Templar" as a "hypothetical fictional group." (van Gerven Oei 2011).

As stated, the manifesto is a bricolage and already the title witnesses the diversity of sources Breivik used.

The manifesto is also highly repetitive, not only in its statements, but also in the use of words and concepts of which Islam is one the most quoted.² One can ask why Breivik, who would possibly have stood up to commit violence in any

² A simple item search in his manifesto gives us the following hits: Feminism: 77; Marxism: 190; Multiculturalism: 469; Multicul: 1164; Islam: 3444; Identity: 109; Christian: 2237; Europe: 4310; Secular: 132; Jihad: 1018. Obviously, Christianity, Jihad, Islam and Europe are central references to his writings.

case, with or without Islam, is that much focused on this religion, and not on, for instance, ecologism or conservatism. Interestingly enough, Breivik's hate toward Islam is not to be situated at the level of the other-as-Muslim but at the level of the European-as-Muslim. He prophesizes that by the year 2083, Europe will be Christian again and "All traces of current and past Islamic influences in Europe will be removed. This includes mosques and Islamic cultural centers. All Muslims will be deported from European territory" (Breivik 2011).

Despite the harshness of the above quote, Breivik's manifesto does not appear directed toward foreigners or Muslims as such but people from within his own community who he believes are too tolerant toward foreigners, Muslims in particular. He argues that because Islam and Europe are not compatible, a new knighthood should take back Europe from the cultural Marxists, humanists, leftists, feminists, or suicidal (read: pacifistic) Christians who, according to Breivik, have already alienated Europe from its true Christian destination. As a result of this thinking, we notice that in his attacks in Norway, Breivik did not intend to kill Muslims but young innate people who are, to his conviction, part of a next generation of what he calls 'cultural Marxists,' leftist people who are unaware but nonetheless guilty of handing over Europe to an ongoing Islamization.

Breivik wants to remind Europeans of their cultural Christian background. He argues, "A re-christening is crucial to leave behind the dangerous and suicidal humanistic, secular and multicultural ideologies of our times" (Breivik 2011), further suggesting that "Europe should stand up again and fight against its enemies which are, as said, stemming from inside" (Breivik 2011). Therefore, we might conclude that first and foremost, the manifesto's baseline is that we (Europeans) are destroying ourselves by allowing others to take over our continent. Moreover, Breivik proposes that the decline of Europe has only been possible because we have forgotten the supremacy of Christian religion and culture. However, he suggests that Christianity has nothing to do with tolerance or peace rather Christians ought to join an ongoing crusade against the threats from inferior cultures currently infecting our great European project.

As previously stated, 2083 is the prophetic date to restore Europe and give it back its old strength and supremacy. According to Breivik, this restoration entails more than re-christening; everyone within Europe who relies upon the wrong ideas or ideology requires re-education. In his delirious description of the phases of the revolution towards a new Europe, Breivik not only foresees breeding programs to increase European population, but also a 'Declaration of Defection' for all Europeans who will be prepared to confess publicly their wrong ideas about the future of Europe as we know it today reciting this phrase, 'I hereby admit and acknowledge that multiculturalism is a European hate-ideology designed to deconstruct European identity, cultures, traditions and nation states. I used to support this anti-European hate-ideology. However, I no

longer support the European cultural and demographical genocide' (Breivik 2011).

Obviously, we should not expect to distill a coherent theoretical framework from *2083*, although the tendency of this manifest appears repetitively clear: Europe can only be Europe if it is restored and if it expels all Muslims or people from other cultural backgrounds, if Europeans are aware of their 'suicidal multiculturalism' and the need of a new knighthood. In short, Europe will be Europe if by 2083 Christianity is restored as its (only) grand narrative. Then and only then, Breivik is convinced of, Europe will rise as the new phoenix of a shining world order.

3. Religion and Politics

Of course, the brief sketch above does not pretend in any way to discuss *2083* in detail. Despite my close reading of many chapters, it would take another volume to present an exhaustive analysis of it, and to be honest; I do not think that would be of an interesting kind, given the repetitive nature of the manifesto. In this article, I only want to discuss one of Breivik's major points in his bombastic compendium, the plead for Christianity as the grand narrative of future Europe for I believe it touches upon one of the major frictions Europe has dealt with for a few decades now: the relationship between the West and Islam, or more general, the relationship between politics and religion. *2083* is far more than an 'absurd' statement because it reflects the difficult relationship between politics and religion (Wessely 2012).

In the last two decades, numerous analyses of the problems between political modernity and Islam have been made-for example see (Cesari 2005; Parvizi 2007; Vaner, Heradstveit et al. 2008; 2009). More often the analysis is presented as a clash of civilizations (Huntington 1996; Hunter and Gopin 1998; Véguez 2005; Achcar and Drucker 2006; Jansen and Snel 2009) or as a battle of enlightened people vis à vis conservative religious people (Bauberot 2007; Goldstone 2007; Mahoney 2010). And of course, the stakes are high. Since 1989 when the former Iranian Ayatollah Khomeini declared a fatwa on Salman Rushdie for his novel, *Satanic Verses*, the West has developed a troublesome relationship with the Islamic religion. Today, huge tensions are evident e.g. the response, to the cartoons in the Danish newspapers, 9/11 and the resultant aftermath, or the recent movie "The innocence of Muslims" in 2012. It would seem fair to suggest that daily frictions between Western societies and Islam have increased over the past decade and now rest at the centre of our societies.

Though useful to be aware of the tensions modernity is dealing with, none of these perspectives touch the problematic relationship between politics and religion in modern society as such. Religious violence or fundamentalism is of course not the privilege of Islam; think only of the violence against abortion doctors in America's bible belt or against homosexuality in some African countries at present such as Malawi and Uganda, or the ongoing provocations of

protestant Orangemen in Northern Ireland. The relationship between secularism and religion is far more complex than an argument that they are simply opposed to one another, as several scholars have demonstrated (Gauchet 1985; Gauchet 1998; Nancy 2005; Alexandrova, Devisch et al. 2012). Imagine only for a second Breivik to be a Taliban militant. Would it not be reasonable for us to expect another round of debates about the 'primitivism' of Islamic militias and their fight against modernity or the West?

With Breivik's actions and manifest, not only have we lost a certain moral comfort – since 9/11 it is common sense to project the current problems with religion as an exclusive problem of 'the other' (the primitive, unenlightened, religious other) –, we are also left behind with numerous blunt questions. This time, the religious violence does not stem from Islam but from someone who claims Christianity to be the only way to lead Europe to its bright and shining future and therefore pleads to get rid of Islam in Europe. Next to that, Breivik brings religion into play in a very particular way; not as a source of anti-enlightenment, but as the only way to retrace Europe onto its enlightened pathway as he sees it.

Therefore, his manifesto does not fit into the mold of the so-called clash between the enlightened west and unenlightened religion, as is common sense, mainly since 9/11. Breivik puts religion into play in a way that seems very inconsistent at first hand; first of all he stresses on the need for Christian religion as a conservative gesture to save Europe and to make us aware of the disasters of both secularism and Islam; on the other hand, he is in favor of individual freedom when it comes down to personal beliefs, which is one of the main characteristics of a secular society. Apparently, Breivik discovers in Christian religion a political and cultural function that it has lost in secular modernity, but as he calls for a re-christening, he describes himself as someone not very religious: "I'm not going to pretend I'm a very religious person as that would be a lie. I've always been very pragmatic and influenced by my secular surroundings and environment" or "Regarding my personal relationship with God, I guess I'm not an excessively religious man. I am first and foremost a man of logic. However, I am a supporter of a monocultural Christian Europe" (Breivik 2011). Having gone through this evolution, it does not stop him from calling Christianity the only platform able to restore Europe in its strength: "As a cultural Christian, I believe Christendom is essential for cultural reasons. After all, Christianity is the ONLY cultural platform that can unite all Europeans, which will be needed in the coming period during the third expulsion of the Muslims" (Breivik 2011).

To Breivik, a 'cultural Christian' can be a Christian practicing, a Christian agnostic, and even a Christian atheist, as long as Christianity is recognized as the only grand monoculture narrative that will situate Europe as the world's cultural and political trendsetter. This may include a sort of secular society, as long as Christianity is put at the forefront. I quote at length:

The European cultural heritage, our norms (moral codes and social structures included), our traditions and our modern political systems are based on Christianity - Protestantism, Catholicism, Orthodox Christianity and the legacy of the European enlightenment (reason is the primary source and legitimacy for authority). It is not required that you have a personal relationship with God or Jesus in order to fight for our Christian cultural heritage and the European way. In many ways, our modern societies and European secularism is a result of European Christendom and the enlightenment. It is therefore essential to understand the difference between a "Christian fundamentalist theocracy" (everything we **do not** want) and a secular European society based on our Christian cultural heritage (what we **do** want). So no, you don't need to have a personal relationship with God or Jesus to fight for our Christian cultural heritage. It is enough that you are a Christian-agnostic or a Christian atheist (an atheist who wants to preserve at least the basics of the European Christian cultural legacy (Christian holidays, Christmas and Easter) (Breivik 2011).

From the aforementioned suggestions, we can discern Breivik's complex stance towards secularism. On the one hand, people are allowed not to believe in God, which is a common secular stance. On the other hand, secularism is being blamed and religion is described as the cement of a monocultural society: "[S]ecularism promotes a more short term and hedonistic attitude towards life. Since secular people have little faith in God or an afterlife, the tendency is for them to adopt the attitude of 'Eat, drink and be merry, for tomorrow we die'. Of course, not all secular people are like that. But in general, secularism promotes such attitudes" (Breivik 2011). It seems as if Breivik is after some kind of religious authority to lead us towards the right path, but then why does he allow people not to believe?

4. The Doctrine of Hell

As stated, Breivik seems very inconsistent or illogical in the way he brings religion into play and his combination of personal freedom and monocultural Christian violence is at best very puzzling since a conservative religious revolution generally leaves no individual option at all to believe or not, since belief is put forward by force. Therefore, the 'simple' analysis that may serve to explain the rise of fundamentalism as a symptom of the clash between freedom versus violence and enlightenment versus religion, is not explaining anything here. What then is at work in Breivik's manifesto and (how) can we explain it? Is the combination of individual freedom and religious violence only the intellectual bricolage of a lunatic or can and need we say more of this awkward relationship between politics and religion?

Of the many philosophers and theologians who have written extensively about 'political theology' and the relationship between politics and religion in ancient and modern society, Hannah Arendt is one of the most remarkable. In her essays "Religion and politics" and "What is authority?" she makes an interesting and original analysis of secularism and the political function

Christian religion used to have in Western history (Arendt 1968; Arendt 1994). These two chapters may help us to understand why Breivik is in fact 'modern' in a very consistent way when he states that we are free to be Christian or not at the same time demanding that all of Europe to become Christian. Moreover, Arendt's thinking on authority helps us to analyze why Breivik calls his sentence 'pathetic' and asks for a punishment in true accordance with the magnitude of his actions.³

From the very beginning of her analysis, Arendt is transparent in her central assumptions. First of all, she outlines the importance of the doctrine of Hell to the ancient polis and Medieval Christianity "But there is one powerful element in traditional religion whose usefulness for the support of authority is self-evident, and whose origin is probably not of a religious nature, at least not primarily – the Medieval doctrine of Hell" (Arendt 1994, 380). Arendt leaves no doubt about the crucial role of hell and further argues that this doctrine has lost its authority in modern society as revealed by her suggestion that "The outstanding political characteristic of our modern secular world seems to be that more and more people are losing the belief in reward and punishment after death, while the functioning of individual consciences or the multitude's capacity to perceive invisible truth has remained politically as unreliable as ever" (Arendt 1968, 100); and also: "Who can deny, on the other hand, that disappearance of practically all traditionally established authorities has been one of the most spectacular characteristics of the modern world?" (Arendt 1994, 383).

I will discuss the doctrine of hell, and then proceed with the loss of its function in modernity and the remaining means by which religion can survive in modernity. I hope that this will guide us through one of the many puzzling questions Breivik confronts us with: is his combination of individual secularism and religious violence a mere atavism or an incoherent kind of idiocy, or is there some sort of 'late secular logic' behind it? Though multiple perspectives are possible to address this complex question, the way Arendt stresses the loss of authority and of the doctrine of Hell in a secular society, and her analysis of the remaining figures of religion in contemporary democracy, is of good help.

The doctrine of hell, Arendt states, goes back to Plato's myth of the Hereafter in which the souls of people who committed atrocious crimes will be subdued to eternal suffering, as a stunning example to the others. In the *Republic* but also in the *Gorgias*, Plato discusses indeed the idea of a reward or punishment in the hereafter:

But of those who have done extreme wrong and, as a result of such crimes, have become incurable, of those are the examples made; no longer are they profited at all themselves, since they are incurable, but others are profited who behold them undergoing for their transgressions the greatest, sharpest, and most fearful sufferings evermore, actually hung up as examples there in the infernal

³ <http://www.ctvnews.ca/breivik-wants-death-penalty-or-acquittal-for-massacre-1.798047>

dungeon, a spectacle and a lesson to such of the wrongdoers (Plato 1994, Gorgias, 525c).

And also:

[...] and that they came to a mysterious region where there were two openings side by side in the earth, and above and over against them in the heaven two others, and that judges were sitting between these, and that after every judgment they bade the righteous journey to the right and upwards through the heaven with tokens attached to them in front of the judgment passed upon them, and the unjust to take the road to the left and downward, they too wearing behind signs (Plato 1994, Republic, X, 614c).

To Arendt, Plato is not alone in inventing a myth on the Hereafter, but his myth is unique because of its explicit *political* function. As Arendt writes, the legend has been “enlisted in the service of righteousness” (Arendt 1994, 382). Plato, Arendt states, needs the myth in order to prevent people from doing something the state has no other means to than frightening people with an even worse punishment than their own crimes. The myth needs to prevent people from killing others in this world. The whole of Plato’s universe is split up into a real world of brightening Ideas and of world of shadowy representations, but these insights, as Plato explains with the Allegory of the cave, are only for the privileged ones. To convince the others, the masses, Plato’s state needs another story that will lead people to set the standards and principles to human behavior in this world. This ‘other story’ becomes the myth of the Hereafter:

We find it somewhat difficult to gauge correctly the political, non-religious origin of the doctrine of hell because the Church incorporated it, in its Platonic version, so early into the body of dogmatic beliefs. It seems only natural that this incorporation in its turn should have blurred the understanding of Plato himself to the point of identifying his strictly philosophic teaching of the immortality of the soul, which was meant for the few, with his political teaching of a hereafter with punishments and rewards, which was clearly meant for the multitude (Arendt 1968, 129).

As Arendt’s reading of Plato’s myth insists on its political importance, she also demonstrates how this political function lives through in Roman and Christian thought:

Just as the derivative character of the applicability of the ideas to politics did not prevent Platonic political thought from becoming the origin of Western political theory, so the derivative character of authority and tradition in spiritual matters did not prevent them from becoming the dominant features of Western philosophic thought for the longer part of our history (Arendt 1968, 124).

The roman triad of religion, authority, and tradition is confronted with a substantial test once the ‘anti-institutional’ Christianity becomes the official religion of the Roman Empire, and needs to be integrated in the ‘secular’ political framework of Roman political thought. This was a major challenge as the Roman

Empire had lost its political authority leaving only God with authority and the king as nothing but a secular power within an empire that was grounded into a transcendent revelation. And secondly, the caesura between the Roman worldly grounding of power and authority and the Christian ideas of revealed truths stemming from a transcendent instance outside the world, was a significant test to the Roman Empire; how to relate the transcendent God to the immanent worldly power?

Only Plato's ideas on the standards for human behavior on top of the world and his myth on the Hereafter, made it possible to understand God's revelation politically: "God's revelation could now be interpreted politically as if the standards for human conduct and the principle of political communities, intuitively anticipated by Plato, had been finally revealed directly [...]" (Arendt 1968, 127). As only from the fifth century on, the doctrine of Hell is of real importance to Christianity, Arendt conceives this as a proof of the political function of this doctrine. The Christian creed, Arendt writes, shows no doctrine of Hell as long as Christianity remains without secular interests and responsibilities. The increase of this secular interest goes hand in hand with the integration of Platonic ideas into Christianity.

The amalgamation of Roman political institutions with Greek philosophical ideas allows Christianity to turn its vague ideas on the hereafter into a dogmatic system of punishments and rewards for human deeds in the hereafter. This framework dominates medieval Europe and it is only once Christianity loses its authority in the modern world, the doctrine of hell is no longer of political relevance. Christianity needed hell to prevent people from killing others by confronting them with a punishment far worse than death: eternal suffering. Arendt therefore stresses the enormous consequences of the loss of this doctrine for modern society and for the relationship between religion and politics in general.

5. Secularization

To Arendt secularization means first and foremost the disappearance of religion from public sphere and the elimination of the doctrine of hell as the only real political element of Christianity. This doctrine is understood as the religious sanction of a transcendent *authority*:

Superficially speaking, the loss of belief in future states is politically, though certainly not spiritually, the most significant distinction between our present period and the centuries before. This loss is definite. For no matter how religious our world may turn again, or how much authentic faith still exists in it, or how deeply our moral values may be rooted in our religious systems, the fear of hell is no longer among the motives which would prevent or stimulate the actions of a majority (Arendt 1968, 135).

Authority is what religion and therefore society has lost today: "Politically, secularism means no more than that religious creeds and institutions have no

publicly binding authority and that, conversely, political life has no religious sanction”(Arendt 1994, 372). In a secular society, the Roman triad of religion, authority, and tradition has evaporated with the end of Christian religion as the grand narrative of medieval and early modern Europe.

From then on, Arendt concludes, political power has only two means of legitimatization left: first of all deliberation which is central to democracy and the way people try to look after consensus, persuasion or exchange of ideas; the other means is violence, rather characteristic but not privileged to totalitarianism, and often used in a context in which the authority has lost its authority and therefore needs violence to install it; authority then becomes authoritarian or even installs a ‘hell on earth.’ Consequently, to Arendt, secularization is more than an evolution of mankind from heteronomy towards autonomy. It is a political caesura in the history of the West: “The political consequence of the secularization of the modern age, in other words, seems to lie in the elimination from public life, along with religion, of the only political element in traditional religion, the fear of Hell” (Arendt 1994, 382).

Arendt conceives this loss as the major political event of secular modernity and she concludes her “Religion and politics” not in a mere optimistic mood. Taming the phantom of religion has advantages, she states, but it also includes risks we should be aware of:

But while in the past the danger chiefly consisted of using religion as a mere pretext, thus investing political action as well as religious belief with the suspicion of hypocrisy, the danger today is infinitely greater. Confronted with a full-fledged ideology, our greatest danger is to counter it with an ideology of our own. If we try to inspire public-political life once more with ‘religious passion’ or to use religion as a means of political distinctions, the result may very well be the transformation and perversion of religion into an ideology and the corruption of our fight against totalitarianism by a fanaticism which is utterly alien to the very essence of freedom (Arendt 1994, 384).

For my analysis of Breivik’s manifest, this is of course an important observation. Given the loss of its authority in secular modernity, to Arendt, the (political) place of religion in modernity is a precarious one. If religion aims at political power, it has only two political means left, she states: deliberation and violence. What Arendt fears the most is not religion as such but the transformation of religion into an ideology, often in the name of a fight against already existing violence. Obviously, today, if we look back upon the first decade of the twenty-first century, it is not that hard to see the danger Arendt is talking about. Not only we have undergone the violence of religious fundamentalism and terrorism but also of the fight against it, which has, often in the name of secular freedom, turned itself into a very violent war excluding civil rights and freedom. As is illustrated in a State of the Union by former President Bush, from 2006: “Abroad, our nation is committed to an historic, long-term goal – we seek the end

of tyranny in our world. Some dismiss that goal as misguided idealism. In reality, the future security of America depends on it.”⁴ The dangers of this ‘state of exception,’ as Giorgio Agamben has analysed so thoroughly (Agamben 2003), were and are still major: the suspension of civil rights, the legitimization of torture, the prison of Guantanamo Bay, all of these initiatives were taken in the name of Western freedom and the fight against Islam fundamentalism, while at the same time, Christian religion was also at the centre of this fight, think only of the words people like Donald Rumsfeld used these days: ‘eternal justice,’ ‘crusade,’ etc.

6. Late Secularism

Let me return now to Anders Breivik and his manifesto. As I have stated, even though Breivik’s crusade against Islamization may reflect similar thinking in our time, we cannot compare his actions with any other political event in contemporary politics. At the same time, neither should we become intellectually lazy and simply condemn any idea he sympathizes with, nor any author quoted by him. That would not make sense. Adolf Hitler being a vegetarian does not suggest vegetarianism to be a national-socialist practice, nor, alternatively, eating meat as a leftist progressive statement.

Having said that, we are still challenged to try to understand what happened. Not only is it extremely shocking that in Norway, an exemplary model of democracy, a Norwegian citizen uses this kind of extreme violence towards his young fellow-citizens at the same time Europe’s self-confidence as being the enlightened continent got pulverized because someone ‘from inside’ appeals to the same ‘pre-modern’ or ‘religious’ violence that up until now has been attributed to ‘other’. An enlightened European democracy can easily pretend to (under)stand primitive tribes slaughtering in Africa, or Taliban Muslims destroying Western cultures; as argued, these are figures of the Other: those of which we pretend to have nothing in common with. In the case of Breivik, things are more complex for he appears as one of us. If such anger and revenge can be generated from within a democracy in such a devastating way: what about it?

This is a very tough and aporetic question, the more Breivik appears to rely upon the context of late secular societies in which individuals have the freedom to choose the religion of their preference. At first sight, Breivik is very inconsistent in allowing people to be agnostic or atheist while pleading at the same time for a Christian Europe. A superficial reading of *2083* would only confirm this inconsistency hereby relying upon the ‘sick mind thesis’ and leave the philosophical analysis behind. However, Arendt’s writings on politics and religion make clear how this combination is perfectly possible and even dare I suggest logical. Probably more than he is aware of, Breivik is the perfect demonstration of the two faces of religion in modern secular society Arendt

⁴ http://www.cbsnews.com/2100-250_162-1264706.html

writes about for he repeats several times in the manifest, he is not really a believer and he would never blame others to be atheists, but at the same time, he counterweights the dangers of Islamization with a violent restoration of the force and authority of Christianity. Only when succeeding in this, 2083 can be the prophetic year of which he dreams.

As Arendt explains, an authority that has lost its authority can only restore itself by violent means. The doctrine of Hell has been lost, but not the danger of violence present at the core of Western religious thought:

The Introduction of the Platonic Hell into the body of Christian dogmatic beliefs strengthened religious authority to the point where it could hope to remain victorious in any contest with secular power. But the price paid for this additional strength was that the Roman concept of authority was diluted, and an element of violence as permitted to insinuate itself into both the very structure of Western religious thought and the hierarchy of the church (Arendt 1968, 132-133).

Maybe in our secular societies, we have been too comfortable in thinking that we had dealt with this violent religious thought and that all there was left of religion in today's Europe was nothing but personal belief. Arendt's analysis makes clear that at least there is another, darker side still present in a secular environment: the hope or tendency to restore what has been lost. It is too easy to distance ourselves from this tendency as were it the mere expression of primitive or lunatic thought coming from somewhere else: from unenlightened people, religious people, sick people, or from the Other in general. Arendt's analysis confronts us with the latent possibility of religious violence stemming from the heart of secular democracy itself. Consequently, the vile logic Breivik relies upon is more than a silly bricolage of extreme ideas. Instead of distancing from such ideas – democracy – should be interested in the political and intellectual contexts that shape expressions within texts such as *2083*. For it would seem that *2083* exemplifies a frightening proof that from within democracy, the 'phantom of late secularism' has never ceased to play its role. Time and again, we remain challenged to think through the difficult relationship between politics and religion, not to mention secularism and multiculturalism. Religion has lost its authority, not its political mean(ing)s. Though multiple perspectives are possible to address this complex question, the way Hannah Arendt stresses the loss of authority and of the doctrine of Hell in a secular society, and her analysis of the remaining figures of religion in contemporary democracy, offers us a promising perspective on Breivik's vile logic.

Starting from a liberal, secular viewpoint concerning our personal belief, one would never expect Breivik also to plead for the possibility and even the need for the restoration of Christian religion as Europe's grand narrative. For from being sick, primitive, or unenlightened, Breivik makes use of the two means Arendt talks about and therefore relies upon violence to reinstall its authority in Western Europe, to address the spread of Islamic religious ideas or Muslim

violence in Europe, of course relying upon the vile logic he develops in his manifest. While Breivik does not stop from warning us of the dangers of Islam, he is convinced his own violence is legitimized by a higher purpose since the future of Europe depends on it. Though he falls back upon the 'medieval' idea that a political community presupposes (religious) homogeneity (Lefort 1981; Lefort 1986), his plea can only happen from within a secular society in which this homogeneity already has been lost. In no way whatsoever, the context of our secular society (en)forced Breivik to do what he did. However, it is my argument that Breivik could only construct his actions and ideas within the (Christian) democratic context within which he lived.

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Good Fit versus Meaning in Life

Wim de Muijnck

Abstract: Meaning in life is too important not to study systematically, but doing so is made difficult by conceptual indeterminacy. An approach to meaning that is promising but, indeed, conceptually vague is Jonathan Haidt's 'cross-level coherence' account. In order to remove the vagueness, I propose a concept of 'good fit' that a) captures central aspects of meaning as it is discussed in the literature; b) brings the subject of meaning under the survey of the dynamicist or 'embodied-embedded' philosophy of cognition; and c) allows the theoretical discussion on meaning to become more focused and systematic. The article addresses two apparent problems with the idea of 'good fit,' namely the fact that both challenges and relations of an agent with the outside world are central to meaning. It is finally pointed out which implications adopting the concept of 'good fit' instead of 'meaning' would have.

Keywords: meaning in life, embodied-embedded, cross-level coherence, agent

Introduction

Meaning is arguably the most important thing in the world. We are speaking here of 'meaning,' not in its linguistic or representational sense, but in the sense of 'meaningfulness,' or 'meaning in life.' If meaning is absent, we are indifferent, apathetic, bored, cynical, alienated, disoriented, frustrated, or desperate; and if we are, it seems that no further thing can have any value for us either.

So one would expect meaning to be a key subject of discussion and research in the philosophy of mind, meta-ethics, normative ethics, and psychology. Now, representational meaning is a mainstream subject of study, and so are separate aspects of meaning, such as depression, intrinsic motivation, attachment, or morality. But not meaning as such: the subject is elusive and the concept is indeterminate, and this makes systematic theoretical discussion or empirical research on meaning difficult.

An approach that nevertheless seems promising is Haidt's account of meaning in terms of coherence (Haidt 2006, Ch. 10). At issue here is so-called 'cross-level coherence,' i.e., coherence among the diverse levels of a person's life and self: body, mind, and society and culture. Haidt claims: "People gain a sense of meaning when their lives cohere across the three levels of their existence" (Haidt 2006, 227).

Haidt's approach, I submit, is attractive for at least three reasons. First, the idea of coherence as generating a sense of meaning seems intuitive. Second, the approach provides a workable angle for people in search of meaning: they can start looking for ways to achieve more coherence, rather than for less achievable things such as fame, a lasting legacy, or perfection. Third, this approach alerts us

to features of our living conditions that make achieving or maintaining coherence difficult for us.

Conceptually, however, Haidt's account remains sketchy. What exactly is it that people with coherent lives gain a sense of? Haidt does distinguish different senses of 'meaning,' and the sense that is relevant here is that of 'purpose within life' (Haidt 2006, 213-217). But what exactly is 'purpose within life' and how does it relate to coherence?

Below, I will myself articulate a coherence-based concept called 'good fit.' 'Good fit' is here abbreviatory for 'dynamically conceived multiple-context fitting-in with an agent's embodied-embedded mode of being.' 'Good fit' is here a seemingly trivial but crucially important notion that rides on the back of an 'embodied-embedded' conception of agency that I will here refer to as 'dynamicism.' I hope to demonstrate that this concept can do at least some of the work usually done by the concept of 'meaning,' but do it better. The concept of 'good fit' is intended a) to capture central aspects of meaning as it is discussed in the literature; b) to bring the subject of meaning under the survey of dynamicism; and c) to allow the theoretical discussion on meaning to become more focused and systematic.

However, the aim of introducing the concept of 'good fit' is not the reduction or elimination of the concept of 'meaning.' For it is precisely the indeterminacy of this concept that keeps us sensitive to important but hard-to-articulate aspects of living well. There is health, and happiness, and beauty, and virtue; and then there is also this further important but elusive thing called 'meaning.' We thus keep our horizons open, and this is a good thing. The 'good fit' account is just an attempt to redeem those parts of the subject matter that do allow of more systematic treatment, but under current dialectical conditions remain hazy.

Below, I will first discuss the concept of 'meaning' as it is currently discussed, and highlight its indeterminacies (Section 2). Next I will, on the basis of Haidt's account (see above) explain the dual idea of psychological health as 'coherence' and meaning as 'good fit' (Section 3). As we will see here, developing a workable concept of 'good fit' requires that two problems be solved. First, our conception of 'good fit' must allow a positive role for anomalies, adversity, discord, struggle, emotional turbulence, and obstacles – all things that typically do not seem to fit an agent well, yet can be very meaningful, and for precisely that reason. Second, 'good fit' must be understood in such a way that it is not just the counterpart of internal coherence, but also that of an agent's relations with the outside world. That is, we must be able to say that these relations can fit the agent well, and this must mean something different from saying that the agent fits in with the outside world. I will try to solve these two problems by outlining what agency and good fit amount to in the theoretical framework of dynamicism (Sections 4 and 5). I will finish by discussing a range of issues that will bring into focus what adopting the concept of 'good fit' would amount to (Section 6).

2. The Trouble with 'Meaning'

Systematic philosophical discussion on meaning (e.g., Baggin 2004; Cottingham 2002; Eagleton 2008; Klemke and Cahn 2008; Messerly 2013; Metz 2014) is rare compared to that on, say, semantics, rationality, happiness, or morality; attempts to make such discussion empirically informed and interdisciplinary (e.g., Baumeister 1991; Flanagan 2007; Wolf 2010) are even rarer; and empirical research on meaning (e.g., Battista and Almond 1973; Reker and Woo 2011; Steger, Frazier, Oishi and Kaler 2006) is scant in comparison to research on health, happiness or wealth.

The point is not that theorists rarely discuss meaning. Many do, as an aside to or part of a different main subject; and meaning – or something like it, or an element of it, or something closely related to it – is also discussed under different names, such as 'authentic happiness' (Sumner 1996), 'intrinsic motivation' (Deci and Ryan 1985), or 'psychological health' (Haidt 2006, Ch. 7; see below). All the same, systematic inquiry into this specific subject is relatively marginal.

The likely reason for this, I submit, is conceptual indeterminacy. For 'meaning' seems to be a concept with too many different, and sometimes conflicting definitions, and it seems very hard to decide, for the purposes of systematic inquiry, on a characterization that is not overly controversial. Consider the following sample of attempts:

Meaning, if there is such a thing, is a matter of whether and how things add up in the greater scheme of things (Flanagan 2007, xi).

Meaning is that special, personal insight of how the world is connected to us. (...) (W)e have to learn who we are and what it is we want, what we need, what each change in reality implies for us – for our goals, our appetites, our fears and desires. (...) Meaning equals importance. It confers the royal kiss of significance on the outpouring of sense manufactured by the cortex. And it grows and consolidates in a nearby part of the brain: not the cortex, but the limbic system (...) (Lewis 2012, 35).

A human person's life is more meaningful, the more that she employs her reason and in ways that positively orient rationality towards fundamental conditions of human existence (Metz 2014, 222).

Personal meaning is defined as the cognizance of order, coherence, and purpose in one's existence, the pursuit and attainment of worthwhile goals, and the accompanying sense of fulfillment (Reker and Woo 2011, 1).

Meaning (...) enables people to interpret and organize their experience, achieve a sense of their own worth and place, identify the things that matter to them, and effectively direct their energies (Steger 2010, 680).

Although these characterizations show some overlap, they do not seem to converge on one thing. And the sample might have included characterizations that stress further aspects of meaning that seem important: being part of

something larger (e.g. Seligman 2011), love (e.g. Wolf 2010), or narrativity (e.g. Velleman 1991).¹

Consider, in this light, how Steger et al. (2006) investigate meaning empirically. They do this by way of a survey called the Meaning in Life Questionnaire. Respondents here have to rate, on a 7-point scale, questions such as: 'I understand my life's meaning,' or: 'My life has a clear sense of purpose.' The authors explain:

We defined meaning in life as the sense made of, and significance felt regarding, the nature of one's being and existence. This definition represents an effort to encompass all of the major definitions of meaning and allows respondents to use their own criteria for meaning (Steger et al. 2006, 81).

Now, it might be that indeed, only a very loose definition like the one just given is workable. Something more specific but not overly controversial may just not be available, and there are three reasons to believe this is the case.

First, as was already suggested, the word 'meaning' is precisely useful as a wildcard that enables us to refer to a range of things that we do have a sense of, but that we find hard to articulate. An example here might be a victim of injustice whose only foothold is the idea that 'there must be some meaning to all this'; or a deeply unfulfilled person who can tell herself that she might someday find 'meaning.'

Second, the word 'meaning' is highly ambiguous. It may refer to the presumed meaning of existence, of all that exists, of life in general, or of human life; to the meaning of a life, or lives, in particular; and also to something internal to lives – meaning in, not 'of' life, or lives. And it may of course refer to representation, reference, symbols, and so on. So, extensive exercises in disambiguation are always necessary. Metz, indeed, opts for

(...) a 'family resemblance' view, holding roughly that theories of meaning in life are united by virtue of being answers to a variety of related and substantially overlapping questions that cannot be reduced to anything simpler (Metz 2014, 11-12).

Third, 'meaning' is a word that carries deep-seated moral intuitions and commitments. So, theorists are not going to let a word with such import be used improperly: on any proposed definition they will see important aspects of meaning fall by the wayside or, alternatively, see irrelevant aspects encroaching, and will protest that the definition fails to describe meaning properly so-called.

The main dialectical fault lines here seem to be the following. Most theorists insist that meaning must be experienced subjectively, but some disagree (e.g. Metz 2014, Par. 10.2.1). Many theorists would say that subjective concern for things that are obviously trivial is not genuine meaning, but some disagree (e.g. Haidt 2007). So meaning is often claimed to involve both objective

¹ Also compare Jackendoff (2012, 32–50). Taylor (1989, 18) speaks of 'polysemy.'

value and subjective concern (e.g. Wolf 2010 and Reker and Woo as cited above). But others (e.g. Nagel 2008) will insist that seen in a broader perspective, even lives of subjective concern for objectively valuable things are pointless. Meanwhile, some (e.g. Singer 1993) hold that morality is the essence of meaning, while others (e.g. Kekes 2008) allow that immoral lives may be meaningful. Also, while virtually all conceptions of meaning seem tacitly anthropocentric (and some frankly so, e.g. Metz 2014, 222), it can – and probably should – be insisted that all living entities can live meaningfully each in their own way.

Viewpoints such as these are not easily reconciled. And the result is that the theoretical debate on meaning does not and cannot proceed beyond ethics and descriptive psychology. Attempts to make ‘meaning’ an explanatory notion in psychology, to discuss meaning in the context of biology, to investigate it empirically, to make it the basis of therapeutical programs, or to make it politically salient and relevant, all may exist in some form, but such attempts are severely hampered by conceptual indeterminacy.

3. Coherence and Good Fit

The idea of ‘good fit’ as a way to break free from this impasse derives from what seems to be a quite intuitive notion, namely that of a coherent agent (i.e., organism or person). A healthy, happy, flourishing or virtuous agent, it seems, is a coherent, wholehearted, or harmonious agent – one not torn apart by conflicting impulses, incompatible goals, self-defeating commitments, inconsistent beliefs, or an unstable self-conception.²

What seems crucial here is that we ourselves, rather than something external to us, should be coherent. That is, the coherence must be existential rather than merely perceived. We can perceive a story, a piece of music, or a piece of furniture as coherent. But this is not the same thing as experiencing oneself as coherent. Mere perceived coherence or non-coherence can either move us deeply or leave us indifferent, depending on whether or not we care about it. But it seems that our own coherence or non-coherence cannot leave us indifferent: experienced coherence will amount to a sense of well-being or fulfillment, while experienced non-coherence will amount to suffering. If we do seem entirely indifferent, for instance, in a case of severe depression, this very fact will be unbearable.

Although a conceptual linkage between coherence and psychological health seems intuitive, one between coherence and meaning is a bit more elusive. Let us first consider Haidt’s construal (Haidt 2006, Ch. 10). Haidt discusses

² Compare Antonovsky’s phrase ‘sense of coherence’ for the experience of one’s world as understandable, manageable, and worthwhile (Antonovsky, 1990). Coherence is also the basis of Haidt’s discussion of posttraumatic growth (Haidt, 2006, Ch. 7). This discussion stresses the formative role that adversity can have, and it dovetails with the idea that agents need challenges.

coherence among three levels in a person's life: a physical, a psychological, and a sociocultural level. By way of an example, Haidt describes the tight integration of the bodily, psychological and sociocultural aspects of Brahmin purification rituals (Haidt 2006, 227-229). The idea is that such rituals will be experienced as meaningful, not just because of a Brahmin's intellectual understanding of what the rituals mean, but also by a regular practice that involves rich bodily experience, due to which the understanding becomes 'visceral' (Haidt 2006, 228). Also contributing to meaning is the Brahmin's sense of belonging to a community and participating in an old tradition. As we already saw, Haidt concludes that "(p)eople gain a sense of meaning when their lives cohere across the three levels of their existence" (Haidt 2006 227).

We could argue here that what Haidt is really discussing is psychological health, not meaning. Or perhaps the coherent person does indeed experience meaning due to her coherence, but then the question arises why this should be so. At this point it should be noted that 'coherence' and 'meaning,' or 'coherent' and 'meaningful' cannot be treated as synonyms, like 'coherent' and 'healthy.' A coherent agent is not a meaningful agent, but rather an agent who experiences meaning. And an agent who experiences coherence does not experience meaning, but rather fulfillment or happiness.

But we can here take the step of conceiving of meaning, not as coherence, but as good fit. Note that good fit is the counterpart of coherence, in the sense that insofar as the constituents of an entity fit together – or insofar as any constituent fits the rest of the entity – this entity is coherent. Then, an event, action, person or object that is meaningful to us can be said to somehow fit us well – say, fit who we are, fit how we see the world, or fit the story we tell about our life. Haidt's idea was that if a Brahmin's life coheres across levels, he will experience a purification ritual as more meaningful than otherwise. We can now explain why this should be so: given such coherence, the ritual will fit the Brahmin better than otherwise. For it will fit him in not just one, but in three respects: a bodily, psychological and socio-cultural one.

We now have a notion of 'good fit' that does the work of 'meaning,' at least as Haidt presents it. But if 'good fit' is to cover the same subject matter as 'meaning,' two issues must be resolved. The first one has to do with the fact that at least for human beings, coherence is typically hard-won: only coherence after some struggle seems really worthwhile. Indeed, some of the most meaningful things in our lives are things that do not fit us well at all: anomalies, difficulties, conflicts, adversity, emotional shocks, and so on – things we may call 'challenges.'

The second issue is about internal and external relations. 'Coherence' and 'good fit' are notions that suggest that agents must cohere internally, regardless of how such agents themselves are related to the outside world. Meaning, however, seems to be very much about agents' relations with the outside world. For instance, having social roles or being part of something larger seems to be

key sources of meaning. Of course, we mentally represent the world around us, which makes it tempting to claim that meaning is about coherent representations, rather than about *de facto* external relations. But such an account is solipsistic, and it invites counterexamples based on scenarios of people who are locked up in webs of illusions. So I assume that appealing to representations is not going to be of much help here.

In the next Section, I will offer a dynamicistic account of agents that resolves the two above issues. We will see here that agents, unlike mere objects or machines, are essentially developmental entities; and that as a consequence, challenges are essential requirements for the coherence-maintaining activity of each agent. In this sense, challenges make of agents what and who they are, and while they do not fit them well the moment they occur, they do fit them well in their quality as developmental beings – in particular, challenge-overcoming and self-narrating beings. We will also see that something analogous can be said about external relations: agents are necessarily embedded entities, so that in their case, internal organisation and external relations presuppose each other, and external relations make of them the beings they are just as much as their internal organisation.³

4. Agents

When thinking about agents and coherence, we may be tempted to draw analogies such as those between agents and smoothly running machines, agents and logically consistent beliefs, or agents and well-made furniture. But agents are entities of an entirely different type than non-agents. This, at least, is what dynamicism teaches us. Accordingly, coherence for agents amounts to something different from coherence for non-agents, and we will see that it crucially involves challenges and external relations.

With ‘dynamicism’ I am referring to a blend of principles from physics, chemistry, biology, mathematics, information theory, and theoretical psychology, that has been articulated in different ways and with different emphases by authors such as Deacon (2011), Juarrero (1999), Kauffman (1996), Kelso (1995), Thelen and Smith (1994), and Thompson (2007). Although I am confident that my account of agents will capture the gist of accounts such as these, I do not pretend it to be the canonical view (if there exists one), or to express the views of any of these authors in particular. Nor will the account be anywhere near comprehensive. For it is intended to distill just those elements of dynamicism that are directly relevant to the subject of meaning.

³ We could note that ‘internal’ is here not a spatial, but an organisational notion: whether a process belongs to an agent is not a matter of whether it takes place inside or outside the skin or membrane, but to what extent it is integrated with the other processes (Millikan, 1993, 159). But I assume that this observation would not resolve the issue of ‘external relations.’

On dynamicism, agents are dynamic entities that are incessantly active in generating, protecting, restoring, and developing their own organisation. Such agents are not objects or systems with a structure, i.e., they are not spatial arrangements of detachable parts, but rather sets of processes with a dynamic organisation. In being process-like entities that are kept going by thermodynamic disequilibria, they are more like fires, vortices or hurricanes than like machines.

True, even lifeless objects appear process-like when studied at a micro-scale. But unlike lifeless objects, which do have a structure, but are entirely passive in their quality as processes, agents actively maintain an organisation. What this means is that processes such as blood flow and brain activity, and subprocesses such as metabolism and cell growth, sustain each other, and that the continuation of each process is required for the continuation of other processes that are also required for the agent's continued existence.⁴

Such organisation is possible due to nested sets of boundary conditions, which are constraints on what processes can and cannot take place in the agent. These boundary conditions are established over time by phylogenetic and ontogenetic processes – the so-called 'architecture' of the heart, or the 'wiring' of the brain – and, in the case of human agents, also by environmental structures such as intensive-care units or books, i.e., by technology.⁵

When we say that an agent maintains its own organisation, 'maintaining' means generating, protecting, restoring, and developing. That is, an agent does not merely keep itself in good shape or heal itself when injured; an agent also grows and changes. The ontology of an agent is thus profoundly temporal: agents are not just process-like, but also developmental beings – beings who remain themselves by changing all the time.⁶ If their development stalls, this does not make them static: in such a case, decay sets in.

Accordingly, the key notions of 'organisation,' and (its opposites) 'disruption' or 'disintegration,' must here be understood dynamically and developmentally. If the key notions here were 'order' (in its information-theoretic sense of 'compressibility of description') and 'disorder' (in its sense of 'randomness,' or 'entropy'), this would amount to thinking in static terms. Indeed, neither instances of order (such as simple shapes, or regular sequences), nor instances of randomness (such as garbage, or noise on the radio) have, by themselves, anything to do with agency. It is true, of course, that agency and

⁴ A phenomenon that is given names such as 'autopoiesis' (Tompson 2007) or 'teleodynamics' (Deacon 2011). Dynamicism is here indebted to Kant's principle of 'intrinsic finality' (Kant 1790, Sect. 64–66), which Deacon (2011, 302) calls 'the most prescient and abstract characterization of the dynamic logic of organism design.'

⁵ This latter example derives from Deacon's discussion of 'teleodynamic work' (Deacon 2011, 360).

⁶ Note that this is not an account of agents as 'space-time worms': on dynamicism, agents are individuated by their organisation, not by their spatiotemporal boundaries.

organisation do require the imposition of order on a substrate. But rather than mere order, organisation is the ongoing renewal, emergence and disappearance of order in many varieties. For an agent's very own dynamics generates tensions that cannot be resolved under present conditions of organisation, and that push the agent into new regimes.

Agents, however, do not just develop by way of such internal tensions. Even more important in this respect is their responding to intrusions of randomness. They need such intrusions for their development, because these give them leverage for change. So short of outright threats or damage, agents need manageable excitements, challenges, and opportunities. That is, apart from a nondisruptive, supportive, nurturing and stimulating environment, agents also need obstacles, nudges and sometimes even severe blows that enable them to keep developing.

Agents thus characterised are also, and necessarily, situated or 'embedded' entities. Not only do agents need to exchange matter and energy with the outside world; 'agent' and 'environment' are also complementary notions, and the nature and identity of an agent cannot be understood in abstraction from what environment the agent has, and how it interacts with this environment. Thus, what constitutes an agent is not just its internal dynamics, but also its interrelatedness – its dynamic couplings or co-evolution – with the outside world. Accordingly, being an agent not only consists in managing one's internal dynamics, but also, so to speak, in managing one's self-nonsel relations. Both make the agent into what and who she is, and neither can be dispensed with.

To sum up: an agent must maintain its coherence, not just by maintaining its internal organisation, but also by maintaining its embeddedness and its development. This is the type of entity an agent is: agents are not machines that are created once and get worn down, or that can be unplugged and sit still. No matter whether agents are in full action or asleep, they are always active. They must renew themselves each moment, must develop, and must keep interacting with their environments. If they do not do so, they quickly disintegrate.

The above should suffice to resolve the two issues that were raised earlier on in the context of Haidt's coherence account of psychological well-being and meaning. If 'good fit' is the counterpart of coherence, not in the sense of static order, but in the sense of dynamic organisation, then challenges and external relations turn out to be key aspects of good fit in the sense that is appropriate for agents. In that sense, an accident or illness can fit a person well – not when it occurs, but in the course of a life –, and so can membership of a community or the continuation of a tradition.

It should be noted that a complete account of agent coherence or good fit would require that we addressed a more profound issue: that of explaining how agents can experience their own coherence. Of course, our own experience suggests that coherence is pleasant and lack of coherence unpleasant. But this

presupposes that agents can assess what does and what does not fit them well, and this in turn presupposes that agents' experience is guided by norms of good fit. Then, what do such norms amount to, and where do agents get them from? However, I will here leave this issue alone.

5. Contexts of Fit

At this point, we have seen that the counterpart of agent coherence or psychological health is good fit; that properly (i.e. dynamicistically) understood, agent coherence (hence good fit) requires challenges, and that external relations define an agent just as much as its internal organisation. But especially human beings, with their intricate minds and complicated lives, need to harmonise many potentially conflicting aspects of their mode of being, and they can be coherent or non-coherent in different respects, involving different contexts of fit: contexts relative to which things can fit or not fit the agent well, hence appear as meaningful. What we have discussed so far is the first and primordial type of coherence, namely physiological or bodily coherence. There are, however, further respects in which things can fit an agent, i.e., there are various contexts relative to which there may be good fit or lack of fit. Good fit will then typically be the fitting in of something (an event, object, experience, activity, person, or whatever) with an agent's mode of being with respect to multiple contexts. Without the pretention of being complete or accurate, let us distinguish the following contexts:

An emotional context. At issue here is emotional health or balance. We are always busy regulating our emotional lives, acknowledging or suppressing, amplifying or damping, approving or disapproving, or choosing to act or not act on this or that emotion. Disruption here consists in inappropriate or disproportionate emotionality, as in depression or phobias. Emotions also have temporal shapes that feel either natural or unnatural (Dewey 1980, 57–58; Johnson 2007, 41). This type of emotional coherence is most salient in music, literature or film: a good piece will give emotional satisfaction, while an ill-chosen abrupt ending of a play, for instance, makes no emotional sense. In our emotionality, the coherence of we ourselves is at stake: while emotions fluctuate and non-disruptive emotional imbalances will be part of who we are, disorganised emotionality is destructive.

An epistemic context. At issue here is the coherence of our web of beliefs, understood as an evolving 'force field' in the sense of Quine (1951). Arguably, the coherence of our web of beliefs is also the coherence of us ourselves: our urge to know and to understand, i.e., to solidify, update and expand our model of the world seems every bit as powerful as our urge to maintain our bodily coherence. Most of us are curious, we dislike contradictions and indeterminacies, and we feel deep satisfaction when things fall into place (Gopnik 2000). And while epistemic disruption (confusion, disorientation, or 'losing one's mind') threatens our identity, epistemic coherence strengthens it.

A practical context. At issue here is the fit among our capabilities, actions and goals. Coherence in a practical respect is what is often called ‘purposiveness’ (e.g. Steger 2009, 680). One way of being purposive is by being competent, or efficacious (Deci and Ryan 1985). Here, the experience of ‘flow’ (Csikszentmihalyi 1991) seems to be a paradigm of practical coherence. Another, more ambitious way is striving not to have lived in vain, i.e. to leave a legacy (Belliootti 2001). Disruption here comes in the form of inactivity, helplessness, frustration, futility, indecision or a divided will. The coherence of ourselves in our very quality of agents seems here at stake: we shape our identities by committing ourselves to values and by making things happen.

A narrative context. At issue here is the self-conceptions we humans develop, not by images or theories, but by way of stories, or self-narratives (Fireman, McVay, Jr. and Flanagan 2003, 4–5). A nonhuman life cycle may take care of itself, but as human beings, we need to know where we come from, where we stand, and where we are going. Our lives are complicated, we need to direct ourselves, and others expect us to justify our choices to them. For this we need a coherent self-narrative, one with continuity, central themes, and a direction. These self-narratives are typically embedded in collective narratives, i.e., the myths that our culture imposes on us.

A social context. At issue here is the way we are connected to others – our emotional ties to specific individuals, our roles in communities, and the coordination of our behaviour with that of others. Disruption here comes in the form of lack of communication, persistent misunderstanding, social exclusion, alienation, loneliness, or Durkheimian *anomie*. Note that this listing does not include challenges (see above), such as temporary misunderstandings, conflicts, or even animosity: these do not disrupt us, but rather form us. Obviously, social coherence is a matter of external relations. But the coherence of ourselves is at stake in our social lives even so. We have a need to belong; the disruption of an intimate bond can be every bit as painful as physical injury; our self-conception and sense of personal identity require recognition from others; and much of our behaviour and cognition is shaped by our education and by the rules and institutions of our community.

6. How Adopting ‘Good Fit’ Will Work Out

The above listing of contexts of fit, and the – admittedly sketchy – account of what is at stake in each case, should demonstrate that the idea of ‘good fit’ can cover a range of things commonly associated with meaning. But to see how adopting it would work out, let us finally apply the idea to some key themes.

Sisyphus. A classic scenario in the debate on meaning, brought up by Camus (1942) and discussed further by Taylor (2000) and many others is that of Sisyphus, condemned by the gods to futile labour. Sisyphus’s life has deliberately been rendered pointless, but we might all be like Sisyphus, and what if Sisyphus

would build a temple rather than just move a rock, or what if he found intense pleasure in pushing his rock uphill?

This scenario is central to the debate on the meaning of life, but it would be much less relevant to debates on 'good fit.' The reason is that virtually all aspects of Sisyphus's life that are relevant to good fit – personality, family and friends, art, faith, and so on – are absent from the scenario. Of course, Sisyphus can be pictured as a forever-lonely prisoner in a gloomy underworld, but then his – utterly dire – predicament does not seem relevant to most real-world agents. From a 'good fit' perspective, the only interesting thing about this scenario is that it reminds us that our living conditions can be perversely rigged – not by the gods but by, say, modern society – in such a way that achieving coherence is made difficult for us.

The experience machine. Another classic scenario, originally developed by Nozick (1974, 42-45) against hedonism, but often discussed in the context of meaning subjectivism, involves a contraption that makes people falsely believe that they are leading interesting lives. People connected to the machine would subjectively experience meaning, or think they did, but it seems that their lives would not really be meaningful at all.

The 'good fit' account, with the part about 'external relations,' would here run parallel with meaning non-subjectivism. In the experience machine, good fit would be lacking because people's beliefs and feelings do not bear on their actual behaviour. We might even say that the scenario is not about 'agents' worth the name. In general, the 'good fit' account is not subjectivistic, since it is not just about coherence among agents' mental states, but about the coherence of agents' embodied-embedded mode of being. And this includes agents' being socio-culturally embedded, i.e., their relating to value systems that are not of their own making.

Trivial concerns. Agents can be absorbed in activities that, from a more detached perspective, seem trivial. Then, can these activities still be meaningful? Some people compulsively shop, play video games, or watch all their online friends' moves. Meaning subjectivists and non-subjectivists disagree here on the relative importance of either subjective concern or objective worth. Again, the 'good fit' account runs parallel with meaning nonsubjectivism: trivial pursuits may be absorbing, but they will not fit most people's considered values well. On the 'good fit' account, however, the – non-solipsistic – perspective of the agent is pivotal and not, as on nonsubjectivist accounts of meaning, objective worth.

This difference becomes visible when we think of a dog who fetches sticks thrown by her boss. Nonsubjectivists might dismiss this activity as trivial. But this activity does seem to fit the dog well: it is challenging yet doable; it is emotionally satisfying; and it strengthens the dog's bond with her boss. Of course, dogs do not seem to self-narrate like humans do, so this type of fit will be absent. But that is just what a dog's mode of being is like. So on the 'good fit' account there will be an important difference between dogs fetching sticks and rich

ladies' fetching expensive handbags, while such a difference will not be salient on 'objective worth'-oriented meaning accounts.

Anthropocentrism. What about tapeworms or plants? Intuitions may diverge on the meaningfulness of the activities of dogs or small children, but speaking of 'meaning' in the case of so-called 'primitive' organisms will sound patently odd.⁷ But this is precisely the problem: 'meaning' is an anthropocentric notion that inherits all the insoluble demarcation problems raised by anthropocentrism. By contrast, the notion of 'good fit' does not seem burdened in this way – compare attributing 'good fit' and attributing 'experience of meaning' to a microbe. An obvious advantage of this non-anthropocentrism is that the study of meaning will be easier to integrate with dynamicist cognitive science and biology.

Perfect coherence. Consider a person who is healthy, well-liked, well-connected, successful, famous, emotionally stable, and who has a track record of heroic feats. It might be thought that such an extraordinarily accomplished agent is a case of all-round and perfect coherence: everything fits, and fits neatly. But such a construal does not make sense here: the confusion is between 'organisation' and 'order.' As we have seen above, organisation is supposed to be 'ongoing renewal, emergence and disappearance of order in many varieties.' This implies that dynamic organisation is incompatible with all-round and perfect coherence, and that the idea of a perfectly coherent agent is an oxymoron.

Immorality. Consider a murderer who does not have a conscience and carries out his crimes in perfect secrecy. The 'good fit' account seems here to run parallel with meaning subjectivism: given the murderer's overall mode of life, the murders may be fitting this agent well. They may not fit him socially – he has secrets to conceal – but the need for deceit and secrecy might be challenges by which he can achieve personal growth – of course by his own standards. But cases of coherent crooks seem largely fictional, hence of questionable relevance. Real-world antisocial people do not seem to be harmonious or free from inner conflicts at all, and they face regular ostracism and punishment. In analogy with Tiberius (2008, Ch. 8) we might say that the 'good fit' account renders morality contingent, but not arbitrary.

Explanation. 'Good fit' might be a notion with more explanatory power than 'meaning.' Consider that there are two categories of human behaviour, namely those of existential decisions and nonmoral values, that may be best explained by an appeal to meaning. The right answers to 'Why did Peter become a priest?' or: 'Why do we love unspoilt nature?' might be, respectively: 'Because Peter was in search of meaning,' and: 'Because unspoilt nature is meaningful to us.' Taken by themselves, however, these answers are not very informative. They each require the telling of a more detailed story.

⁷ Not, though, of 'pointlessness': see, e.g., Wiggins (1988), Nagel (2008), or Taylor (2000).

Now compare: 'Because becoming a priest fit Peter emotionally, epistemically, socially, and narratively,' and: 'Because unspoilt nature fits us bodily, emotionally, and narratively.' These answers, too, require a more detailed story, but they do seem more specific than the 'meaning'-answers.

Scientific study. As already suggested, the concept of 'good fit' seems to allow for more theoretical integration with other fields than one of 'meaning,' so it might make meaning more empirically tractable. For instance, a Good Fit Questionnaire, in analogy with the Meaning in Life Questionnaire by Steger et al. (2006) might contain specific questions about the fit, or lack of fit, respondents experience between their beliefs, emotions, values, actions, social life, and life story.

That said, neither 'meaning' nor 'good fit' seems to lend itself particularly well for straightforward quantification and measurement. The appropriate methods for studying good fit will be of a hermeneutical and phenomenological, rather than quantitative character.

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Blame for Nazi Reprisals¹

George Schedler

Abstract: I examine the blameworthiness of the resistance for Nazi reprisals in three morally disturbing cases which occurred in Nazi occupied Europe. I have organized my argument in the following way. After describing the cases, I propose a set of criteria for assessing the degree to which actors are blameworthy for the deaths of innocents. Using these criteria, I then explore the blameworthiness of the resistance members in these cases. I follow this analysis with an application of the doctrine of double effect. My conclusion that some resistance members are blameworthy using my criteria is confirmed by the application of the doctrine of double effect.

Keywords: blame, blameworthiness, murder, resistance movements, Nazi reprisals

This paper explores the question of whether the resistance in Nazi occupied Europe shared some blame in certain circumstances for Nazi reprisals following their operations, based on historical events Rab Bennett described in his monograph, *Under the Shadow of the Swastika*. He observes at the outset that

“[p]recisely because the moral case against Nazism was so overwhelming, there has been ... an understandable tendency to treat resistance as *a priori* right ... Resistance acquired ... an aura that became difficult to question” (Bennett 1999, 29).

Of course, the Nazis’ murders of innocent hostages are unjust, but Bennett emphasizes the morally relevant fact that the Nazis made it clear beforehand that innocents would be murdered in reprisal for the specific actions which the resisters undertook. This circumstance raises doubts about whether the resisters are blameless for the Nazis’ inevitable murderous reactions. In light of Bennett’s research, I offer some answers to the difficult question of the extent to which the resisters may have been blameworthy.

I examine two of the many disturbing reprisals he recounts, as well as a third case, not of a specific operation, but of a subset of ideologically motivated resisters. In the first case, the Nazis carried out a threat they issued in advance using hostages they were already holding and then murdered more hostages when the resister did not surrender as they demanded. In the second case,

¹ This is a revised version of the presidential address which I delivered on December 27, 2014 at the American Society for Value Inquiry annual meeting in Philadelphia. It benefited from comments from attendees, especially Professor David Benatar (University of Cape Town, South Africa) and from subsequent criticisms of Professor Stephen Kershnar (State University of New York, Fredonia).

reprisals were not threatened ahead of time but easily predictable given the Nazi track record. Third, I examine the blameworthiness of resisters for whom the reprisals had instrumental value in achieving their long-range goals. I have organized my argument in the following way. After describing the cases, I propose a set of criteria for assessing the degree to which actors are blameworthy for the deaths of innocents. Using these criteria, I then explore the blameworthiness of the resistance members in these three cases. I conclude that some resistance members are blameworthy using my criteria.

I. Cases

Case1. In August, 1941, after the killing of a naval cadet in occupied Paris, the Nazis warned, via infamous black and red posters displayed throughout the city, that an appropriate number of hostages would be shot should there be a recurrence. The ratio was officially set at ten to one. Subsequent to the display of the posters, a Nazi commandant was murdered. Given the high rank of the officer, fifty hostages were rounded up and shot. Another fifty hostages were threatened with death unless the perpetrator came forward. This did not happen, so a total of 100 innocents perished (Bennett 1999, 131-132).

Bennett raises the question of whether in such a case the resister had a moral obligation to surrender and thereby save the lives of the hostages (Bennett 1999, 132). Given that the resister did not surrender, the murders of 100 hostages actually raises two questions: whether the resistance members who carried out the murder can be blamed for the deaths of the first fifty innocents in view of the threatened and entirely predictable reprisal, and whether the assassin can be blamed for failing to save the second set of hostages. I address the first question, but not the second (since it raises blameworthiness for an omission rather than for overt planned action which is my focus here). I will henceforth refer to this case as the "Nazi officer case."

Case 2. The sabotaging of railway lines in Ascq in 1944 was a more nuanced case, in which the Nazis made no threats in advance as in the Nazi officer case, but three preceding acts of sabotage at this location made it clear that a fourth instance would surely result in reprisals against nearby villagers. Bennett describes the appalling fallout from the operation.

"The forgotten epilogue to such tales of derring-do was the list of names of hostage and reprisal victims: for example, the 15-year-old boy, and the 76 other innocent reprisal victims killed after an act of railway sabotage at Ascq in 1944. This particular stretch of track on the main line from Antwerp to Paris had been sabotaged three times at the same inhabited spot. Even the most unimaginative resisters must have realized that their actions endangered the local population. It has been suggested that they could have considered moving their operations to the forests south of Lille where the Germans had no easy pretext for reprisals" (Bennett 1999, 34).

The resistance knew that Nazi reprisals for sabotage were visited upon inhabitants living near the site, so, had they blown up the track in an unpopulated area, they would have spared the lives of the victims of the Nazi reprisal. Henceforth, I will refer to this as the “rail sabotage case.”

Case 3. Here my focus is not on one operation but on a philosophically interesting group of resisters with a shared ideological orientation. The communist faction of the resistance did not simply carry out operations to weaken the Nazi occupying powers but also regarded the Nazi reprisals for their operations as a method of recruitment of new members to the communist cause. Their reasoning was that the severity of the Nazi reprisal policy would alienate the population and produce ten new members for every hostage the Nazis murdered (Bennett 1999, 136). Though that result did not seem to have materialized, nevertheless the communist faction regarded the Nazi reactions to their operations as a recruitment tool.

One example was an action in Rome, thirty-three SS men and ten civilians were killed by a bomb which was most likely detonated by the communist faction of the Italian resistance. Not all passers-by were warned, to avoid the German troops being alerted by an evacuation of the densely populated section of Rome in which the explosion occurred. The non-communist members of the resistance disclaimed responsibility for the attack.

Hitler’s immediate reaction was to order that section of the city to be destroyed and fifty hostages be shot for every German soldier killed. This was reduced to a ten to one ratio, so 330 people were chosen, the youngest being 15, the oldest, 74. The Vatican denounced both the bombing and the reprisal (Bennett 1999, 137-138).

Leaving aside the question of blame for loss of those passersby whom the communists left in the dark, does all the blame for the 330 innocents whom the Nazis murdered in retaliation lie with the Nazis? Or can the communist resisters be blamed as well? I will henceforth refer to the resisters who carried out these kinds of operation (resulting in the deaths of innocents due to reprisals) with the long-range goal of increasing recruitment to the resistance as the “communist resistance” or sometimes as “communist resisters.”

A major stumbling block in answering these questions is whether the resistance can be blamed for the hostage murders without also finding them responsible for the resulting injustice. It seems odd to conclude that they did not act unjustly but were morally blameworthy for the reprisals, but, on the other hand, it seems odd to say that they along with the Nazi enemy were to blame for the murders. Despite the oddity, the rightness or wrongness of an act depends in my view on whether the consequences of the act are good overall in the real world, whereas blameworthiness depends on the reasonableness of the beliefs of the actor. An act is wrong, in other words, when there is an alternative act that brings about the same amount of overall good but reduces the bad consequences, regardless of whether the actors examined alternatives before acting. The

blameworthiness of actors depends on whether they either intended the bad results—as the Nazis clearly did—or perhaps not intending them carried out the action knowing Nazi reprisals would occur.

II. Criteria for Moral Blameworthiness

Specifically, one is morally blameworthy for the deaths of others, when one intentionally brings about those deaths without justification or excuse. Intentional mental states include purpose, knowledge, or recklessness. An actor who unintentionally causes the death of another may, on some accounts, be morally blameworthy to an even lesser extent, where one's negligence causes the death. I am unsure about using negligence as a basis for blameworthiness for deaths of innocents, but I will include it for the sake of completeness. Before setting out the criteria, let us make some preliminary remarks about the scope of this inquiry and my terminology.

We need to bear in mind that this is not a general inquiry into blameworthiness for just any act but a narrow inquiry into the use of deadly force against regimes using terrorist tactics by resisters who plan their operations in advance. What applies here may not apply to acts or omissions causing deaths in other situations, such as, operation of heavy machinery or motor vehicles. There is no question, for example, that a resister can at the time appreciate the wrongfulness of the act, unlike other cases where impairment may be an issue.²

When I refer to a resister who causes the death of another (who is a member of the regime), I have in mind one who, acting alone or engaging in a joint effort, directs deadly force (which is known to kill or seriously injure others) toward another, and the other dies as a result.³ An excuse for an act (such as, provocation or duress) reduces the blameworthiness an actor would otherwise have for the act. Justification for an act (such as self-defense or defense of others) changes the nature of the act or renders it morally permissible. I take it for granted that one is justified in using deadly force to protect oneself or others from the threat of death by an unjust aggressor.⁴

A resister may *arguendo* be negligently blameworthy as well for the deaths of civilians. If so, the resister is blameworthy to a lesser extent for failing

² Appreciation of the wrongfulness is sufficient for blame but not necessary, because in other cases, such as, intentionally failing to take medications resulting in unconsciousness while operating a motor vehicle, one may still be said to have had a fair opportunity to exercise one's capacity to appreciate the wrongfulness of one's actions and be held criminally liable for the injuries to others, as Hart argued in *Punishment and Responsibility* (Hart 1968, 152). However, no scenarios like this are at stake in the actions of the resistance under discussion here.

³ This is sufficient but not necessary since one can cause the death of another by setting in motion a sequence of events not known to cause death or serious bodily injury, but a case like this is not at issue in resistance actions, all of which involve the use of deadly force.

⁴ Where the aggressor is a just defender then deadly force is not a permissible response.

in the planning of the use of deadly force to take precautions against the deaths of innocent civilians when those deaths could have been avoided without altering the reasonably perceived balance of good and bad consequences of the act. Following H.L.A. Hart, I dub this a “fair opportunity to avoid the deaths of the innocents” (Hart 1968, 152). The act itself is of course right or wrong depending on the overall consequences, but the negligence of the planners is limited to the consequences which we can reasonably expect them to foresee. Consequences occurring decades later, for example, may render their act or omission wrong in the long run, but would not render the actors blameworthy so long as these consequences were not foreseeable at the time. Given the controversy over Hart’s theory of liability here, I only note in passing how resisters can be blamed for negligence; my interest is limited to the basis for holding them blameworthy for intentional acts.⁵

In assessing blame for the reprisals, I ignore the real-world complication of the actors’ and planners’ differing intentions and extent of their knowledge of relevant facts and future consequences. Aside from a common commitment to drive the Nazis out of occupied countries, different resisters undoubtedly had different intentions and varying amounts of information, so I can only reach hypothetical conclusions about blame. I presume the individual actors have certain morally relevant mental states and degrees of knowledge. Aside from the shared commitment of the communist resisters to establishing a society consistent with Marx’s vision, it would be unrealistic to make further assumptions about shared intentions and the extent of information available to the resisters.

Here, then, are the criteria that capture an intuitively plausible set of elements of moral blameworthiness of resisters for deaths of innocents. A resister whose actions result in the deaths of innocents:

1. is fully to blame if she intentionally caused their deaths without sufficient justification or excuse (I refer to this as “level 1 blameworthiness”),
2. but is not fully to blame if she, *ceteris paribus*, had a partial excuse, such as, provocation or duress, or, without intending their deaths, either knew the innocents would die or recklessly caused their deaths (“level 2 blameworthiness”).

Taken together, these are sufficient for moral responsibility for such deaths. It might also be noted that an actor can be blameworthy for trying unsuccessfully to kill innocent people, but my focus is on the successful cases.

Before we focus on the resistance, we should note that it is quite clear that the Nazis met all the conditions of joint intentional action to kill innocents; they have level 1 blameworthiness of intentional actors. We will assess the

⁵ See for example Claire Finkelstein, “Responsibility for Unintended Consequences” (Finkelstein 2005).

blameworthiness of the resistance by resolving the following issues: whether the resisters can be said to have caused the deaths of innocents; whether the resisters have a justification for their actions; whether they have an excuse available; whether the foreseeability of the Nazi reprisals constitutes an intervention which reduces or vitiates blameworthiness.

III. Application of the Criteria

A. Causation

First, the Nazis certainly engaged in a joint effort, and innocent people died as a consequence. Given that the Nazis were the murderers, there is some question as to whether the resistance can be said to have caused the deaths. Clearly, one can be blameworthy for a murder without actually committing it, as is the case, for example, when a mafia don doesn't actually carry out a murder but with the use of mafia slang, such as, "put him on ice" or "whack him" or "do him up," make it clear to a contract murderer that he is ordering that a murder take place. Of course, I am not suggesting that the resistance did order the murders in this way; the point is that merely because they did not use deadly force against the innocents does not of itself show they had no causal role in the deaths.

One response to this is that the Nazis murdered the innocents, regardless of whether the Nazis issued advance threats of reprisals (as in the Nazi officer case). Merely because the resistance performed acts that resulted in the Nazis murdering innocent people does not show the resistance was to blame. The Nazis, not the resisters, it might be said, committed the murders.

This defense overlooks the high degree of probability that the resisters knew that the Nazis would, as a result of the resisters' actions, kill innocent people. At the very least, the resisters knew they were risking Nazi reprisals, even if they neither intended reprisals nor could be sure they would take place.

Despite the likelihood of the Nazi response, intuitively at least, the resistance's blameworthiness, if any, cannot rise to the level of blame they would have for directly killing the hostages. For example, let us suppose the resistance in Nazi disguise had staged a mass murder comparable to the St. Valentine's Day massacre, where they donned Nazi uniforms, used Nazi weapons, spoke in German, and so on, in hopes of bringing down the regime sooner. Their blameworthiness could only be distinguished from that of the Nazis themselves by the resisters' loftier intention of bringing down the Nazi regime. That is, given that blameworthiness is in part a function of intent, and the disguised resisters' intent of a massacre of civilians would be a speedier end to Nazi occupation, they might be less blameworthy than the Nazis whose intent is to kill civilians and continue repression. This suggests that blameworthiness of the resistance for any of the actions in the three cases cannot rise to the level of blameworthiness for the Nazi decision to carry out reprisals.

However, this defense loses plausibility in cases where the collaborationist Vichy regime invoked it. The Nazis went so far as to ask one official in the regime to approve a list of the hostages to be killed. He disapproved, but said nothing in response to a second shorter list which the Nazis presented to him. They of course interpreted his silence as assent (Bennett 1999, 59). He viewed himself as not participating in the murders (Bennett 1999, 60). Clearly, he could not plead that he was blameless because he did not shoot the hostages himself. Although the resisters were not collaborators, that the Nazis would murder innocents after Vichy assent to the list of names was equally as inevitable and certain as Nazi reprisals after the resistance operations.

Unlike the collaborators, the resistance was not given the opportunity to disapprove of Nazi murders of specific people; nor did they engage in a joint effort, as collaborators did. They may have a causal role in some indirect way, but it does not follow that for that reason alone they are morally blameworthy at any level. We will explore another basis for blameworthiness due to the predictability of the Nazi response when we consider whether the Nazis were interveners who were solely to blame (in D below), but for now the resistance cannot be said to have caused the deaths of innocents directly.

B. Justification

But do the resisters have an overriding justification for actions resulting in the murder of innocents? Surely, the resisters in all three cases would point to the innocent lives saved by the more timely defeat of the Nazis as the justification. The more quickly the Nazi occupation is ended, the more innocent lives will be saved (which would otherwise be lost by deportation to death camps), and the Nazis will be defeated more quickly if the resistance recruits more members.

If the resisters' actions are justified in this way, their operations must be a reasonable means to this end. If, for example, refraining from mounting these operations or mounting some alternative operations would *ceteris paribus* save more lives, then their actions could not reasonably be said to save more innocent lives.

It is not clear that the justification succeeds in the case of the communist resisters. Their intention was to recruit more members to the communist faction of the resistance and ultimately convert France to a communist regime. To the extent that this faction was Stalinist and ultimately committed to converting France into a Stalinist style regime, with all the horrors and innocent loss of life that this would entail, the long-term loss of innocent life would likely be as great for France as it would under Nazi occupation. To the extent that this faction of the resistance overlooked this or realized this but ignored the consequences, the saving of innocent life loses its appeal as an overriding justification.

The justifications in the Nazi officer and the rail sabotage cases would not be ideological, so let us set the intention of establishing a Stalinist regime aside and ask whether the justification of shortening the Nazi repression and returning

France to some form of consensual government (whatever that might be) is plausible. In the Nazi officer case, the Nazis killed ten times more innocents than they antecedently threatened to murder, so it is plausible to suppose that, had the officer been of a lower rank, or, even more likely, had the victim been a soldier of lower rank, the final ratio would only have been the officially promulgated at ten to one. Of course, the counter to this is that the death of a higher ranking officer is more disruptive to the Nazi regime, but the ultimate question is whether fewer lives are lost because of the assassination of a Nazi of such high rank. It is certainly not obvious that more lives were saved in the long run, and it is likely that the Nazis would have killed only ten had the soldier chosen been of a lower rank.

There is a bit more clarity in the rail sabotage case, because the Nazis routinely rounded up hostages in the locale where sabotage had taken place. They presumed that local residents must have cooperated with the resisters. The Nazis most notorious massacre, two years prior to the rail sabotage, was the reprisal for the fatal wounding in Prague of Reinhard Heydrich. The Nazis publicized the complete destruction of two nearby villages, which was roundly condemned worldwide, so the French resistance would have been aware of the Nazi practice (Bennett 1999, 261). Had the resisters chosen to sabotage the rail lines in a remote location, the sabotage would likely have been equally effective and deprived the Nazis of their usual source of hostages. Resisters in the rail sabotage case, then, by failing to avail themselves of the opportunity to reduce the risk to civilians, voluntarily put civilians at risk, knowingly or at least recklessly.

C. Excuse

If the justifications do not succeed, could the resistance, in all three cases above, be blameless or less blameworthy at level 2, due to an excuse? First, it might be argued that they acted under extreme duress, given the savagery of the Nazi treatment of the French populace.

One difficulty with this is the question of whether the members of the resistance acted under threats or duress in the usual sense. The usual cases of duress go something like this: Jones is properly said to have been under duress to do act X (such as, surrendering his own valuable property or performing some seriously morally wrong act), because he told that, if he refused to do act X, Jones (or some innocent third party) will in turn suffer serious bodily harm or be murdered. But the Nazis did not order the populace to engage in seriously morally wrongful acts; they simply demanded that the populace refrain from resisting the occupation. They made what might be called a conditional threat: if anyone engages in acts of resistance, innocent people will be murdered.

However, a close examination of the cases shows that the Nazi threats are just as coercive as the duress cases and similar in morally relevant respects to the usual cases of extortion and armed robbery. Consider how similar to the

threat in the Nazi officer case is to a loan shark's prediction that if Jones fails to repay his loan with interest on time, one of Jones's family members will suffer broken bones, or the armed robber's threat to shoot Smith's companion if Smith does not surrender her wallet. Let's consider how the loan shark's and robber's threats are just as conditional and equally as coercive as the Nazi threats of reprisals.

The threat in the Nazi officer case is clear: the Nazis publicize in advance that if a Nazi soldier is murdered, innocent hostages will be killed. In case 2, the rail sabotage case, we have no explicit threat as in Nazi officer case, but the Nazis' past practices of reprisals against local residents in other acts of sabotage combined with the pattern of sabotages at that location, make an implicit threat clear. The communist resistance is similar in this regard. So, the similarity between the usual hypothetical threats and our three cases is clear.

However, there is morally significant difference between compliance with robber's or loan shark's demands and the resistance which renders duress morally irrelevant to the resistance operations. Duress would be an excuse for compliance with Nazi threats; it cannot not be employed as an excuse for refusal for complying either with the robber's and loan shark's demands or Nazi demands, as in the three Nazi resistance cases. That is, if investigators should ask Smith why she surrendered her wallet to the robber, the excuse would be that the robber threatened to kill her companion. If Smith refused to surrender the wallet with the result that the robber carries out the threat to murder her companion, the excuse is unavailable. The question morally analogous to our Nazi cases is whether Smith or Jones would be to blame for the injuries to others if they did **not** comply with demands of the loan shark or the armed robber. (My intuition is that they would be.) Regardless of the answer, duress is not an appropriate excuse for refusal to comply with the Nazi threats, though it may be invoked by those who did not resist the Nazis, just as it can be invoked by those victims who comply with the robber or the loan shark.

D. Foreseen Intervention

If duress is not available to the resisters as an excuse, their blameworthiness could at least be reduced by arguing that the Nazis' intervention brought about the deaths of innocents. The reasoning would be that when a consequence, C, occurs as part of a causal sequence resulting from the action of an initiating actor, but part of the sequence is an intervener's independent, voluntary action resulting in C, then, even if the initiating actor foresees C, the initiating actor is not to blame for C. A variation on an example Warren Quinn offers about intentional action will shed light on this. Smith is using a loud mower on his lawn. If his response to the question of what he intends to be doing by riding his mower is "mowing the lawn," or "cutting the grass," then we can posit that as his intention (Quinn 1989, 340). If we were to point out that the noise disturbs his neighbor, he would presumably say that it is not his intention to cause that even

though it has happened in the past. Such a response would be comparable to the non-communist resisters' avowal that they intend to disrupt the Nazi supply lines in the railroad sabotage case or to disrupt the Vichy regime by killing a Nazi officer, but they can foresee reprisals in both cases.

However, the communist resisters' intent to increase recruitment to their cause through the inevitable reprisals would be comparable to something like the following twist on the lawn mowing story. In the past, Jones always became so upset by the Smith's mowing that afterward he played heavy metal music so loud that it disturbed the entire neighborhood. Smith's intention in riding the mower is not only to maintain his lawn but to turn the entire neighborhood against his next door neighbor, Jones, by moving Jones to play loud heavy metal music. If Smith can be blamed for Jones's retaliatory blasting of loud music, then it would seem that the communist resisters can be blamed for the reprisals, given that the reprisals are just as much a part of the aim of recruitment for the communists as is Jones's loud music a part of Smith's plan to turn the neighborhood against Jones.

A slightly different argument can be made for finding the resisters in the Nazi officer case to blame for the deaths of the hostages. Even though the resisters did not intend that the hostages be sacrificed, they nevertheless murdered the officer despite the threats and likelihood the Nazis would follow through. The resistance could have passed up this opportunity and saved the hostages.

There is a slightly different analogy which offers a rationale, however attenuated, for blaming the resisters in the rail sabotage case. Suppose that Smith could cut his lawn equally effectively if he installed a muffler on his mower (at no cost to him and with no detrimental effect on the machine), with the result that Jones would not be driven to blast loud music. Smith doesn't install the muffler because he is indifferent to the mower's upsetting effects on Jones. But his indifference shows that he knows these ill effects would not occur if he installed the muffler, so he is to blame in part for the ill effects on the neighbors. Jones, of course, is also blameworthy for his failure to control himself.

Similarly, as Bennett suggests, the resisters could have equally effectively sabotaged the rail lines south of Lille, so their failure to do so renders them blameworthy at level 2, given that they knowingly ignored this in their planning. If the resisters in rail sabotage case never considered the risk to civilians living nearby, they were negligent because of the predictability of Nazi reprisals against the nearby local populace. The same can be said *mutatis mutandis* in the Nazi officer case: if the resisters considered murdering a lower ranking officer but rejected it, they were blameworthy at level 2, given the likelihood of an excessively brutal Nazi response. If the possibility did not occur to them, they were blameworthy for their negligent failing to consider the effect on civilians

IV. Conclusion

Therefore, despite the absence of a direct causal sequence between reprisals and resistance operations, we can blame the communist resistance for the deaths of hostages, given the integral role of reprisals in recruitment to their cause. The noncommunist resisters who could have made different choices in the location or target of their operations share some blame for the loss in the reprisals of some innocent lives. In the Nazi officer case, for example, knowing the threats which the Nazis issued in advance, the resisters are blameworthy in part at least for the number of murders which the Nazis threatened ahead of time, though arguably not for the unpredictably excessive murders consequent on the resistor's refusal to surrender. The resisters in the rail sabotage case share some blame, assuming they recklessly or knowingly chose to ignore the risk to the local populace. Alternatively, if the resistance in the rail sabotage and Nazi officer cases did not consciously ignore the risk of reprisal, it can be blamed for neglecting to consider such obvious risks.

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Information about Authors

Fred Adams is Professor of Linguistics & Cognitive Science and Professor of Philosophy at the University of Delaware. He has published well over 100 articles and chapters on cognitive science, epistemology, philosophy of language, philosophy of action, and related topics. He is also co-author of *The Bounds of Cognition* (Wiley-Blackwell, 2008, with Ken Aizawa) and of *An Introduction to the Philosophy of Psychology* (Cambridge University Press, 2015, with Daniel Weiskopf). Contact: fa@udel.edu.

Arnold Cusmariu received his Ph.D. in Philosophy from Brown University under Ernest Sosa and has published articles in metaphysics, epistemology and more recently aesthetics, available at Academia.edu. Also a sculptor, Dr. Cusmariu is working on a set of lectures placing his artwork in the context of what he calls "The Prometheus Challenge" alongside Manet, Degas, Picasso, Dali, and Lipchitz. Contact: bravo323@gmail.com.

Ignas Devisch, PhD (1970), is Professor in Ethics, Philosophy and Medical Philosophy. He holds a position at Ghent University, department of Primary Care and Family Medicine, and is affiliated with BIG (Bioethics Institute Ghent) and Artevelde University College, Belgium and was researcher for five years at the Radboud University Nijmegen. He publishes in the fields of medical philosophy, philosophy and ethics. Contact: Ignas.Devisch@UGent.be.

Wim de Muijnck, PhD, is an Assistant Professor at the Faculty of Social Sciences, Nijmegen University, The Netherlands. In his research, he explores the intersection of metaphysics, metaethics and 'embodied-embedded' philosophy of mind. Among his publications are: "Valuation by Behaviour," *Philosophical Explorations* 13, 2 (2010), and: "The meaning of lives and the meaning of things," *Journal of Happiness Studies* 4 (2013). Contact: w.demuijnck@kpnmail.nl.

George Schedler was Professor of Philosophy at Southern Illinois University, Carbondale where he specialized in philosophy of law. He served as chair of the Department for eleven years before retiring in 2014. He received his PhD in philosophy from the University of California, San Diego, and his J.D., *cum laude*, from Southern Illinois University School of Law. He is the author of *Racist Symbols and Reparations; Philosophical Reflections on Vestiges of the American Civil War* (Rowman & Littlefield), and articles on the retributive justification for punishment, apologies and reparations for American slavery, sex selection abortion, and use of addictive drugs during pregnancy. Contact: geosched@siu.edu.

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Symposion

Charlotte Shreve is a UTA (Undergraduate Teaching Assistant) studying Philosophy and Cognitive Science at the University of Delaware. She can be reached at Cshreve@udel.edu.

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