MATHEMATICAL STUDY OF THE MECHANICO-STRUCTURE OF THE CRANIAL VAULT (APPLIED TO A CRANIUM DISCOVERED AT VĂDASTRA DATING FROM THE TRANSITION PERIOD FROM THE NEOLITHIC TO THE BRONZE EPOCHS)

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The archeological excavation undertaken in 1970 at *Vădastra* (Romania) on the hillock which the natives call 'la Drăgan' revealed a tomb belonging to the *Coţofeni* culture (the period of transition from the neolithic to the Bronze epochs). In the tomb there were two skeletons : that of a child of the *Infans II* age and that of a woman aged 16—20 mutilated, dismembered[1] (pl. 1/1).

The tomb discovered is a part of a small cemetery of the Coţofeni cultural bearers, situated in the western part of the settlement on *Dealul Cişmelei* (The Fountain Hill) in the proximity of 'Măgura Fetelor' (The Maidens' Hillock)[2]. Later Romanians of the 14th century were also buried on the 'la Drăgan' site while in the 17th-18th centuries, the hillock was again inhabited. In 1952, several tombs belonging to the people of the Coţofeni culture and to Romanians of the 14th century were destroyed by the activity of an improvised brick-kiln[3].

The anthropo-morphological investigation carried out on the mutilated skeleton at Vădastra constituted the object of a few papers[4] and studies[5], in which stress was laid on the morphologic aspect. The present work supplemented and very thorough, dwells on the methodology which establishes the typology through mathematical methods limiting the investigation performed to the cranium only. From the outset, we mention that the researches were carried out on a mutilated and deformed cranium, which induced us to utilize a new method of measurement devised by the authors. This method constitutes the main object of the present paper.

METHODOLOGICAL PRINCIPLES

If the typological study of a complete and well preserved cranium can be performed without any great difficulty, the same cannot be said when we have to study an incomplete, fragmentary material. Specialized studies, it is obvious, should be substantiated on as veridical a basis as possible and when certain parts are absent they should be very correctly reconstituted. The fragmentary and deliberately mutilated aspect of the skeleton of a grown up female found at Vădastra, allows us to dwell in this technical study on the procedure employed for reconstituting and establishing the anthropological type, providing, at the same time, to archeologists a multi-discipline study based, in the first place, on *mathematics*.

[1] Corneliu N. Mateescu, Materiale, 10, 1973, p. 22—23.

[2] The settlement on the *Dealul Cişmelei* (The Fountain Hill) with the hillock named 'la Drăgan' is situated on a fragment of the middle terrace of the Danube. In this part of the settlement the stratigraphy has the following appearance : the *Vădastra I* stratum underlies the culture strata, and above it the *Vădastra II* stratum is superposed (both belonging to the middle neolithic epoch). The *Sălcuţa* stratum (the late neolithic epoch) lies above the Vădastra II stratum. The last stratum with *remainders of the 17th—18th centuries* was destroyed in the course of time by the flowing (streaming) waters by deep ploughing and by frequent levelling of the ground. The pit of the tomb containing the broken up skeleton as well as other pits of tombs belonging to the same interval of time has penetrated through the Sălcuţa and Vădastra II strata, its bottom part reaching the base of the Vădastra II stratum.

[3] Corneliu N. Mateescu, SCIV, 6, 1955, 3—4, p. 452; idem, Materiale, 10, 1973, p. 19.

[4] H. Dumitrescu, Cornélius N. Mateesco, *Contribution à l'étude des sépultures de la civilisation Cotzofeni : Un squelette déchiqueté découvert à Vădastra (Roumanie)* in *IX[e] Congrès de l'UISPP, Nice 13—18 Septembre 1976, Résumés des Communications*, 1976, Nice, p. 398 : Sylviu Comănescu, Horia Dumitrescu şi Corneliu N. Mateescu, *Studiu Matematic de mecano-structură a bolţii craniene (aplicare la un craniu din perioada de tranziţie de la neolitic la bronz)*, Annual Scientific Session of the Museum of National History and Archaeology Constanţa, October, 22, 1977 (unpublished paper).

[5] Horia Dumitrescu und Corneliu N. Mateescu, PZ, 1981 (in the press).

DACIA, N.S., TOME XXV, 1981, P. 33–40, BUCAREST

It is common knowledge that mathematical studies, through deductive reasoning on the properties of entities of any kind, establish at the same time the relations between these entities. Likewise, the multi-discipline studies of the cranial vault — for that is what our study refers to — can benefit by the exactness of mathematics not as an aim in itself, but as a research methodological instrument. Since the cranian vault is a specific volumetrical form, it goes without saying, that we have to resort to the methods of both solid and plane geometry.

To reconstitute an incomplete cranium, two mathematical methods may be employed : a) *the numerical* and b) *the graphic* methods. The numerical method implies direct measuring and the deductive calculation of certain geometrical elements : lengths, angles, arcs. Its results yield a very precise reconstitution. The graphic method utilizing instead of numerical calculations, solutions given by drawings is more expeditious, but its results are rather less precise. The choice of the mathematical method depends on the aim of the research, contingent to what the archeologist wishes to demonstrate, hence the necessity of a close collaboration with the anthropologist and the mathematician.

To start a mathematical reconstitution of a cranium, we must get to know its structures and be able to infer its form as a resultant of *the mechanico-structure laws.*

On the basis of these laws, the structure of a bone appears as the result of the interaction between the traction and the pressure forces which intersect under a constant angle. Thus, two fundamental notions have to be explained, which, for the cranium have been masterly analysed by Gr. T. Popa[6] :
a) the notion of mechanico-structure connected to the fundamental form of the cranium, is common to all vertebrates, and has thus a *genotypic* hereditary character, and b) by the notion of functional form which is individual, thus of *phenotypic* character. When reconstituting a cranium, account should be taken of both these elements, the former universal, the latter individual.

The general form of the cranium has been compared graphically to a ship : the calvarium represents the 'deck' while the base of the skull represents the 'keel'[7]. This graphic image justifies — according to L.H. Farabeuf — the defining of the movement that can be applied to the cranium movements as follow : it tacks, it pitches, it sails[8].

To reconstitute the cranial vault, five complete arcs and two incomplete arcs in the sagittal direction should be establised, as well as five complete occipital and two incomplete frontal arcs in the transversal direction from the intersections of which there results the form of the vault (pl. 1/2). For the base the dimensions and the angle of two pairs of beams constituted by the small wings of the sphenoid and by the rocks of the temporal bones with reference to the medial beam consisting of the body of the sphenoid bone and the basilar apophysis of the occipital bone[9] have to be precisely defined.

REMAINDERS OF THE CRANIUM OF THE VĂDASTRA SKELETON

The preserved parts of the skeleton of a female discovered at Vădastra[10], analysed typologically are : a fragment of the frontal bone *(os frontale)*, the two parietal bones *(os parietale)*, a part of the occipital *(squama occipitalis)*. The recovered bones of the facial skeleton are : the ascending apophysis *(processus frontalis)* of the left maxillary prolonged by the supra- and infraorbital edges *(margo supra-* and *infraorbitalis)*, a part of the body of the left maxillary corresponding to the cannine fossa *(fossa canina)*, the nasal bones *(os nasale)* as well as two teeth : the second upper right premolar (D_5) and the first lower right incisor tooth (d_1).

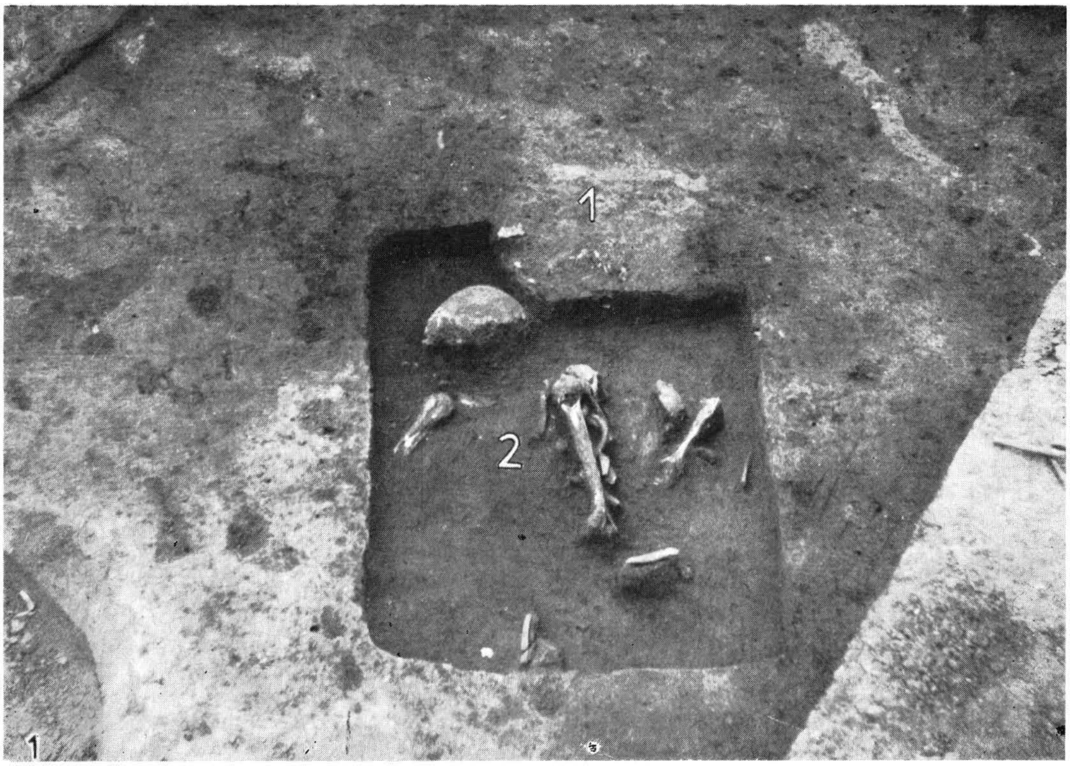
[6] Gr. T. Popa *et al.*, *Les Méninges, leurs dépendances et leurs relations*, Analele Academiei Române, Memoriile Secţiunii Ştiinţifice, III series, 16, 1940, 7, p. 19—21.

[7] Angelo Cesare Bruni, *Compendio di Anatomia descrittiva umana*, I, Milano, 1940—XIX, p. 227.

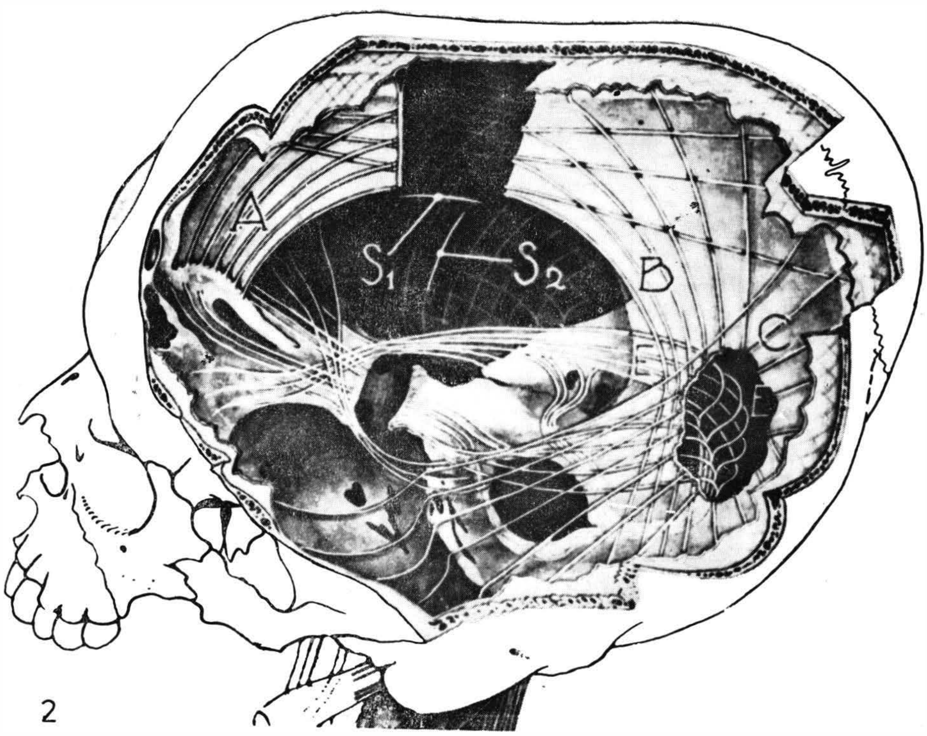
[8] L. H. Farabeuf, *Précis de Manuel opératoire*, nouvelle édition, Paris, p. 845.

[9] Gr. T. Popa *et al.*, *op. cit.*, p. 93—95.

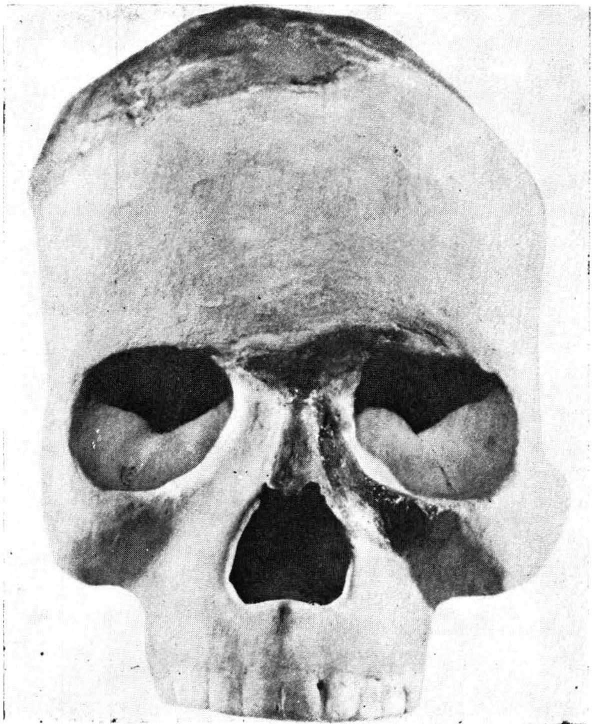
[10] The parietal humps (*tubera parietalis*) middling developed, occipital middling pronounced, the bulging forehead with intermediary crests (within limits), slenderness and dimensions of the bones, the 'sutural condition', morphological data of the external and internal structure as well as the two teeth rove, for certain, that the skeleton belonged to a woman aged 16—20. Due to a certain ritual practice the woman, in perfect health was dismembered and then a part of the corpse was buried according to the ritual, together with her child. On the woman's cranium there were spots of red ochre and near the two craniums there were a few little bits of red ochre.



1. *Vădastra.* The pit of the tomb wild the two skeletons; (Photo C.N.M.).
1. That of a child; 2. That of a woman dismembered due to a ritual practice.



2. The mechanico-structure of the cranium (the cranium + the *dura-mater).* Schematic drawing of the traction and pressure fibres, system oriented (after Gr. T. Popa *et al.*, *op. cit.*, pl. XLIV).



1

Woman's cranium discovered at Vădastra, reconstituted. 1. *Norma facialis.* Transverse-frontal index $\frac{(ft - ft \times 100)}{co - co} =$ $= 86.2$; frontal-parietal index $\frac{(ft - ft \times 100)}{eu - eu} = 76.3$; frontal-zygomatic index $= 83.3$. Middling orbital obliquity ($\measuredangle$ 51.0°). The inclined glabella-orbital plane of 93°, with respect to the horizontal. Orbital index 74.7, nasal index $=$ $= 38.0$.

2

2. Left *norma lateralis.* Height of the calotte along the ob — n-i perpendicular = 111.6. Length-height index = 65.8 (after Horia Dumitrescu und Corneliu N. Mateescu, PZ, 1981, in the press).

RECONSTITUTION OF THE CRANIUM

Actually, the reconstitution of the cranium started with delimiting the vault and fixing the anthropometrical points. Thus, for the *neurocranium* the following points were taken into account and introduced into the mathematical calculation: (n) nasion (a reference point) and (m) metopion, (b) bregma, (ob) obelion, (1) lambda, (i) inion situated on the median line and the longitudinal axis, considered collinear points; on the width axis, (eu) eurion, (ft) frontale with the fronto-nasal suture as starting point, considered the fundamental basis. Delimiting vectors of the arcs and chords were drawn for here (fig. 1).

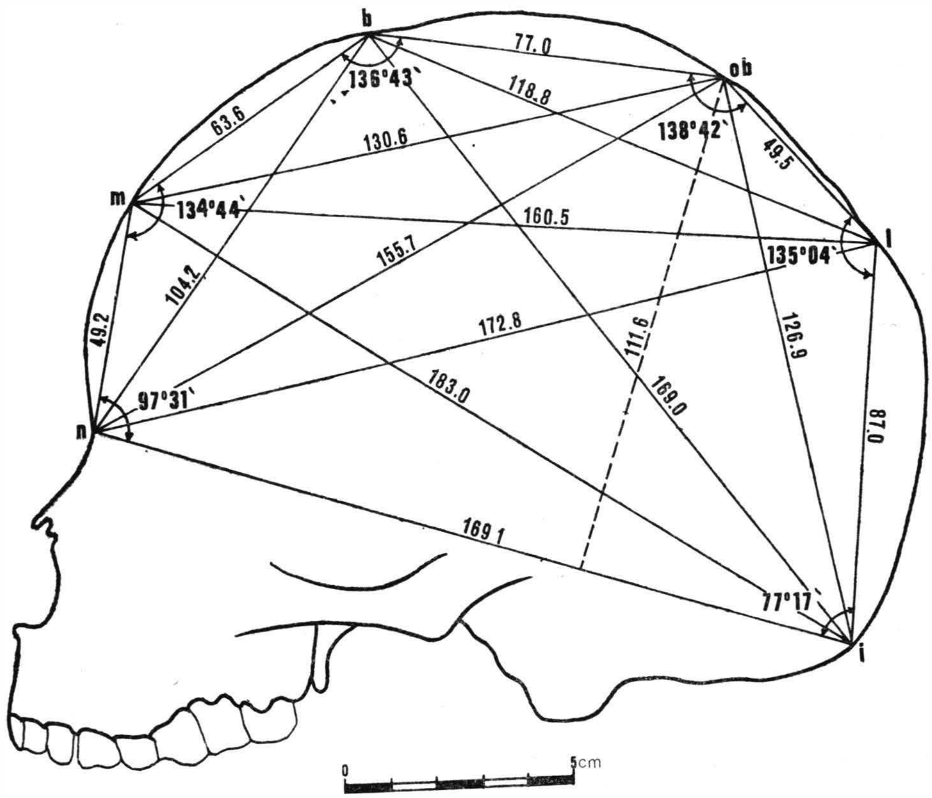


Fig. 1. — Reconstitution of the woman's cranium discovered at Vădastra, with the anthropometrical points on the sagittal line (after Horia Dumitrescu und Corneliu N. Mateescu, PZ, 1981, in the press).

To get a clear image of the whole, the base of the cranium was remade, maintaining the base-vault ratio to an angle of 22°, utilizing for the vault the inclined planes of the parietal and occipital bones[11]. The mastoid apophyses are situated on the line descending from the obelion along the nasion-inion line, and the sphenoidal wings on the straight line descending along the sphenoidal sinus from the bregma[12]. The vault was consolidated by means of the median sagittal arc and the lateral fronto-parietal and occipital arcs : the *linea temporalis sup.* and *linea temporalis inf.*, to which is added the malar arc. The median arc starts from the level of the fronto-nasal suture towards inion, having the three derivations towards the mastoid apophysis : *linea nuchae suprema*, *linea nuchae sup.*, *linea nuchae inf.*

Then the most difficult phase of the reconstitution had to be tackled i.e. *the connecting of the facial massif to the neurocranium.* The position of the malar bone is of great importance since it is the point where the fascicles of the reinforcing pillars meet in the four directions : upwards towards the base of the cranium, downwards towards the upper alveolar arch, forwards, towards the fronto-nasal area, backwards through the zygomatic apophysis towards the temporal lines.

The basic structure of the facial pyramid is the reinforcing pillar starting from the second upper premolar tooth (D_5) on the right. The image can be projected in a looking-glass on the

[11] L. Testut, *Traité d'Anatomie Humaine*, I, Paris 1911, p. 173.

[12] V. Fetter, M. Prokopec, J. Suchý, Svatava Titlbachová, *Antropologie*, Praha, 1967, p. 134, fig. 131.

left. The facial massif pyramid is connected to the neurocranium (according to P. Legoux[13]) starting from the pillar, i.e. from the teeth, not from the cranial vault. The form of the maxillaries and of the mandibles is inferred from the direction of the worn facets of the teeth in the two arches. Consequently the presence of the two teeth was essential for the reconstitution of the facial bones and for connecting them to the base and vault of the cranium under consideration[14] (pl. 2/1—2).

METHODOLOGY, MEASUREMENTS AND CALCULATIONS

The study of the mechanico-structure elements of the cranial vault (points, curves etc.) requires, in the first place, that they should be determined geometrically. As the cranial vault is of a completely irregular geometrical form and can be represented through drawing only on the strength of certain measurements (angles or lengths) we propose a simple mathematical and graphic method by which our object can be attained. Thus, knowledge of the form of the cranial vault is gained by means of profiles chosen especially in relation with the characteristic points of interest concerned.

It is well known that a curve is drawn with greater precision as more points of the curve are determined through calculation or graphically, leaving in the end only small portions to be approximated between the points. As the analytical working out of the determination of the multitude of points of the curve is arduous, we suggest the curve should be drawn through an optical and graphic method, using a conical projection, under conditions similar to the object in view. Thus the following operations are necessary :

— marking all the characteristic points on the calotte with small discs of paper stuck on the respective areas along a profile, and on which the point proper is to be pricked;

— measuring of the elements of length between all the points marked in the chosen plane containing the profile chosen;

— on the plane established the suitable uniting through straight lines of all the points marked, which will lead to the obtaining of an irregular polygon, with all the possible diagonals. Thus the polygon is inscribed in the characteristic curve of the cranial calotte, having its apexes of the curve we intend to determine;

— calculation of all the elements of the irregular polygon (angles, perimeter etc.) by means of simple fundamentally trigonometrical and geometrical formulas, applied to the irregular triangles formed inside the polygon;

— drawing the polygon thus previously determined at a suitable scale, on a white piece of cardboard which will also serve as a screen for projecting the characteristic curve in which the polygon is inscribed;

— placing in suitable positions of a pin-point source of light, the cranial calotte and the projection screen, so that the cast shadow of the characteristic curve of the cranial calotte should coincide with the points of the polygon. In this situation, the projection screen will show a curve similar to the one on the calotte, at the chosen scale. The contour of the cast shadow is then drawn on the screen with a pencil, this being, in fact, the object of the method.

To exemplify this method, we shall state the operations performed as well as the results obtained :

On the calotte of the cranium under study (fig. 1) we marked by pricking 6 mm paper discs the characteristic points of the profile in the sagittal plane of the calotte : nasion (n), metopion (m), bregma (b), obelion (ob), lambda (l) and inion (i). We measured with a pair of compasses the lengths between all the points marked (15 measurements); by uniting them suitably, they form an irregular hexagon with its 9 diagonals. The distances between the points of the compasses

[13] P. Legoux, *Détermination de l'âge dentaire de fossiles de la lignée humaine*, Paris, 1966, p. 267.

[14] The reconstitution was performed on the basis of the mechanico-structure laws of the cranium through measurements with a Martin-Saller apparatus. For the verifying and most faithful representing of the cranial curve mathematical calculation was introduced.

were measured against a precision rule graduated in halves of millimetres; smaller fractions were approximated. The results of the measurements are the following:

The side or the diagonal		Length in mm	
n—m		49.2	
	n—b		104.2
	n—ob		155.7
	n—l		172.8
n—i		169.1	
m—b		63.6	
	m—ob		130.6
	m—l		160.5
	m—i		183.0
b—ob		77.0	
	b—l		118.8
	b—i		169.0
ob—l		49.5	
	o—i		126.9
l—i		87.0	

In calculating the elements of the irregular hexagon, the condition was set that the effects of the errors in measuring should be as small as possible, and account was taken of all the lengths measured, so that all the possible combinations of triangles formed in the interior of the hexagon were considered: 120 triangles (6 × 20 triangles) 20 of which are totally distinct, namely: n l i, n ob i, n b i, n m i, n ob l, n b l, n m l, n b ob, n m ob, n m b, ob l i, b l i, m l i, m ob i, b ob i, m b i, b ob l, m ob l, m b l and m b ob.

To calculate the angles of every single triangle the following formulas were utilized:

$$(1)\quad \cos\alpha = \frac{b^2 + c^2 - a^2}{2\,bc} \qquad (2)\quad \sin\beta = \frac{b \sin\alpha}{a} \qquad (3)\quad \gamma = 200^c - (\alpha + \beta)$$

To exemplify we render the numerical calculation in the n l i triangle (fig. 2).

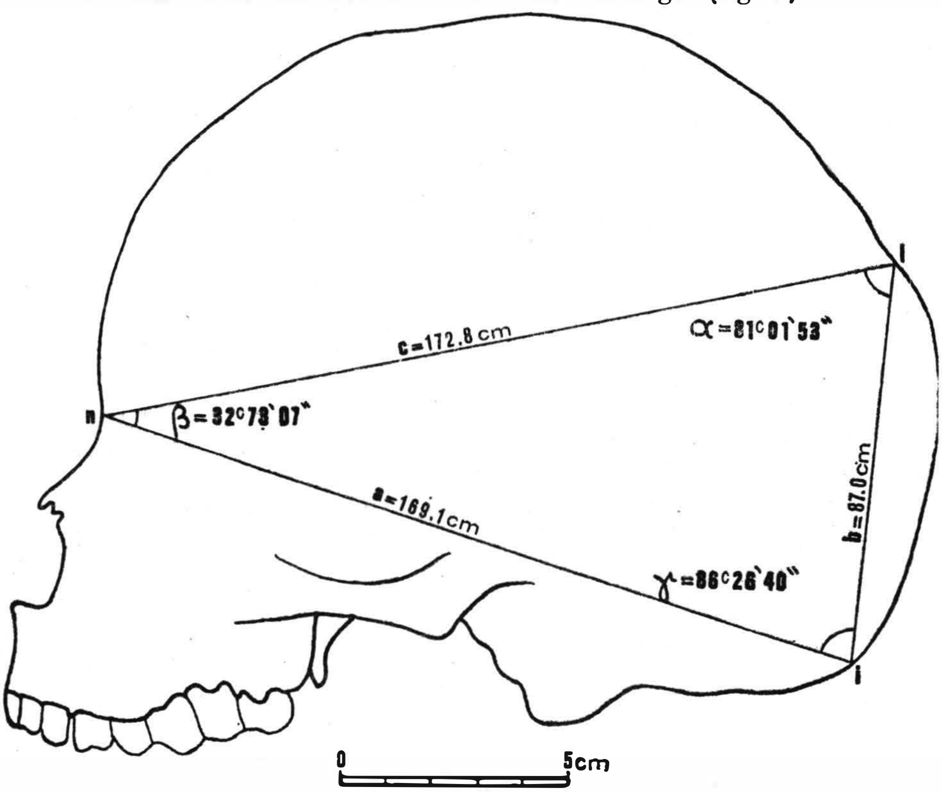


Fig. 2.— The n l i triangle.

$$\cos \alpha = \frac{7{,}569 + 29{,}859.84 - 28{,}594.81}{30{,}067.2} = \frac{8.834.03}{30{,}067.2} = 0.293810 \qquad \alpha = 81^c\ 01`53``$$

$$\sin \beta = \frac{87.0 \times 0.955863}{169.1} = \frac{83.160081}{169.1} = 0.491780 \qquad \beta = 32^c\ 73`07``$$

$$\gamma = 200^c - (81^c\ 01`53`` + 32^c\ 72`07``) = 86^c\ 26`40``$$

The calculations for the other of 19 triangles are effectuated in the same way, obtaining totally 60 angle values, that is, 10 angle values for each apex of the hexagon. As a consequence, a suitable combination of these value yields the value of the angle in every apex as the average of 6 primary values.

The calculation of the average value for the angle in point n apex is as follows:

	$108^c\ 31`\ 43``$
	$107^c\ 61`\ 91``$
	$108^c\ 63`\ 38``$
	$107^c\ 67`\ 67``$
	$108^c\ 44`\ 93``$
	$109^c\ 24`\ 48``$
the average	$108^c\ 32`\ 30``$

Similar calculations were made for obtaining the average values of the angles in the other apex of the hexagon. A calculation was then effected for compensating the errors of closing of the interior angles in the hexagon, utilizing the following formulas:

$$(4)\quad \Sigma_n = (n-2) \times 200^c \qquad (5)\quad U_n = U_1 + U_2 + U_3 + \ldots$$

$$(6)\quad \Sigma_n - U_n = \Delta_u \qquad (7)\quad C_u = \frac{\Delta_u}{n} \qquad (8)\quad U_R = U_1 + C_u$$

The numerical values calculated are:

— error of non-closing;

— correction for every angle is: $C_u = 0^c\ 03`\ 46``$.

The final values rounded off to the minute of the angles, in the apexes of the hexagon are:

Designation of the apex	Value	
	Centessimal degrees	Sexagessimal degrees
n	108^c 36`	97° 31′
m	149^c 71`	134° 44′
b	151^c 89`	136° 43′
ob	154^c 09`	138° 42′
l	150^c 07`	135° 04′
i	85^c 88`	77° 17′

By applying graphical methods (fig. 3) the irregular hexagon was drawn on a white sheet, at a scale of 2 : 1; it was placed vertically to serve as a projection screen. Approximately, on a horizontal line from the centre of the hexagon, on a stand, and at a proper distance, a reflector was placed, which emits a cone-shaped light beam. On another stand whose supporting table is movable in the three directions of the space, the cranium was placed conveniently, so that the profile on the sagittal plane of the calotte should be the contour line of shadow cast by the cranium calotte, superposed on the 6 apexes of the hexagon. This line of contour was thus drawn on the screen, it being the curve of intersection between the cranian calotte and the sagittal plane.

Another important anthropometrical element of the cranium has been also calculated, the length of the perpendicular from the obelion (ob) along the nasion-inion line (n—i) utilising the trigonometrical relations in the n o i, triangle, after having previously compensated the angular non-closing error :

(9) $h_a = b \times \sin \gamma = 126.9 \times 0.877637 = 111.372$ mm

(10) $h_a = c \times \sin \beta = 155.7 \times 0.717479 = 111.711$ mm

(11) $h_a = \dfrac{b \times c}{a} \sin \alpha = \dfrac{126.9 \times 155.7}{169.1} \times 0.955250 = 111.615$ mm.

By averaging these 3 values, a value of 111.6 mm was found (fig. 4).

Any other element in the hexagon can be determined in a similar manner.

On the sheet of paper on which the sagittal section of the cranial calotte was drawn, the sagittal lengths were measured with a curvimeter, obtaining the values (reduced to the scale) :

— nasion-metopion (n—m) = 49.0 mm
— metopion-bregma (m—b) = 69.5 mm
— bregma-obelion (b—ob) = 80.0 mm
— obelion-lambda (ob—l) = 54.0 mm
— lambda-inion (l—i) = 90.5 mm

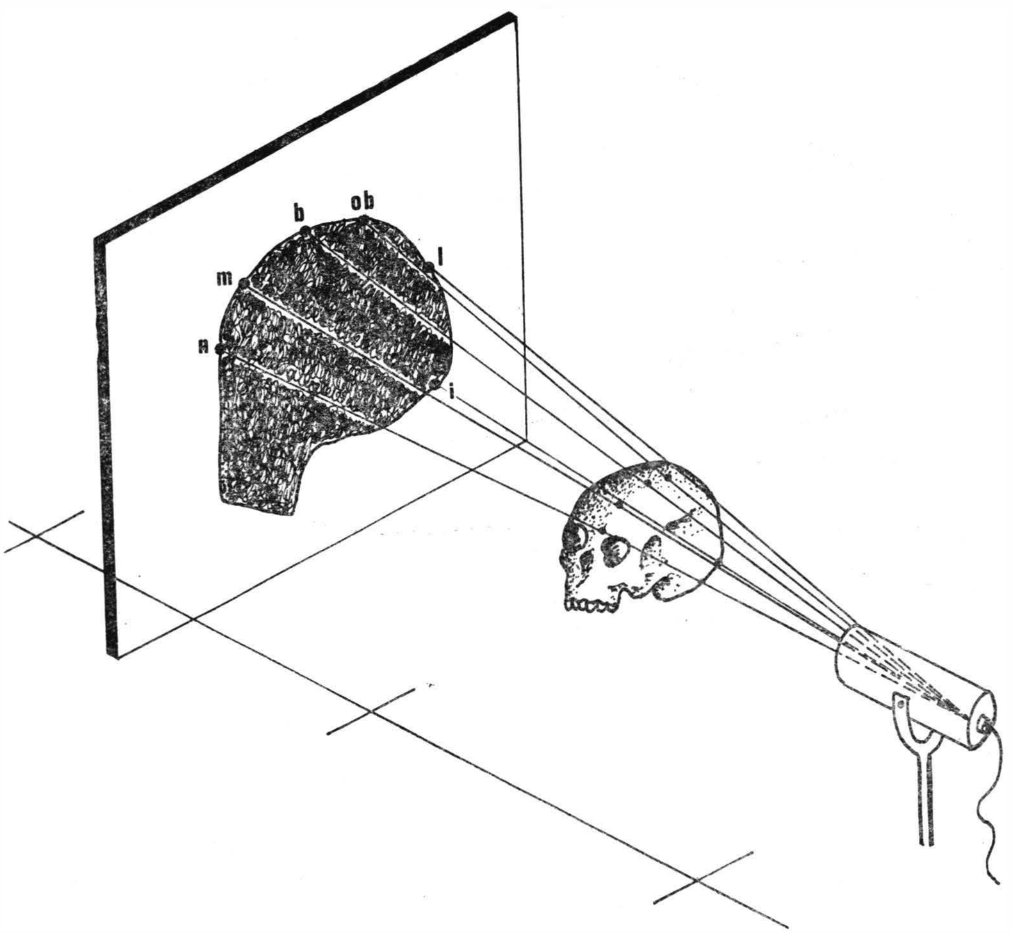


Fig. 3. — Scheme of the optical and graphical device for the drawing of the cranial calotte.

The centessimal system and the table of natural values[15] were utilized in the calculations, the operations being performed on an electronic computer[16]. One can do the same for any of the sections of the cranial calotte with characteristic planes.

The mathematical method utilized, which we named the method of 'converging angles' yielded the following :

— the value, thus the form of the median sagittal curve of the cranium ;

[15] D. Clurileanu, *Tabele trigonometrice*, Editura Tehnică, București, 1950.

[16] With the electronic computer 'Litronix 2210' no. 42, 359.

— the verification of the measurements effectuated at the anthropometrical points taken with the Martin Saller sliding callipers and subsequently with a pair of compasses, compared to the graded line;

— by totalizing the arcs on the meridian in the sagittal plane, an image results which is nearest to reality.

We mention that by this method, utilizing the optical and graphic projection to the cranial vault, small curves can be analysed and respectively the very tiny surfaces between characteristic areas situated near each other.

This mathematical method very much restricts the possibility of errors in the appreciation of reality, and consequently the elements obtained will contribute to the specialized anthropological study that will lead to conclusions of archeological and ethnic nature.

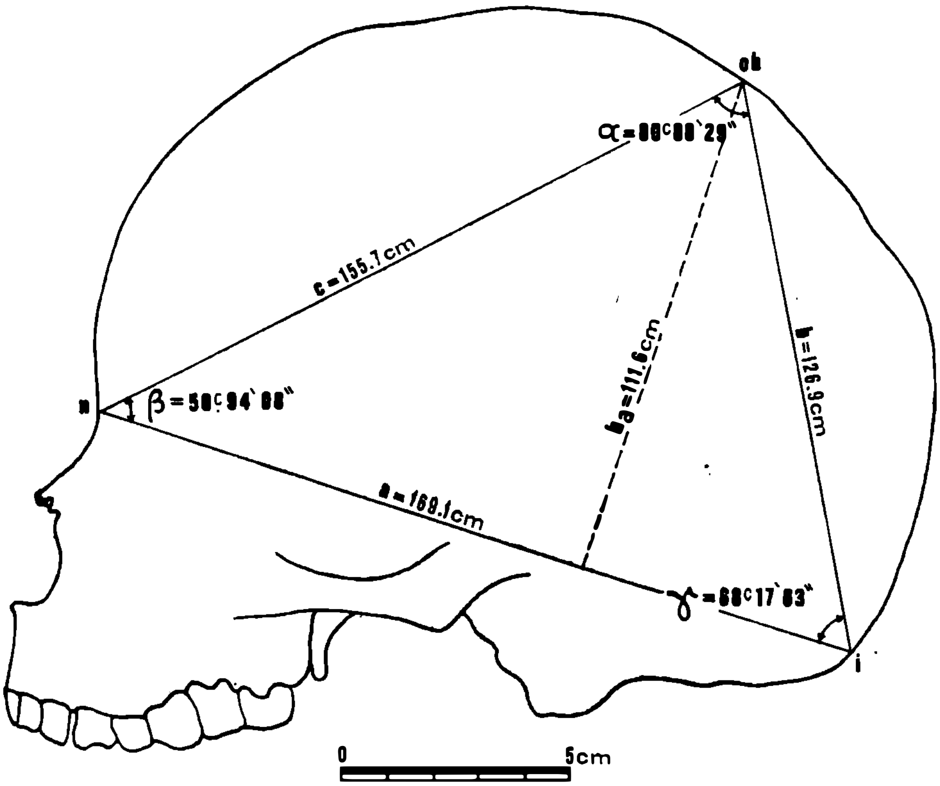


Fig. 4. — Scheme of the irregular polygon in sagittal plane.

CONCLUSIONS

The following conclusions result from the above study:

— in the excavation carried out in 1970 at *Vădastra*, on the hillock locally known as 'la Drăgan', a tomb was discovered belonging to the *Coţofeni* culture (the transition from the neolithic to the Bronze epochs) in which two skeletons were discovered: that of a child (of the *Infans II* age) and the mutilated skeleton of an adult person;

— the cranium of the adult (a woman aged 16—20) was broken up and mutilated; it was reconstituted according to a method based on the application of mathematical proceedings to the data of mechanico-structure;

— the mathematical methods were based on the drawing of certain arcs chards and angles: starting from measurable data, the unknown elements were inferred, on the principle of converging angles and the determination of the triangles on a median sagittal plane;

— to determine the curves, 6 points were established which joined by lines form the irregular reference hexagon. By optical projection, the determined apexes of the hexagon are joined together, drawing conveniently the line of the cranial curvature;

— the application of this method enabled the precise reconstitution of the cranium studied, and, likewise to establish, on the basis of the measurements performed, the anthropological type: dolico-mezzocran, typological Mediterrano-Easteurepoid;

— the method can be successfully utilized for research work on fragmentary craniums discovered in archeological excavations.