REMARKS ON THE GEOMETRIC PRINCIPLE IN THE ROMAN ARCHITECTURE. THE RHYTHM OF COLUMNS WITH VITRUVIUS

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It has long ceased to be a novelty that behind the *Vitruvian numbers* one can find irrational numbers, expressions of the geometric constructions that had constituted an older compositional matrix of Hellenistic origin[1]. Here we shall analyse the geometric support that has generated the type of rhythm of columns, as Vitruvius conveyed it in his Books[2]. The mainstream of the analysis is that the *internal* geometry or the geometry of the design – an expression of the way the "activation" of the architectural composition is understood – has to be directed in such a way as to be achieved starting from a single unit, measurable. In this case such a unit is the diameter at the base of the column, that is the modular unit. This way not only the internal unit of the composition is ensured, but also the commensurability of all the parts according to a unit of measure, as it is enough for the starting unit (the module) to be expressed depending on it.

In view of the present study an involved dimension with a generating function in the geometric support has to be expressible, either directly arithmetically (rational numbers), or geometrically, depending on the generic starting unit (in this case the module). We shall take into account that the transparency of the geometric support can be – from an exterior point of view, which belongs to the observer – more or less concealed by those rounding off operations (a phenomenon that must have been constant not only with Vitruvius[3]) allowing for a geometric reality to be observed as a simple arithmetical relation (while the dimensions resulted from geometric operations to be conveyable in execution by "round" quotas"[4]).

We shall try to find those internal relations which ensure the correlation between all the elements of the composition and the starting unit of the design. To this end it is not enough to put to good use the various geometric relations between groups of two dimensions each. The evaluation of the arithmetical expression of these types of relations is the first stage to be studied, but it constitutes only an emphasis on the relations derived from the generating geometric matrix. The final goal of the present study is assuming certain hypotheses for the evaluation of these geometric matrices forming the generating core of the composition.

A. THE IONIC STYLE[5]

1. *The dyastyl rhythm.($I=4D$; $H=8.5D$; [$Di=9.394$])* (Fig.1)

P. Gros remarked the existence of a diagonalizing relation linking the height of the column to the dimension of the intercolumniation: $8.5/3=2.833\approx2.828=2\sqrt{2}$[6]. The diagonalizing relation is not limited to that, as it extends over the interaxis (I), pointing out the connection to the height of the column (H), as $H/I = 3\sqrt{2}/2$ (Fig.1a). Therefore, the simple diagonalizing procedures entirely comprise the reciprocal

[1] Gros, 1976.

[2] We shall use to this end Gros, 119.

[3] In Vitruvius' case, v. Gros, 119.

[4] Gros, 119.

[5] Notations: I=dimension of the interaxis; H=height of the column; D=diameter at the base of the column; Ln=side of the inscribed regular polygon (circumscribed) in the circle. The calculus of the side of the regular polygon with "n" sides (Ln) was done according to the formula $Ln=2R\sin\alpha/2$, where R is the radius of the circle and α is the measure of the angle in the centre of the polygon; *Di=diameter-diagonal (H:I)*.

[6] Gros, 119 The diference resulted in this assessment is 0.005D.

relations between the height of the column, the width of the interaxis (I) and the diameter at the base of the column (D).

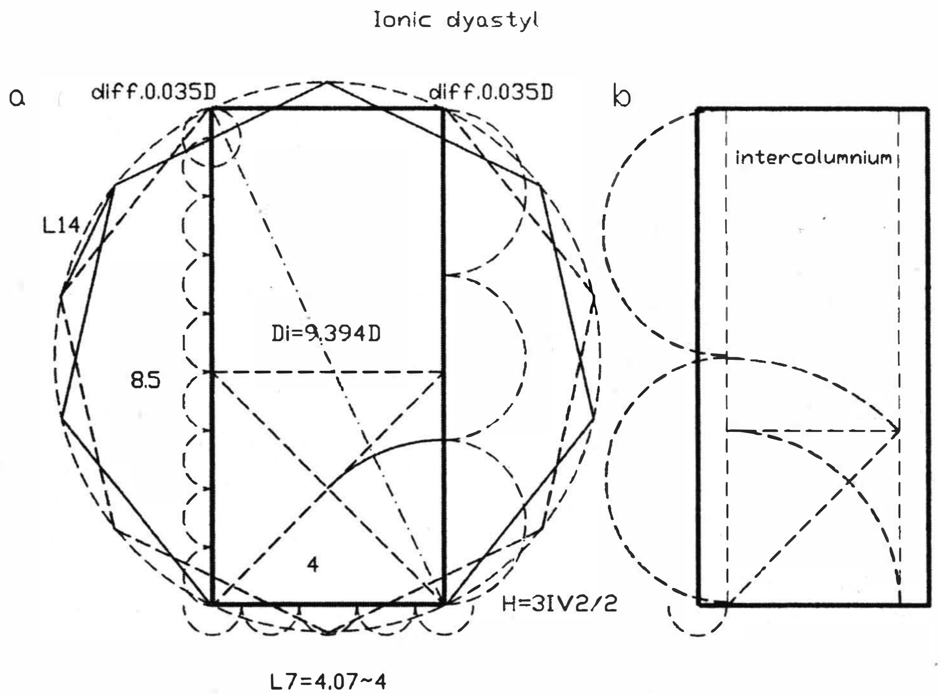

Fig. 1. Ionic diastyl.

Under the circumstances, the expression according to the diameter at the base of the column (D) of the fundamental rectangle (I:H) is *4D : 6√2D*[7] or *2 : 3√2*. The regular heptagon[8] inscribed in the circle passing through the tips of the rectangle (H:I) has the side equal to the dimension of the interaxis: $L_7=4.07D\approx 4D=I$.

The origin of the differences as compared with the values conveyed by Vitruvius. A part of the differences can be the result from the rounding off (adjusting) procedures. Taking into account that the geometric support originally was not a mounting applied from the outside, we can presuppose that the radius of the circle *generating the fundamental rectangle (H:I)* was an arithmetically measurable value, according to the module unit. Therefore, the initial value of the *diameter-diagonal (Di)* could have been 9.3(3). If associated to an interaxis of 4D, a column height of 8.495D results. Implicitly, it results that the column height was rounded off, from 8.495D to 8.5D.

2. *Systyl rhythm (I=3D; H=9.5D; [Di=9.962])* (Fig.2)

a) The geometric support is more elaborated than in the case of the dyastyl, and as we shall see, may form the geometric matrix for other types of rhythms[9]. The rectangle (H:I) is characterized by a relation of simple diagonalizing, but distinctly singled out as compared with that governing the *dyastyl*. One may say that the rhythm of the *systyl* occurs under the auspices of the number *10*. The ratio I/H is 9.5D/3=3.166 ≈3.162=√10 (Fig. 2b). In other words, the height of the column is obtained according to the module unit (directly involving also the dimension of the interaxis) as hypotenuse in a rectangular triangle with catenaries equal to 1D and 3D.

The geometric figure sprouting this type of rhythm is the decagon inscribed in the circle passing through the tips of the fundamental rectangle (H:I) (Fig. 2a). The side L_{10} is equal to the dimension of the

[7] If we accept the level of the diffrences involved in the relations between H and I, then 8.5D ≈ 6√2D = = 8.484D.

[8] The construction of the heptagon is extremely simple, either by the assessment of its side (L7=0.867R) according to the radius of the circle as $L_7\approx 7R/8$ (the difference as compared to the ideal dimension 0.008R) or by relating it to the apothem of the hexagon inscribed in the same circle:L_7=1.735R/2= =R√3/2 = a_6 (0.003 R).

[9] Nevertheless, we note the reversible nature of the geometric contructions: consequently, we should also consider the reciprocal hypothesis.

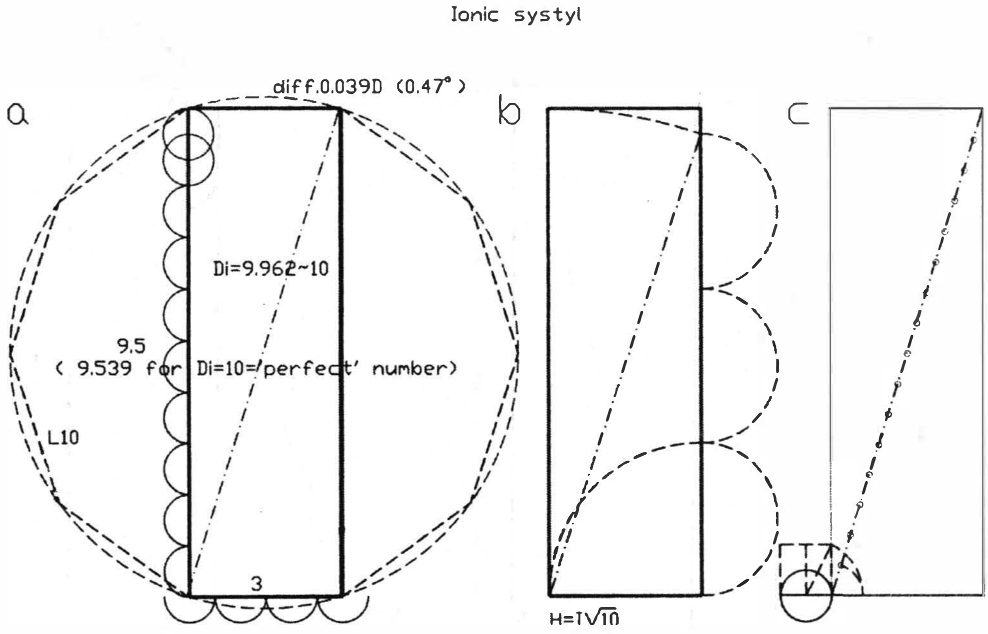

Fig. 2. Ionic systyl.

interaxis[10], while the height of the column is given by the length of the cords coming down from the tips of two opposite sides each.

Origin of the differences as compared with the ideal dimensions As we consider the *diameter-diagonal*, next to the dimension of the interaxis, as a unit directly correlated to the module, we might be led to believe that its value was 10 D. The repartition (I:H) turns into (3D:9.539D), therefore the rounded off value was that of the column height dimension: from 9.539D to 9.5D, as conveyed by Vitruvius.

b) One of the types of rhythms comprising in the peculiarities of its geometric support the *systyl matrix*, rendering explicit the Hellenistic Age origin of the Vitruvian grills, can be encountered at the *Naiskos of the Didymaion* (Fig. 3). Here the height of the column reaches 9.5D, and the dimension of the interaxis 2.8D[11]. Therefore, *the diameter-diagonal is* 9.904D, which makes it expressible as $16\varphi_1 D$ (radius=$8\varphi_1 D$)[12]. The dimension of the interaxis expresses a geometric support resulted by a simple diagonalizing being assessed at $2\sqrt{2}$D. The same peculiarity is expressed also by the value of the intercolumniation, as 1.8D can be considered to be $\sqrt{13}$D/2 (namely half of the hypothenuse of a rectangular triangle with the catenaries 2D and 3D). It becomes clear how it is possible to reach geometrically the type corresponding to the Vitruvian grill from a type like the one expressed at Didyma, namely the Vitruvian type 3/9.5 / [9.962], namely from the Naiskos distribution of the 2.8/9.5/9.904 type. The transformation could not have been a stranger to the idea of simplifying the geometric construction (from the polygon with 11 sides to the decagon) possibly reinforced by the fascination for the perfect number 10.

c) The exceptional quality of the geometric construction lying at the base of the *systyl* is that it can contain the *dyastyl* rhythm as well (Fig. 4). In other words, the *systyl geometry generates dyastyl geometry*. By scaling the two constructions according to the diameter at the base of the column (D), one can see how the *dyastyl* can be considered to be a transformation of the *systyl*: by considering the circle and inscribed decagon corresponding to the systyl, one can construct the tangent circle inside the decagon. The dyastyl rhythm is generated by the heptagon inscribed in this circle. Maybe not by chance, the numerology of this transformation contains the numbers 7 and 10. Their relation, frequently used for the approximation of $\sqrt{2}/2$, in fact describes one of the principles governing the dyastyl.

[10] The side L_{10}=3.07D≈3D (dif.0.07D).

[11] Gros, *Commentaire*, 105,119 (indication referring to the intercolumniation, equal to 1.8D).

[12] Because 9.904D/16=0.619. This approximation of the medium and extreme ratio corresponds to the ratio 13/21.

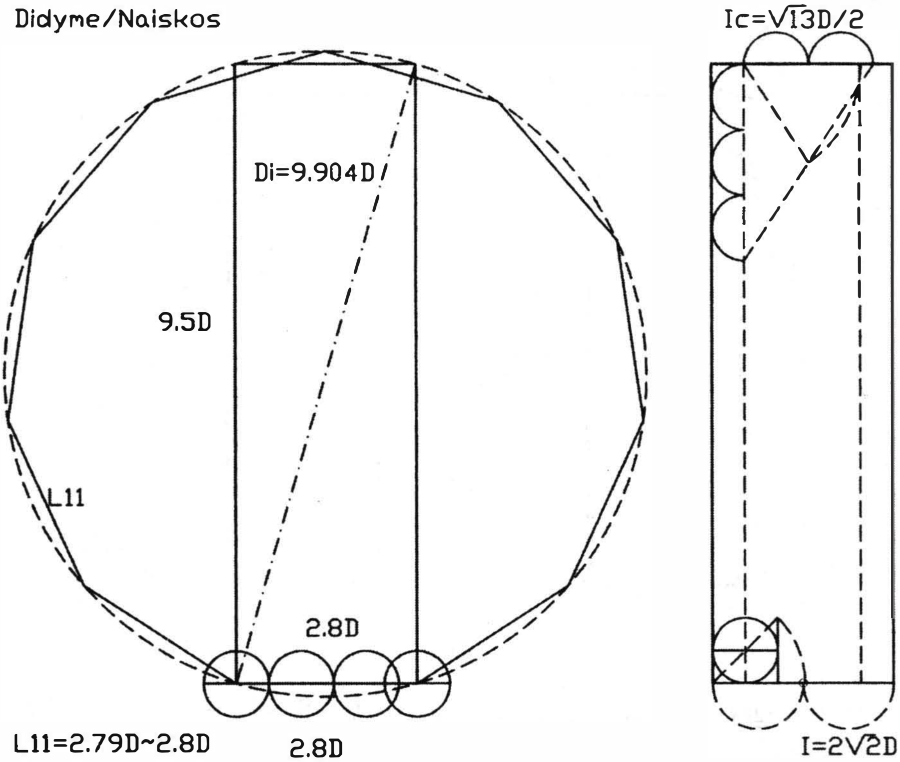

Fig. 3. Didyme/Naiskos.

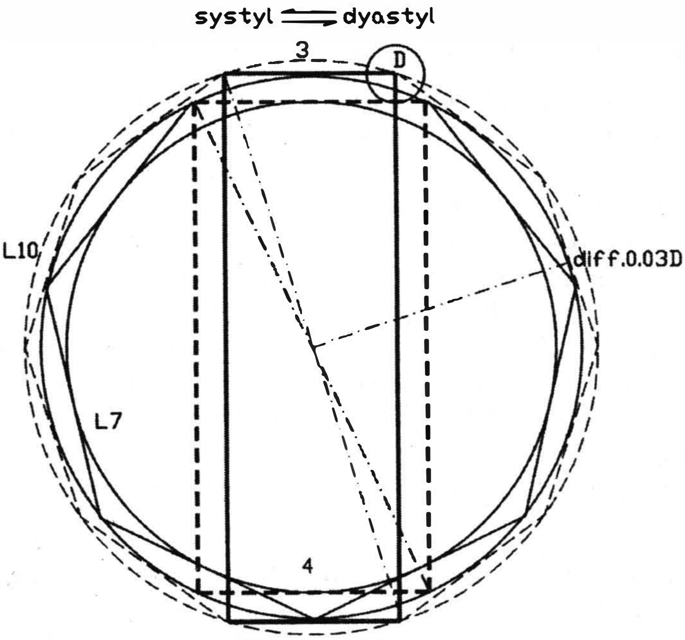

Fig. 4. Ionic systyl-dyastyl.

3. *The rhythm of the eustyl (I=3.25; H=9.5; [Di=10.04])* can be a transformation, or simply a *variant of the systyl*: it results by the increase in the dimension of the interaxis in such a way as instead of being equal to the side of the *inscribed* decagon (the systyl case) it gets closer to the side of the *circumscribed* decagon (diff. 0.0017D), and the *diameter-diagonal* remains extremely close to 10D (exactly 10.04D)[13] (Fig. 5a).

a) If we consider the Vitruvian grill as a result of a rounding off of modular numbers, we might think that the value of the interaxis originally involves the *gold section applied to the modular unit (D),* as 3.25D/2=1.625D (Fig. 5b). As this value can originate in the approximation of the *medium and extreme ratio* (that is in φD =1.618D) it results that the dimension of the interaxis could have tolerated a rounding off from 3.236D to 3.25D. In other words, a *geometric measurability was transformed into a simpler, arithmetical one.* As one extends this conjecture it gets clearer that the height of the column may be regarded as a result of a diagonalizing involving the number 10, as 9.5D/3=3.16(6)≈3.162=√10. In this variant the *diameter* of the circle dressing up the fundamental rectangle results as 10.03D, namely ≈10D.

[13] Therefore, while in the systyl the diagonal ≤10, in the eustyl case it is ≥10.

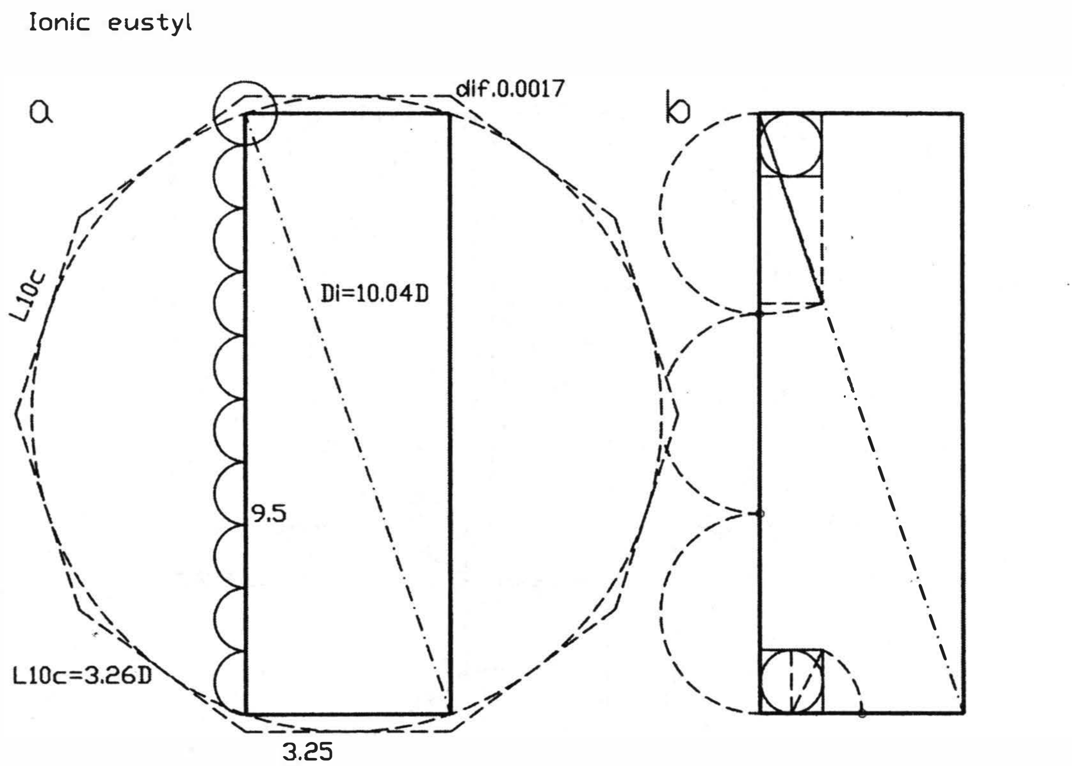

Fig. 5. Ionic eustyl.

b) A variant in which the *diameter-diagonal* has a value identical to10D is the one in which the interaxis being exactly 3.25D – according to the Vitruvian provisions – the value of the height of the column becomes 9.457D. It is well understood that such a value is conveyable – by adjustment as 9.5D.

c) At the same time it is possible that the origin of this type of rhythm might be recognized in the intention of numerological subordination depending on the symbols of the number (10). An ideal geometric support could be described by a repartition (I:H) of the ($\sqrt{10}$D: 3$\sqrt{10}$D) type, namely 3.16 D: 9.486D. Also in this case the *diameter-diagonal* has a value identical to 10D.

d) It might be stated that, apparently, the difference between the *eustyl* and *systyl* can be reduced to a simple problem linked to options regarding the rounding off irrational numbers, either φ or $\sqrt{10}$ (or $\sqrt{2\sqrt{5}}$), both involved in the construction of the pentagon and decagon. But, this fine movement of rounding options is the expression of a compositional conception in which the beautiful relations and, maybe, a particular reflex of the symbols of numbers are important protagonists. The scarcity of the means of achieving the geometric support – viewed through the passing from the inscribed decagon of the systyl to the circumscribed decagon of the eustyl – stands up to the extremely fine mutations undergone by the concrete forms of the rhythm.

4. *The pycnostyl rhythm (I=2.5D; H=10D; Di=10.307D)* (Fig.6)

a) The interaxis is assimilable to the side of the regular polygon with 13 sides inscribed in the circle comprising the fundamental rectangle of the rhythm. The side L_{13} is 2.466D (roundable to 2.5D). This value emphasizes a *medium and extreme ratio* governing also here the geometry of the rhythm, between the dimension of the interaxis and the module-diameter: 2.466D/4=0.6165≈0.618=φ_1.The rounded value of 2.5D contains in its turn the reminder of this ratio by assessing φ_1 at 0.625.

b) If the *diameter-diagonal* was commensurable arithmetically, then the closest rounding off is from 10.307D to 10.33D. Consequently, for an interaxis of 2.5D, there is a fine increase in the height of the column which becomes 10.02D.

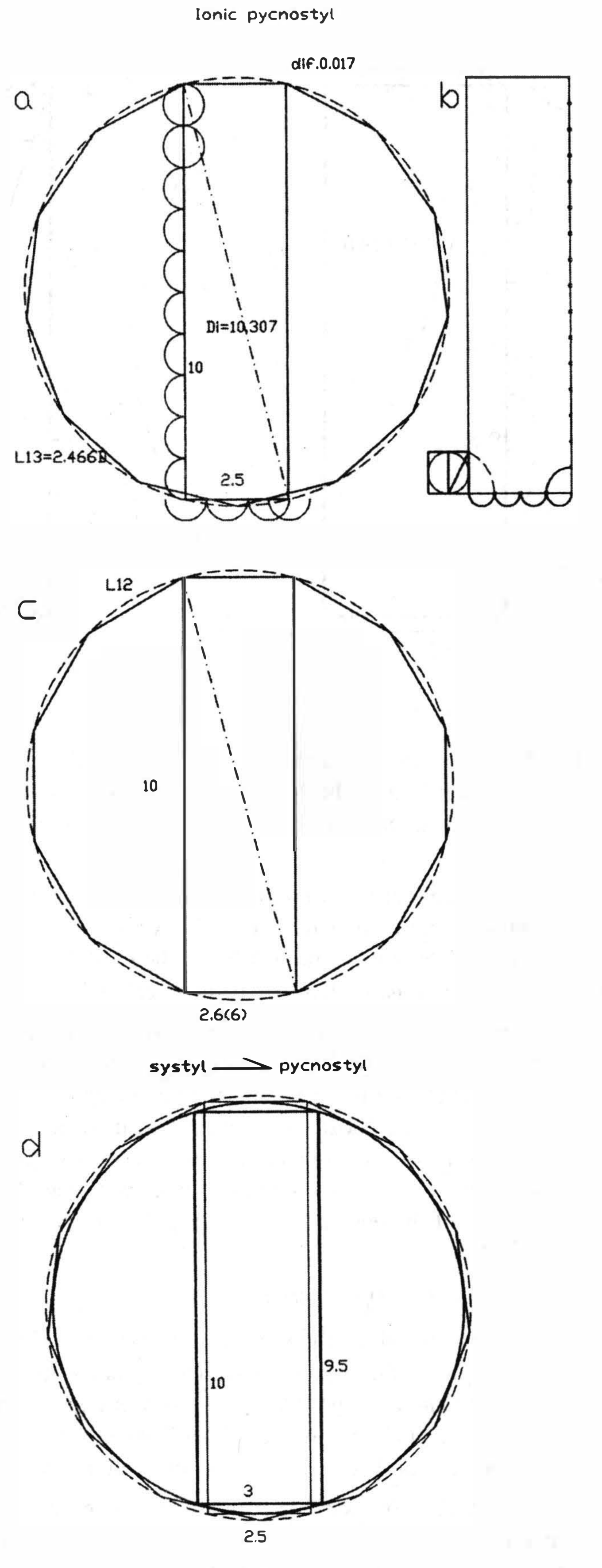

Fig. 6. Ionic pycnostyl.

This variant proves to be *adequate to an architectural reality*, that may be exemplified by the concrete case of the *Artemision in Sardes*[14], where the height of some of the columns is 10.02D, and the interaxis 2.5D. The *diameter-diagonal* has the value of 10.327D≈10.33D.

c) One can imagine a theoretical variant – that might have been at the origin of the Vitruvian pycnostyl – where the value of the interaxis comes from another expression of the medium and extreme ratio, namely from the rounding of the irrational φ^2 (or $\varphi+1$), from 2.618 to 2.66D. We remark this possibility as by it one can emphasize a way to transform the geometric support: it is described by the polygon with 12 sides (or two turned round hexagons) and not by the one with 13 sides (Fig. 6c). The value of the *diameter-diagonal* becomes10.34D, roundable to 10.33D .

d) The pycnostyl can be in its turn considered sprouted by the systyl geometry, as it may be found circumscribing a polygon with 13 sides to the circle corresponding to the systyl (Fig. 6d). According to this hypothesis, it would be necessary for the geometric support of the *Artemision in Sardes* to be regarded as a transformation of the one of the *Dydyme/Naiskos* type. Indeed, if we overlap their geometric supports[15] one may find that, by circumscribing the polygon with 13 sides to the circle corresponding to the fundamental rectangle at Dydyme, one can obtain the geometric support of the Artemision (Fig. 7a). The level of the differences between the ideal solution and the real one is insignificant (Fig. 7b).

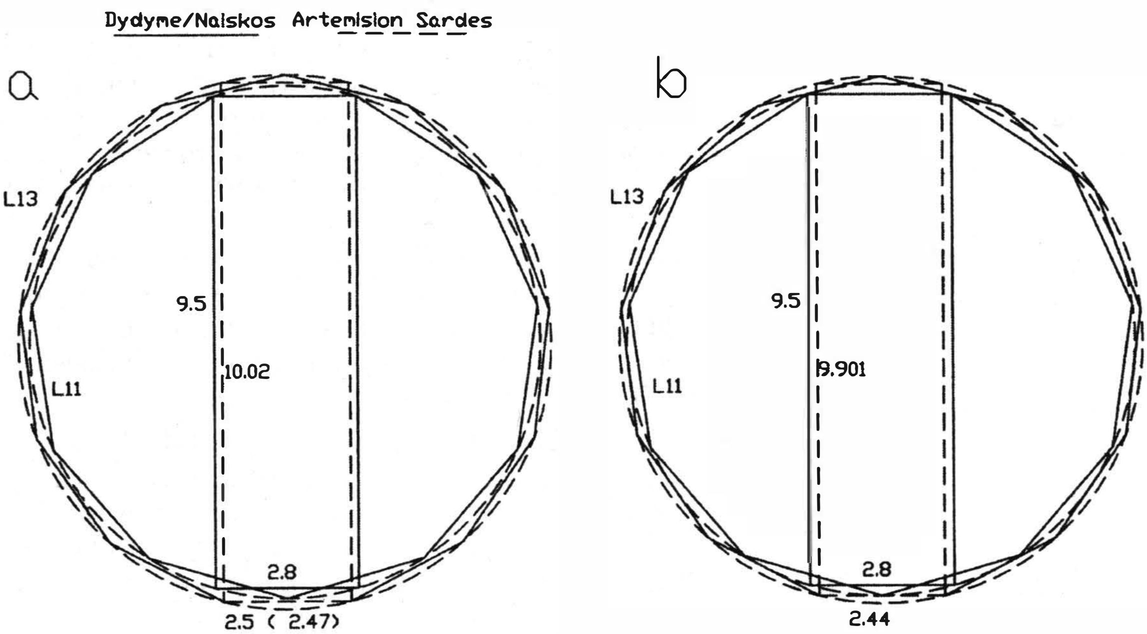

Fig. 7. Dydyme/Naiskos and Artemision Sardes.

5. *The areostyl rhythm* (Fig. 8)

The Vitruvian grill becomes ambiguous as regards this type of rhythm. While the dimension of the height of the column is clearly specified (8D), the choice of the dimension of the interaxis leaves a great deal of liberty, as the only requirement is its value should be higher than 4D[16]. We shall analyse in short a few possible cases, by proposing for the dimension of the interaxis a few values – expressible by whole numbers ratios – ranging between 4D and 5D.

a) *I =4.25D; H = 8D; Di=9.05D.* In the geometric support there is a new element: the polygon characterizing the rhythm is the one inscribed with 13 sides, but the dimension of the interaxis is no

[14] For the values of the height of the column and the size of the interaxis cf. Gros, 105,119.

[15] After we have scaled it first according to a common unit (in this case the *modular unit* considered to be equal to the unit=1, in both cases).

[16] Gros, 117.

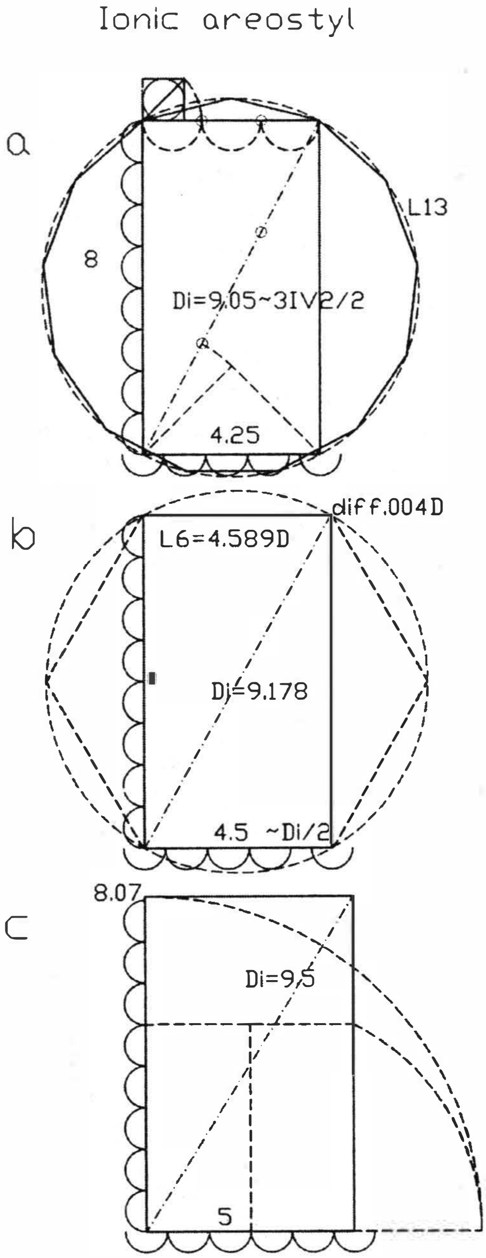

Fig. 8. Ionic areostyle.

longer equal to its side but with the dimension of the cord joining the tips of two contiguous sides (Fig.8a). Among the peculiarities of the geoemetric support it is worth mentioning that the rectangle (I:H) lies under the auspices of the simple diagonalizing (by √2): *diameter-diagonal* as compared with the interaxis (Di=3I√2/2), and the interaxis itself as compared with the modular unit: I=3√2D[17]. It is clear that the diameter-diagonal (Di) can be rounded from 9.05D to 9D[18].

b) *I=4.5D*[19]; *H =8D;(Di)=9.178* (Fig. 8b). This variant can be considered to be a practical result of a geometric support based on the hexagon: the dimension of the interaxis is assimilable to the side of the hexagon inscribed in the circle of the fundamental rectangle, as L6=4.589D, roundable to 4.5D

For an ideal solution, whenever a perfect identity is achieved between the side of the hexagon and the dimension of the interaxis, the rounding has to be done by addition, to 4.625D: in this case the diameter-diagonal becomes 9.24D, practically conveyable by rounding off to 9.25D. The side of the hexagon is 4.62D.

c) *I=5D*[20]; *H=8D*; *Di = 9.433D*. Taking into account that 8/5=1.6, it results that such a rhythm can originate in an application of the medium and extreme ratio (Fig.8c): by rounding the dimension of the diameter-diagonal to 9.5D, the height of the column becomes 8.07D (conveyable as 8D), and the ratio H/I becomes 8.077/5=1.615≈1.618=φ[21]

B. CORRELATION WITH THE COMPOSITION OF THE IONIC CAPITAL[22]

If the rhythm of the column succession, by the type of correlation between their height and the distance between them, is generated by a geometric matrix, it is expected that the defining element of the style, the capital, should be correlated by its composition to the geometric support of the fundamental

[17] Because 4.25D/3=1.416≈1.414=√2.

[18] Following the rounding off of the diagonal, the height of the column "decreases" to 7.933D, without changing in the possibility to be conveyed as 8D.

[19] The lower limit of the interaxis according to A. Choisy, W. B. Dinsmoor, O. Bingöl (Gros, 117).

[20] Limit according to H. Riemann (cf. Gros, 117).

[21] In the ideal case (φ=1.618) the height of the column should reach 8.09D.

[22] Notations: A=length of the façade; B=length of the cushion; F= distance of the eyes of the volutes; D_1=width of the volute; G=height of the volute; E=distance between the volutes; H_1=diameter of the lower surface of the capital here presupposed to be equal to the upper diameter of the column; H_2=lower diameter of the column (Module); N= height of the lower tangent of the spiral; Ln=side of a polygon with"n" sides; an=apothem of a polygon with"n" sides; Hcap=height of the capital.

rectangle (I:H). According to those announced at the beginning of this study, the geometric composition as a whole must be, at last resort, expressible by geometric procedures from a unique starting unit, namely be measurable arithmetically or geometrically as related to it. In our case the starting unit is the *module-diameter*. Therefore, in order to analyse the geometric correlation of the Ionic capital with the fundamental rectangle of the rhythm, it is necessary to study first the compositional geometry of the capital regarded as a distinct whole and the possibilities of expression according to the *module-diameter of the elements making it up*[23].

1. *Geometry of the Ionic capital*[24] (Fig. 9). The geometric support of the façade (Fig. 9a) of the Ionic capital is based on the decagon inscribed in the circle comprising the fundamental rectangle (G:A), the side (L10) being assimilable to the height of the volute (G). The plan, remarkable by the way the square has been achieved (F:B), correlates the width (D) of the volute to the apothem of the hexagon (a_6) inscribed in the circle of the lower surface (Fig. 9b) The octogon circumscribed to the circle built along the façade settles the size of the distance between the volutes (L_8=E) (Fig.9b)[25].

The volute spiral. The position of the inner tangent (N) is settled by the pentagon inscribed in the circle comprising the rectangle (F: G)[26]; between the main diagonal (dp) and the secondary diagonal (ds) there is a relation analogous to that between the radius of a circle and the apothem of the inscribed hexagon[27] (Fig. 9c).

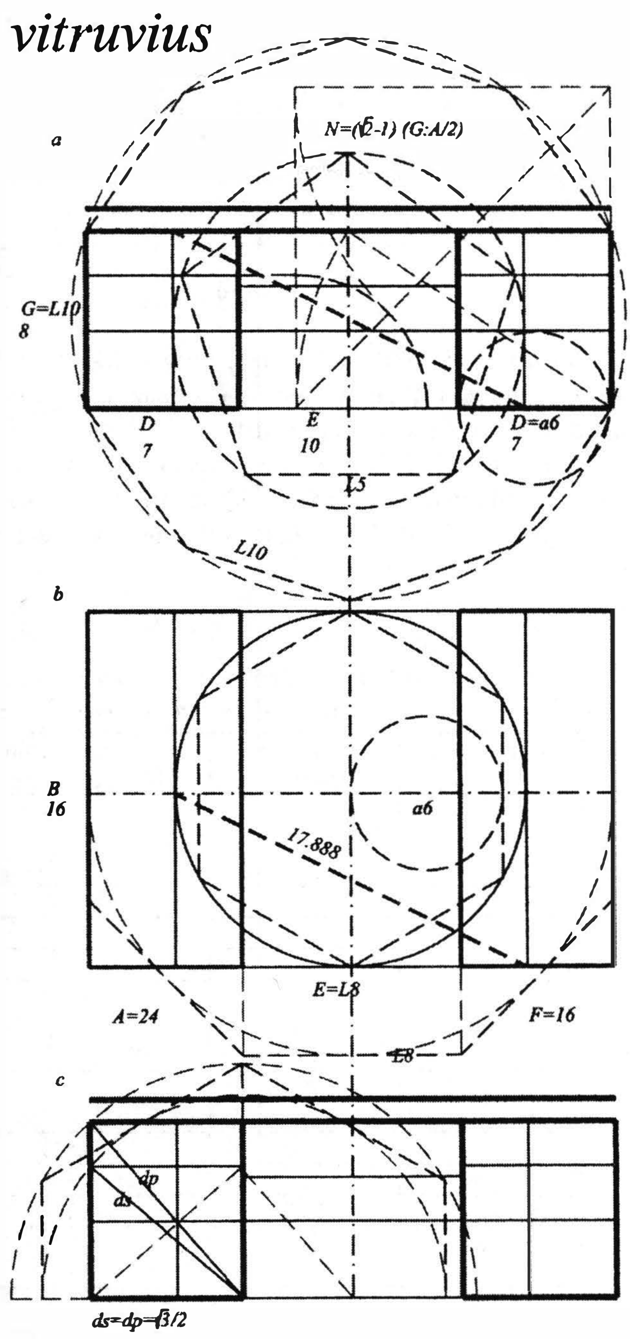

Fig. 9. Vitruvius capital; geometric support.

2. *Expressing the "Vitruvian numbers" according to the diameter at the base of the column (the module unit)*. The ratio between the upper diameter of the column[28] and its lower diameter is not constant: the diminishing of the upper diameter decreases as the height of the column increases[29]. We shall analyse the cases corresponding to the variations of the *contractura* (H_1/H_2), whose limits range between 0.833 and 0.875 (Table 1).

[23] The Vitruvian numbers (units –*partes*) were assessed starting from *1pars= H_1/ 16*.

[24] Partially described in Mărgineanu-Cârstoiu, 2000, 322; exhaustively in Mărgineanu-Cârstoiu, 2002–2003.

[25] The square network pattern of the composition, referring to the paradigm that stood at the origin of the Vitruvian design (the capital of Pytheos from Halicarnassus) was accounted in Mărgineanu-Cârstoiu, 1997, 217.

[26] Directing the composition according to the pentagon and decagon generates ratios derived from the medium and extreme ratio; an example is the possibility to express the height (N) according to the semidiagonal of the façade (without abacus): N=(√2-1) (G:A/2)(fig. 9a). For some derived constructions endeavoured Bingöl, 1980, 150-153.

[27] See. Mărgineanu-Cârstoiu, 2002–2003.

[28] The premise from which we are compelled to start is the dimensional equality between the circumference of the lower surface of the capital with the circumference of the upper surface of the shaft.

[29] Gros, 121-122.

Table 1

(*apud* P.Gros, *Vitruve*, p.122)

Height of the column H (in Foot)	Upper diameter /lower diameter $=H1_1/H_2=H_1/D$
-15	10/12=0.83(3)
15-20	11/13=0.846
20-30	12/14=0.857
30-40	13/15=0.86(6)
40-50	14/16=0.875

Ratio H_1/H_2=10/12 (Table 2, Fig. 10a-c'). Although the main dimensions can be expressed by rational numbers, their geometric origin is obvious. The geometric directing with the module unit[30] is linked to a construction derived from a harmonic ratio (√2-1)., so that the distribution *G :B: A* can be described as *(√2-1) / 2(√2-1) / 3(√2-1)*. The width of the volute (D_1) is correlated to the side of the pentagon circumscribed in the module-circle (L5c) so that the distribution $D_1 / E / D_1$ is the $L_{5c}/2$: *√3/10* : $L_{5c}/2$ type (Fig. 10b). The height of the inner tangent of the spiral involves the medium and extreme ratio directly.

Table 2

Ratio H_1/H_2=10/12=0.833 ; L_{8m}=side of the octogon inscribed in the module-circle =0.382
L_{8c}=side of the octogon circumscribed in the module-circle=√2-1; H_2=D=1

Dimensions	Vitruvian numbers	Transformation Into module-unit H_2	Geometric support	Approx. irrational numbers and diff.
A	24	1.25	5/4 3 (√2-1) 3L8c	√2≈1.416
B	16	0.833	2(√2-1)	√2≈1.416
D_1	7	0.364	L5c/2	Diff.0.001
E	10	0.52	φ^2	φ≈1.612
F	16	0.833	5 /6 2(√2-1) 2L8c	√2≈1.416
G	8	0.416	√2-1 L8c	√2≈1.416
H_1	16	0.833	5 /6	
N	6	0.312	φ_1 /2	φ_1≈0.62
Diagonal (F:G)	17.888	0.931		
1pars	1	0.052	1/19 L8c/8	Diff. 0.0004 Diff. 0.00025

Generating geometric matrix. Noticing that (√2-1) represents the side of the octogon circumscribed to the module-circle, it results that the geometric support generating the values in Table 2 is made up of the succession of two square groups each (turned round[31]) circumscribed and inscribed in the module-circle (Fig. 10c'): the *octogon circumscribed to the* module-circle settles, by its side (L_{8c}) the repartition of the elements in the façade so that G/B/A becomes *L8c/ 2L8c/ 3L8c, and the pentagon circumscribed settles the width of the volute Figs. 10 (a-c)*. The distance between the volutes is settled by the circle comprised between two sides of the stelar pentagon (L_{5o}) (Fig.10b*) The unit pars can be assimilated as 1/19 of the module-diameter, but its origin is geometric and represents the eighth part of the generating construction (√2-1), namely the eighth part of the side of the octogon circumscribed to the module-circle .*

Ratio H_1/H_2=11/13 (Table 3, Figs. 10d-f"). Although the repartition G/B/A is of the previous type (Figs. 10f,f"), and the width of the volute the same geometric correspondent (L_{5c}) in the geometric design

[30] Noted with H_2 or D.

[31] Thus generating the octogon.

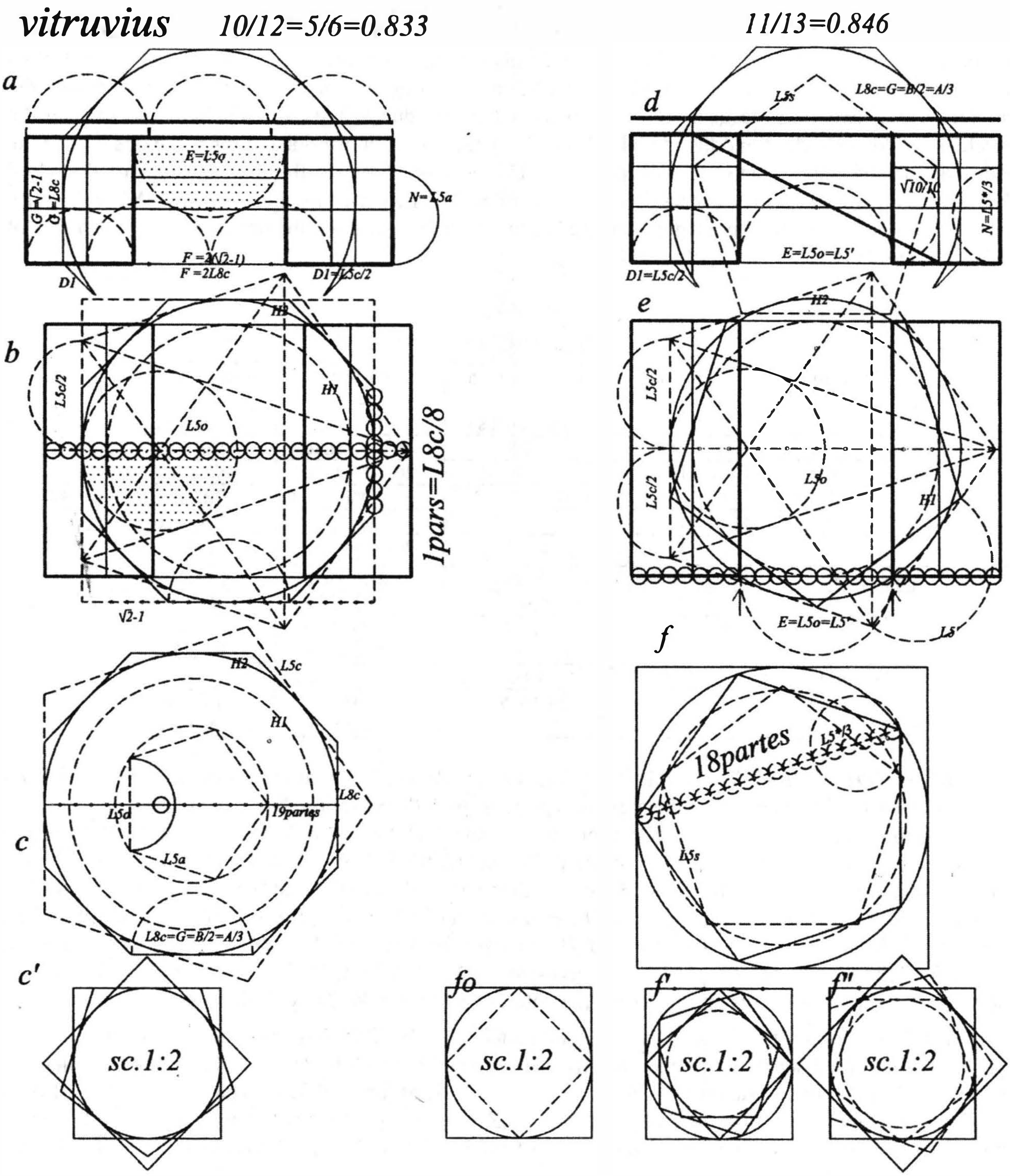

Fig. 10. Vitruvius capital; geometric support and module-circle (H_2).
10a-c': Ratio H_1/H_2= 10/12.
10.d-f'': Ratio H_1/H_2=: 11/13.

undergoes an important transformation: *the module circle passes through the tips of the rectangle (B:E)* (Fig. 10e), *therefore it determines with accuracy the distance between the volutes*[32]. (Fig. 10e). This simplification of the geometric construction reflects a very refined transformation occurred in the generating geometric matrix, having at the base the pentagon circumscribed to the modular circle: a succession of pentagons directed by the *ad quadratum* succession of the modular squares (Figs. 10f,fo,f') comprise all the characteristics of the compositional distribution of the capital. For instance, the simplification involving the distance between the volutes is due to the particular relation between the circle of the lower surface and the modular circle: the pentagon circumscribed to it has the tips laid on the sides of the starting pentagon (Figs.10e, f,f'')[33]. The pentagon settling the inner tangent of the spiral belongs to the *ad quadratum* succession (Figs. 10 f,f'). .The height of the inner tangent (N) is the third part of the side of the stelar pentagon inscribed in the module circle, and the unit *pars* is the 18th part of this side (Fig. 10f).

Table 3

Ratio $H_1/H_2=11/13=0.846$; $L8m=0.382$; $H_2=D=1$

Dimensions	Vitruvian numbers	Transformation into module units	Geometric support	Approx. irrational numbers and diff.
A	24	1.269	$3(\sqrt{2}-1)=3L8c$	$\sqrt{2}\approx1.423$
B	16	0.846	$2(\sqrt{2}-1)=2L8c$	$\sqrt{2}\approx1.423$
D_1	7	0.37	**L8m**	Diff.0.012
E	10	0.528	$\sqrt{10}/6$	$\sqrt{10}\approx3.168$
F	**16**	0.846	$2(\sqrt{2}-1)=2L8c$	$\sqrt{2}\approx1.423$
G	**8**	0.423	$\sqrt{2}-1=L8c$	$\sqrt{2}\approx1.423$
H_1	**16**	0.846	$2(\sqrt{2}-1)=2L8c$	$\sqrt{2}\approx1.423$
N	6	0.317	$\sqrt{10}/10$ **$L_5*/3$**	$\sqrt{10}\approx3.17$ diff.0.0000
Diagonal (F:G)	17.888	**0.945**		
1 pars	1	**0.0528**	**1/19** L8m/8	Diff. 0.0002 Diff. 0.0011

Ratio $H_1/H_2=12/14=6/7$ (Figs.11a-c,c'). The distribution G /B /A and the correlation of the unit *pars* can be considered to comply with the previous types, as the approximation for $\sqrt{2}$ is rough.

In Fig. 11b one can see that the geometric mutation making two opposite sides of the hexagon inscribed in the modular circle tends to overlap the lines of the eyes of the volutes generating the double geometric correlation: the distribution G/ B/ A is directed by the apothem of the hexagon (a_{6m}) inscribed in the module circle (a_{6m} : $2a_{6m}$: $3a_{6m}$). *The unit pars tends towards equality with the 8th part of the apohtem of the hexagon (a_{6m}) and towards 1/19 of the module diameter*[34]. The geometric matrix is framed in the modular square: the *ad quadratum* succession (Figs.11 b,c,c'), led to the stelar octogon – whose stelar core determines the distance between the volutes – is accompanied by the hexagon.

Ratio $H_1/H_2=13/15$ (Figs. 11d-f,f'). Announced in the previous case, the geometric support generated by the hexagon is achieved ideally (with insignificant differences[35]), due to the fact that the opposite sides of the hexagon overlap lines of the cyes of the volutes). The repartition G/B/A is $a_{6m}/2a_{6m}/3a_{6m}$, and the repartition $D_1/E/D_1$ is $L_{8m}/\varphi/3/L_{8m}$.(or L_{8m}/ Lo /L_{8m}). The unit *pars* is the eighth part of the apothem (a_6). The size of the height of the inner tangent of the spiral can be settled by the

[32] At the same time the length of the cushion (B) and the H_1; transposed in the façade (with the circle in the intersection point of the diagonals (G:A), the modular circle determines also the height of the capital considered an abacus (Fig. 10d) These peculiarities can ease significantly the information for execution.

[33] The distance between the volutes corresponds exactly to the large segment determined on the side of the pentagon (L5c) by the pentagon inscribed to it.

[34] For a value of ~ 90cm for the diameter of the lower surface, the difference resulted for *1 pars* is 0.08 cm, involving an error-difference of 0.6 cm for the height of the volute; for smaller diameters, the difference as compared with the ideal value decreases.

[35] That can be seen in the approximation of $\sqrt{3}$ according to Table 5.

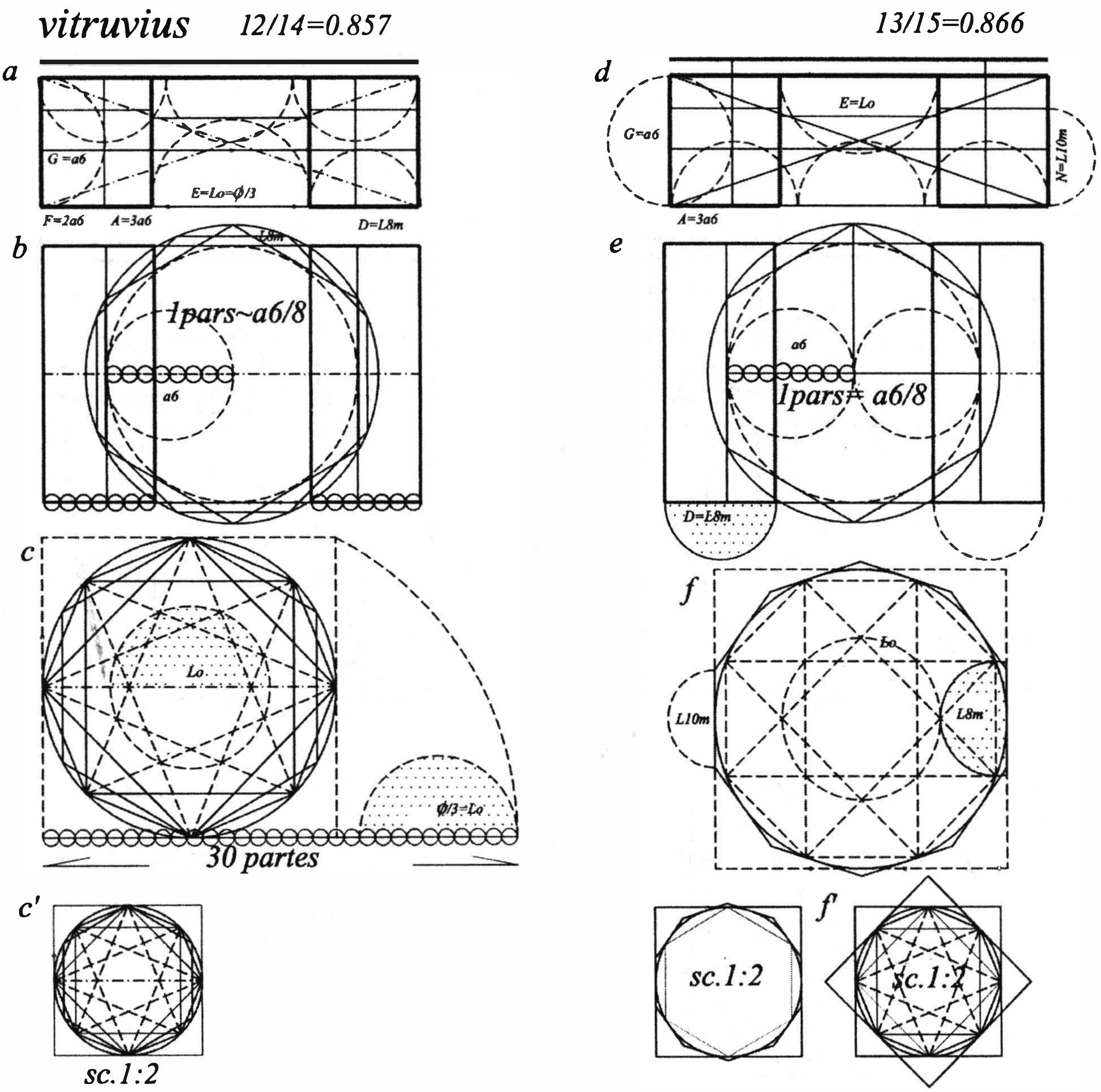

Fig. 11. Vitruvius capital; geometric support and module-circle (H_2).
11a-c': Ratio H_1/H_2=12/14.
11d-f': Ratio H_1/H_2=13/15.

medium and extreme ratio, reflecting its origin in the side of the decagon circumscribed to the module-circle (L_{10}). The geometric matrix is of fhe type previously described, singled out by its being accompanied by the decagon (Figs. f-f').

Ratio H_1/H_2=14/16=7/8 (Table 6, Figs. 12a-c'). It contains the correlation of the capital to the module-diameter through the golden section (the distribution G/B/A is $\varphi^2/6$: $\varphi^2/3$: $\varphi^2/2$ or a6m : 2a6m ; 3a6m, and $D_1/E/D_1$ is L_{8m} : $\varphi/3$:L_{8m} or L_{8m} : Lo :L_{8m}), but the geometric matrix can be concentrated in the diagram analogous to the previous one (Figs. 12c-c'). The correlation of the unit *pars* is exclusively geometric, depending on the geometric figure governing the distribution G/B/A: either on the medium and extreme ratio, as the eighth part of ($\varphi^2/6$), or as ($a_{6m}/8$).

The most important reality of the geometric matrix, common to the three cases (12/14; 13/15; 14/16), is the tendency to concentrate the relation between the diameters H_1 and H_2 in the hexagon inscribed in the module-circle (H_2): that is at the same time the hexagon circumscribed to the circle *(H_1)*.

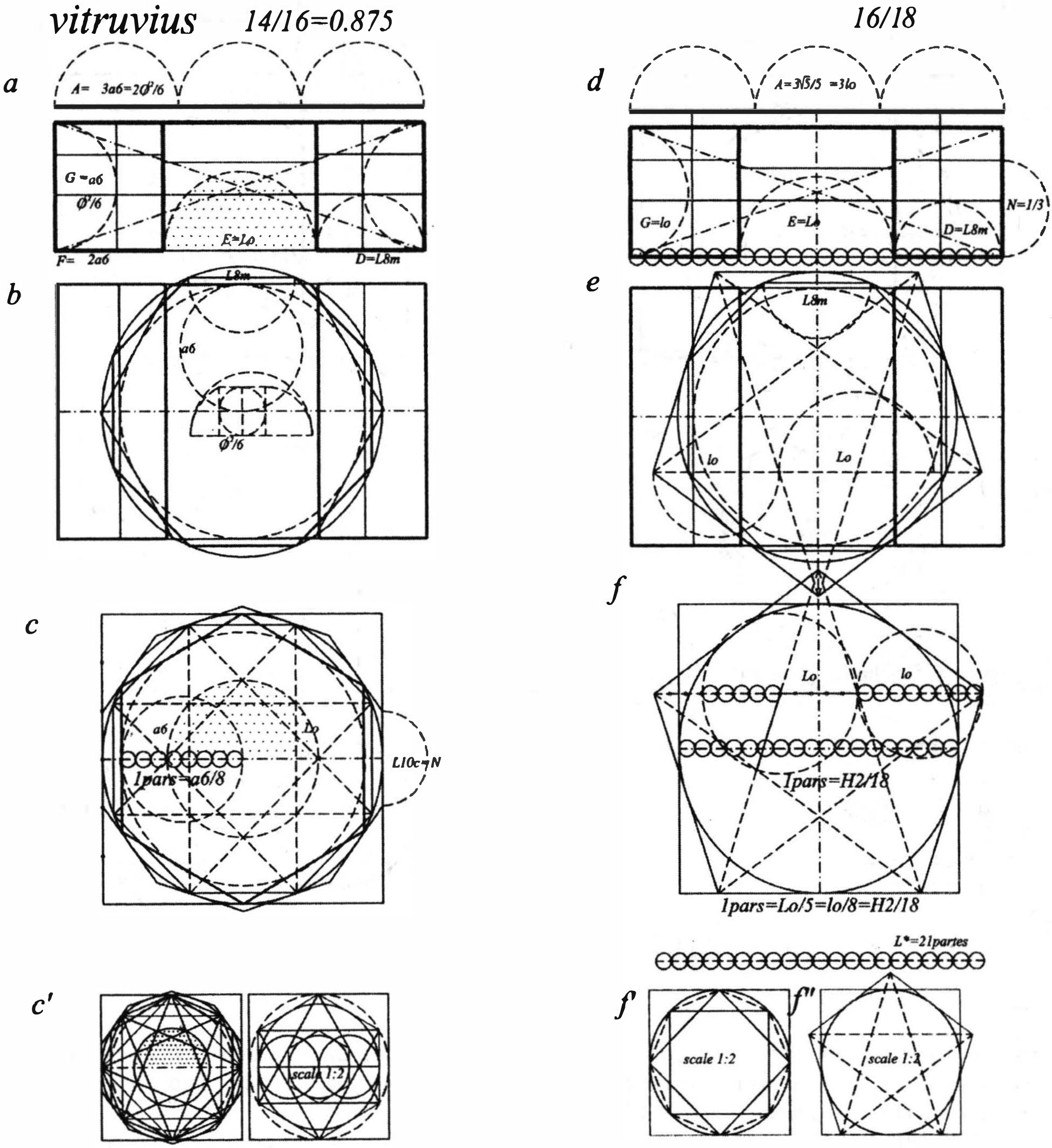

Fig. 12. Vitruvius capital; geometric support and module-circle (H_2).
12a-c': Ratio $H_1/H_2=14/16$.
12d-f'': Ratio $H_1/H_2=16/18$.

Table 4

Ratio H_1/H_2=12/14=0.857; L8m=0.382;
a6m=apothem of the hexagon inscribed in the module-circle= 0.433; H_2=D=1

Dimensions	Vitruvian numbers	Transformation into module units	Geometric support	Approx. irrational numbers
A	24	1.285	(√2-1)/3 ~3√3/4	√2≈1.428 √3≈1.713
B	16	0.857	2(√2-1) √3/2	√2≈1.428 √3≈1.713
D_1	7	0.375	**L8m**	Diff.0.007
E	10	0.535	φ/3 Lo	φ≈1.606
F	16	0.857	2(√2-1)	√2≈1.428
G	8	0.428	√2-1 √3/4	√2≈1.428
H_1	16	0.857	2(√2-1)	√2≈1.424
N	6	0.321	φ/5	φ≈1.605
Diagonal (F:G)	17.888	**0.958**		
1 pars	1	**0.0535**	**1/19** L8m/8 a6m/8	Diff. 0.0009 0.0018 0.0006

Table 5

Ratio H_1/H_2=13/15=0.86(6); L8m=0.382;
a6m=apothem of the hexagon inscribed in the module-circle= 0.433; H_2=D=1

Dimensions	Vitruvian numbers	Transformation into module units	Geometric support	Approx. irrational numbers/diff.
A	24	1.299	3√3/4 3a6m/2	√3=1.732
B	16	0.866	√3/4 a6m	√3=1.732
D_1	7	0.379	**L8m**	Diff.0.004
E	10	0.54	φ/3	φ=1.62
F	16	0.866	√3/2	√3=1.732
G	8	0.433	√3/4 a6m/2	√3≈1.732
H_1	16	0.866	√3/2	√3=1.732
N	6	0.324	φ/5 L10	φ=1.62 diff.0.0009
Diagonal(F:G)	17.888	**0.968**	1	
1 pars	1	**0.0541**	**a6m/8** **1/18.5**	Diff.0.0000

***Hypothesis:* the ratio H_1/H_2=16/18**. From the previous tables one may find that the value of the unit *pars*, expressed acording to the module-diameter, increases as the *contractura* increases: if in the cases corresponding to some heights of columns ranging between 15 and 20 foot, the unit *pars* can be assessed at 1/19 of the Module, at the columns with heights ranging between 20 and 50 foot the value grows from 1/18.5 to 1/18.3. Theoretically, this growth can be considered to have as a limit 1/18 of the Module (Table 7). The directing of the correlations is done in this case by √5, approximated by irrational fractions emphasizing a well known ratio in Greek ancient architecture (4/9)[36]. The repartition G/B/A is *√5/5 : 2√5/5 :3√5/5;* the repartition $D_1/E/D_1$ – where the correlation procedure destined to the width of the volute proves to preserve a remarkable constancy – becomes L_{8m} *: √5/4* L_{8m}. The unit *pars* is the eighth part *of the "geometric unit"*, namely of √5/5. *The geometric matrix can be synthesized by the ad quadratum*

[36] It is also the Parthenon ratio, for instance between the diameter of the column and the distance between the columns (Gruben, 1966/1976, 167) and expressed a correlation by simple diagonalizing: √5/5.

succession accompanied by the pentagon. The side of the stelar pentagon sprouts the distance between the volutes (through the side Lo of the homothetic pentagon) and the height of the volute (through the stele sides) (Figs. 12f-f''). The unit pars is the eighth part of the stele side (lo/8), at the same time assessed as 1/18.

Table 6

Ratio H_1/H_2=14/16=0.875; L_{8m}=0.382;
a6m=apothem of the hexagon inscribed in the module-circle= 0.433; H_2=D=1

Dimensions	Vitruvian numbers	Transformation into module units	Geometric support	Approx. irrational numbers
A	24	1.312	21/16 $\varphi^2/2$ 3√3/4	φ≈1.62
B	16	0.875	7/8 $\varphi^2/3$	φ≈1.62
D_1	7	0.382	**L8m**	**Diff.0.000**
E	10	0.546	φ/3	φ≈1.63)
F	16	0.875	7/8 $\varphi^2/3$	φ≈1.62
G	8	0.437	$\varphi^2/6$	φ≈1.62
H_1	16	0.875	7/8 $\varphi^2/2$	φ≈1.62
N	6	0.328	φ/5	φ≈1.64
Diagonal (F:G)	17.888	**0.978**		
1 pars	1	**0.0546**	1/18.3 $(\varphi^2/6)/8$ **a6m/8**	 0.0007 0.0005

Table 7

Ratio H_1/H_2=16/18=0.8(8)≈2√5/5; L_{8m}=0.382; H_2=D=1

Dimensions	Vitruvian numbers	Transformation into module units	Geometric support	Approx. irrational numbers
A	24	1.3(3)	**4/3** 3√5/5	√5≈2.2(2)
B	16	0.8(8)	**8/9** 2√5/5	√5≈2.2(2)
D_1	7	0.38(8)	**L8m**	Diff.0.006
E	10	0.5(5)	√5/4	√5≈2.2(2)
F	16	0.8(8)	**8/9** 2√5/5	√5≈2.2(2)
G	8	0.4(4)	**4/9** √5/5	√5≈2.2(2)
H_1	16	0.8(8)	**8/9** 2√5/5	√5≈2.2(2)
N	6	0.3(3	1/3	
Diagonal(F:G)	17 888	**0.993**	**≈1**	Diff.0.007
1 pars	1	**0.0555**	**1/18**	Diff.0.000

This theoretical model compels us to pay attention to the value of the diagonal i (F:G). It is obvious that *its value reaches the value of the module-unit as the height of the column increases: in the case of the ratio 16/18 the proximity is maximum.* According to the geometric model proposed *in Table 7,* it results that the ratio H_1/H_2=16/18 in fact contains an *approximation* of √5. Consequently, the ideal values of the original model have to observe the geometric construction in a perfect way. In *Table 8* one can see that in such a model (√5≡2.236), the value *(F:G) is identical to the module-unit.*

Table 8

Ratio $H_1/H_2=16/17.888=0.894$;

Dimensions	Vitruvian numbers	Transformation into module units	Geometric support	Approx. irrational numbers
A	24	1.341	3√5/5	√5=2.236
B	16	0.894	2√5/5	√5=2.236
D	7	0.391	**L8m**	Diff.0.009
E	10	0.559	√5/4	√5=2.236
F	16	0.894	2√5/5	√5=2.236
G	8	0.447	√5/5	√5=2.236
H_1	16	0.894	2√5/5	√5=2.236
N	6	0.335	1/3	
Diagonal (F:G)	17.888	**1**	**1**	Diff. 0.000
1p	1	**0.0559**	**1/18**	Diff.0.0004

It is not out of the question for such a model (Fig. 13) to have its origin in Hellenistic architecture: as a matter of fact, according to *Table 1,* the value 0.894 does not appear in the case of the Roman columns. However, it can be found in Hellenistic architecture, for instance at the Temple of Athens in Priene[37], whose architect created at the mausoleum in Halicarnas the paradigm of the Hellenistic Ionic capital which, having passed through Hermogene's capital, was later conveyed by Vitruvius. Therefore, the correlation between the diagonal (F:G) and the diameter at the base of the column at the capital of the Temple of *Artemis in Magnesia*[38] *, where (F:G)/H_2= 141.2 cm/140 cm=1.008! will not be surprising*[39].

3. *The method used for the study of the correlation between the geometry of the capital and the interaxis of the columns.* Taking into account that a *module-square* contains all the information on the geometry of the Ionic capital (synthetic diagrams in Fig. 10c'; Fig. 11f"; Figs. 12f'-f"; Fig. 13f'), for the fundamental rectangle of the rhythm of the columns to be compared with the fundamental rectangle of the capital it is enough to correlate the interaxis to the Ionic capital. In order to do that we are obliged to express geometrically the dimension of the interaxis. That can be done by the sequences of the modular squares. This way the geometric organizing principle appears governed – in all the types of rhythms – by *the grill of the Module squares*[40]. In order to look into the correlations, the fundamental rectangle of the capital will overlap the interaxis by its being placed in the centre of the modular rectangle[41]. It will be seen that, in all the cases, the geometric matrices can be described by *ad quadratum* successions.

a) *Correlating the composition of the Ionic capital of the interaxis in the Systyl rhythm* (Fig. 14). The diagonals of the fundamental rectangle of the façade of the capital overlap the diagonals of the modular rectangle. That indicates that the rectangle of the façade of the capital has been obtained by a scale reduction (geometrically) of the modular rectangle. The number 10, governing the rhythm (through the side of the decagon inscribed in the circle of the rectangle H:I), can be found in the decagon inscribed in the circle of the modular rectangle: its side is equal to a Module. The beauty of this system reaches its final expression in the geometric correlation to the *intercolumniation*, governed by the number seven. The heptagon inscribed in the *intercolumniation* circle has the side equal to the Module; the heptagon succession

[37] H_1/H_2 at Athenaion is 115 cm/128.5 cm=0.894! (the values H_1 and H_2 *apud* Hoepfner-Schwandner 162/Fig. 160).

[38] Dimensions after Hoepfner, 1968, 214.

[39] We cannot expect that in the case of the Hellenistic columns, the variation of the *contractura* should observe Roman patterns (at the Temple of Artemis the ratio H_1/H_2= =117 cm/140 cm=0.835). If at the capital in Halicarnassus the diameter of the column had been equal to (F:G) – which is 106.8 cm –, its values should be very close to that value: see also Mărgineanu-Cârstoiu, 2000, 315.

[40] In each type of rhythm results a rectangle built up by the *module-squares* juxtaposition, according to the values of the interaxes. Conventionally, we shall define such a rectangle as a *modular rectangle.*

[41] The opportunity of this way of overlapping the "geometries" of distinct subjects with a view to comparing, is based on the essential role of the diagonals in drawing the geometric support (application in Mărgineanu-Cârstoiu, 2001/A, 260, 265, 270); by this method we could find that the compositional design of Hermogene's capital is copied after a capital at the mausoleum in Halicarnassus, being a simple scale value by its geometric method (Mărgineanu-Cârstoiu, Dacia, 2002–2003).

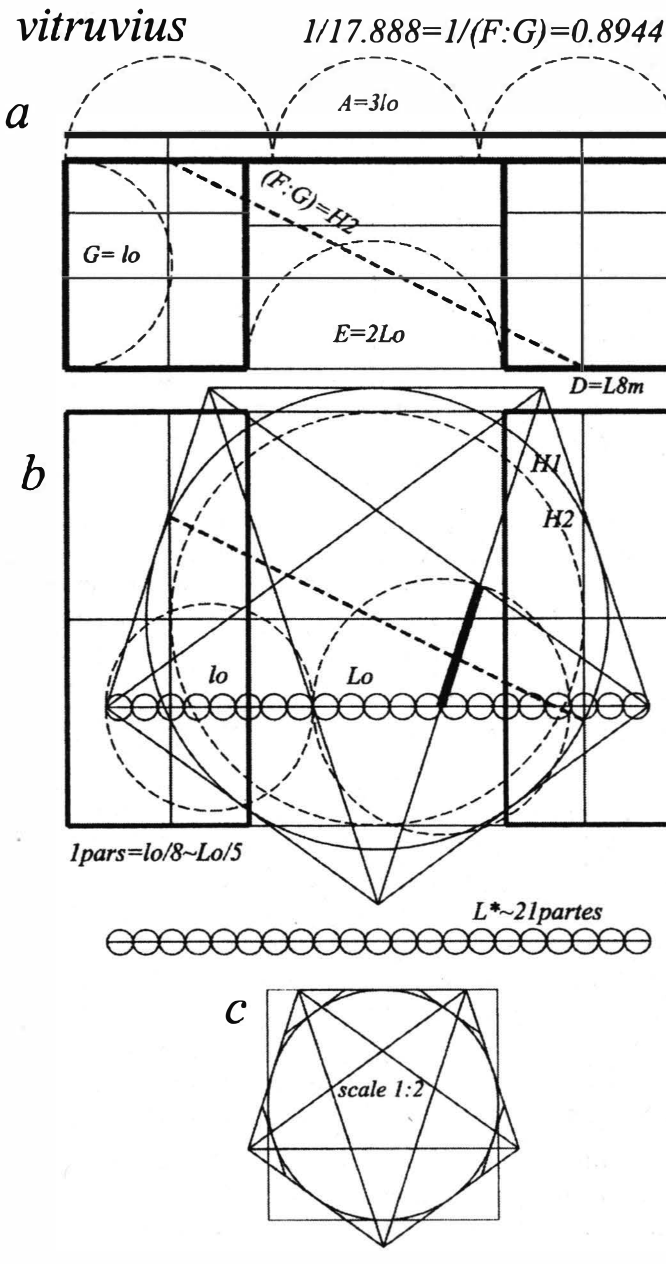

Fig. 13. Vitruvius capital; geometric support and module (H_2); Ratio $H_1/H_2=1/17.888$.

directed by the stelar pentagon comprises the rectangle of the façade of the capital (Figs. 14a-b). *The concentrated geometric matrix expression reveals an ad quadratum succession* (Figs. a'-b') *with a core hexagon* (Figs. a'-b'). Probably it is not by chance that the ratio of the numbers expressing the type of polygons involved (ratio 7/10) is the basis for a common approximation in antiquity, for the *irrational* $\sqrt{2}$.

b) *Correlating the composition of the Ionic capital to the eustyl interaxis* (Fig.15). The number 10, governing in *systyl* the rectangle of the rhythm (H:I) through the decagon *circumscribed* to the circle (H:I), can be found in the decagon inscribed in the inner circle of the modular rectangle, as the side L_{10} is equal to the module. The diagonals of the rectangle pertaining to the façade of the capital do not overlap the diagonals of the modular rectangle: they head towards the corners of the decagon. The façade of the capital is correlated to the intercolumniation according to the same principle of homothety, but it appears simplified by the dependence on the hexagon. The geometric support has a correspondent in an *ad quadratum* succession (Figs. a',b').

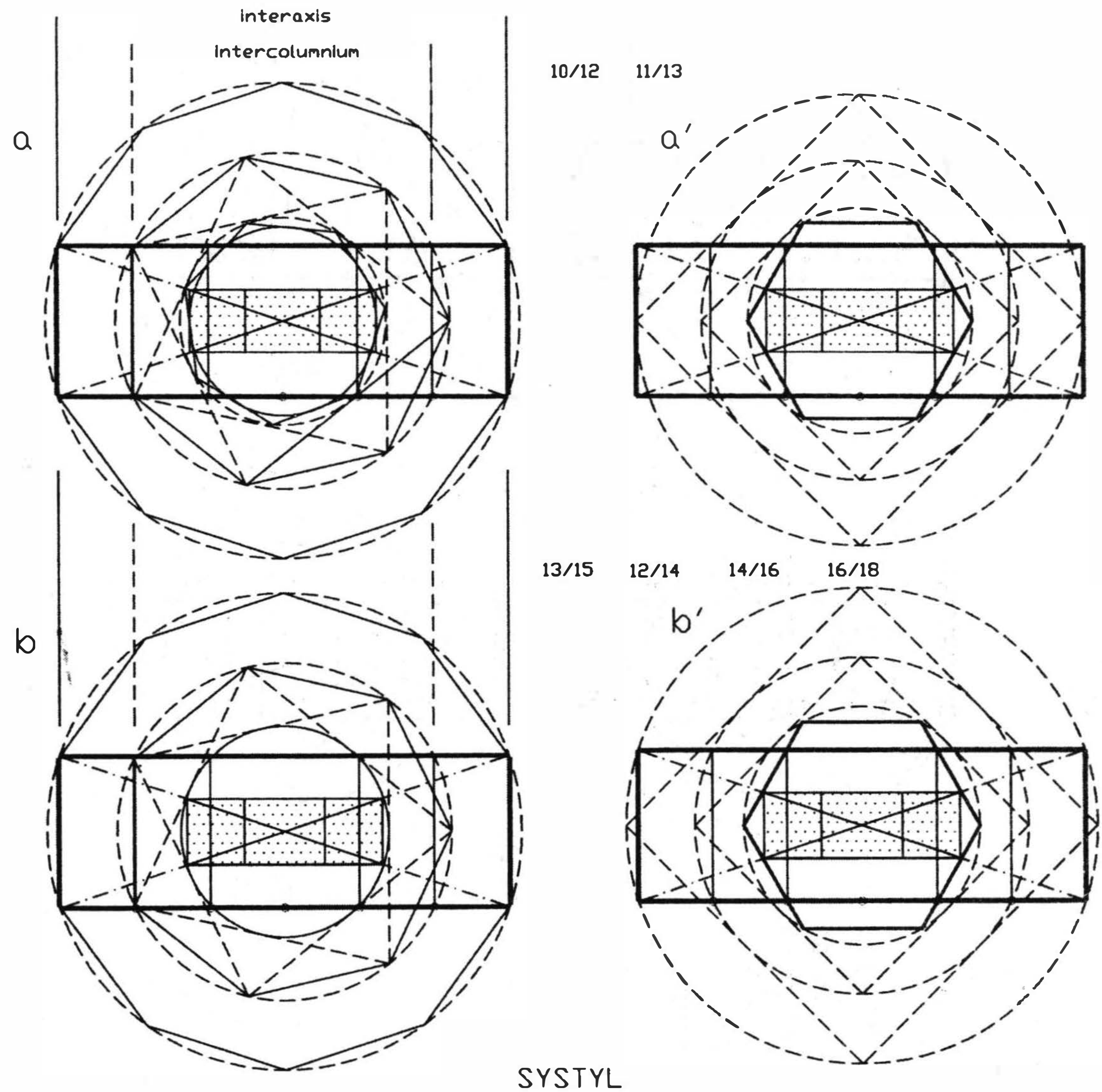

Fig. 14. Correlating the Ionic capital of the interaxis: systyl rhythm.

c) *Correlating in dyastyl* (Fig. 16). The polygon corresponding to the modular rectangle is the one with 13 sides[42].

The decagon inscribed in the rectangle of the intercolumniation has the side approximately equal to a module, settling by its diagonals the rectangle of the façade of the capital (the diagonals of the façade converge with the tips corresponding to the decagon). When the ratio is equal to 11/13; 12/14; 13/15 the circle of the homothetic pentagon – inscribed in the overlapping circle of the intercolumniation – is tangent (interior) to the sides of the rectangle of the façade of the capital (Fig. 16b). In the case of the ratio 10/12 the tangency is not perfect. The geometric matrix is described by an *ad quadratum* succession (Figs. a'-b').

d) *Correlating* in *pycnostyl* (Fig.17).This is the most simply correlated system. The rectangle of the façade of the capital is comprised in the square inscribed in the circle overlapping the rectangle of the intercolumniation. The diagonals of the façade converge with the tips of the decagon circumscribed to the circle of the intercolumniation and with those of the decagon inscribed in the circle of the modular rectangle (Figs. 17b-c). The modular rectangle has a correspondent in the octogon inscribed in the overlapping circle (Fig. 17a). The geometric matrix has a correspondent in an *ad quadratum* succession (Figs. a'-c').

[42] For the modular rectangle to be closed two polygons with 13 turned round sides are necessary.

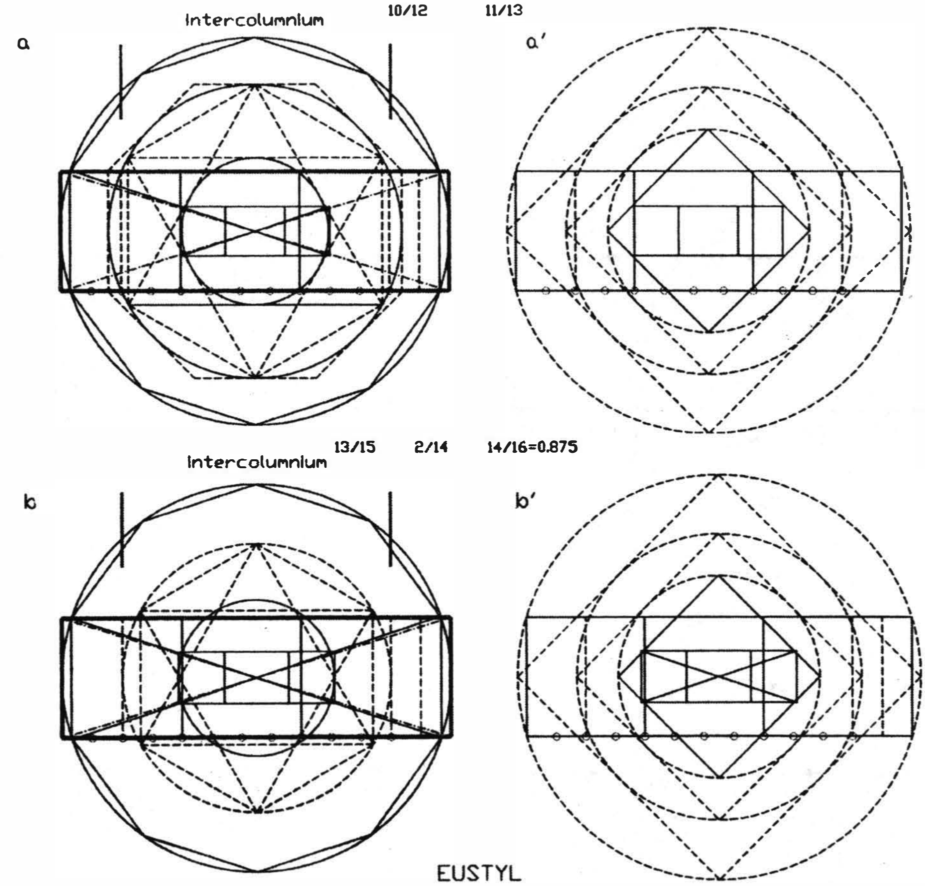

Fig. 15. Correlating the Ionic capital of the interaxis: eustyl rhythm.

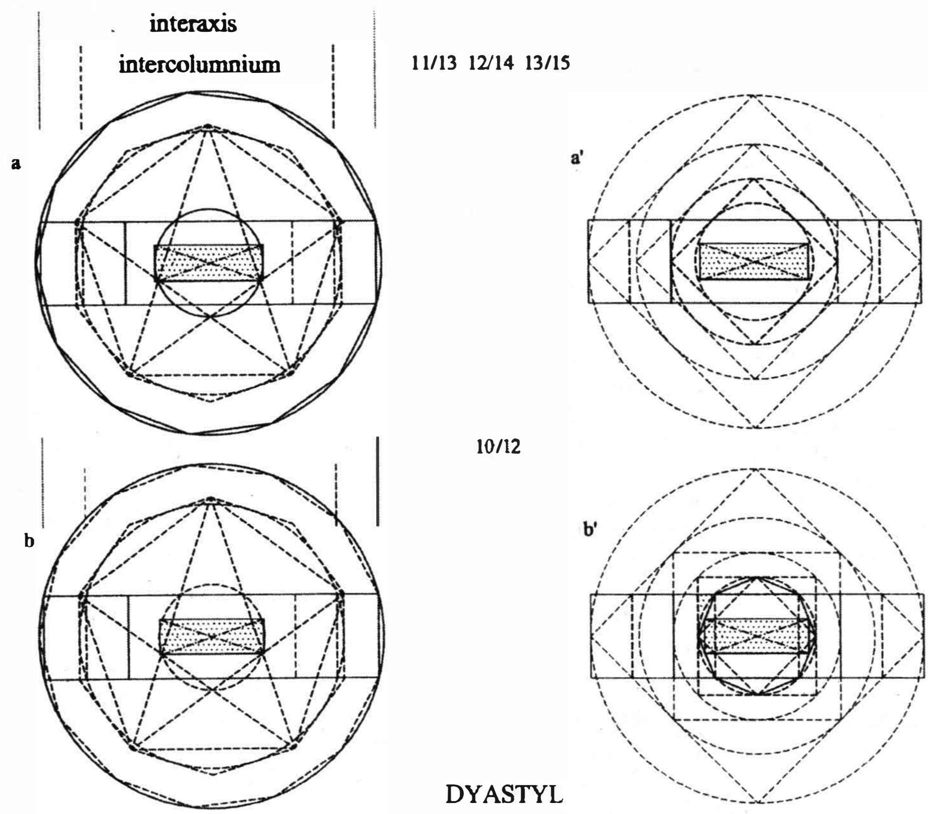

Fig. 16. Correlating the Ionic capital of the interaxis: dyastyl rhythm.

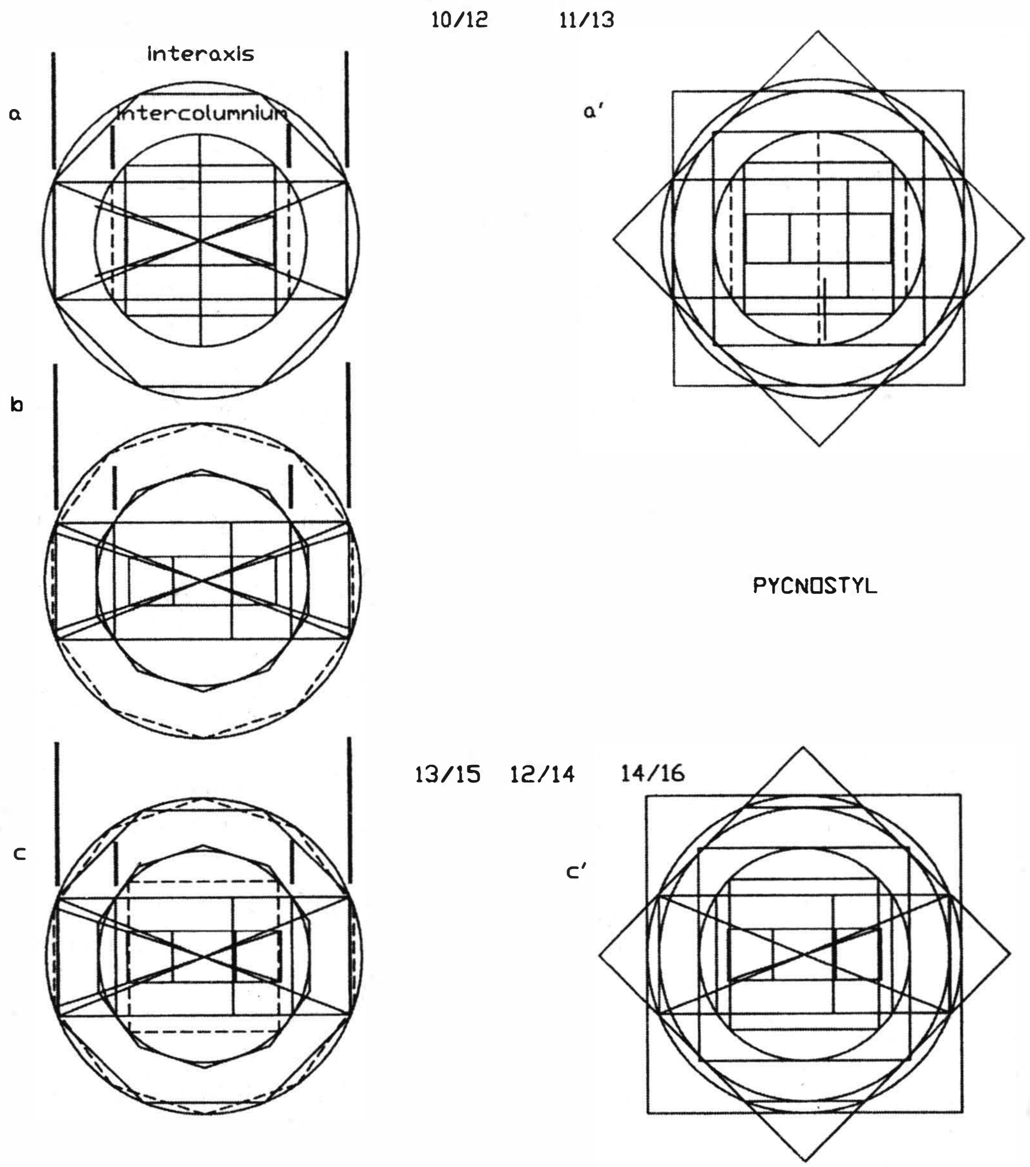

Fig. 17. Correlating the Ionic capital of the interaxis: pycnostyl rhythm.

C. THE CORINTHIAN STYLE

As we know, the Vitruvian system refers mostly to the Ionic style[43]. In the case of the Corinthian, the change in the ratio between the height of the capital and the diameter of the column base entails corresponding variations of the height of the column[44].

1. *Pycnostyl (Fig.18a). H=10.66D; I=2.5D; Di=10.955D.* The polygon with 14 sides determines the fundamental rectangle; between the height of the column and the interaxis there is a simple diagonalizing relation:: $H=3\sqrt{2}$ I. For the circle ray to be measurable it suffices to round off the value 10.955Dla 11D, involving an extremely fine increase in the height of the column, from 10.6(6)D to 10.7D.

2. *Dyastyl (Fig18b-b'). H=9.166D; I=4D;Di=10D.* It is possible to consider that the geometric support is governed by the octogon inscribed in the circle containing the fundamental rectangle (H:I). Keeping the value of the diameter-diagonal at 10D and aiming at a dimension of the interaxis perfectly equal to the side of the octogon, the interaxis should be 3.827D, and the height of the column 9.239D.

[43] Gros, 118-119.

[44] For the grill of Corinthian rhythms, Gros, 119.

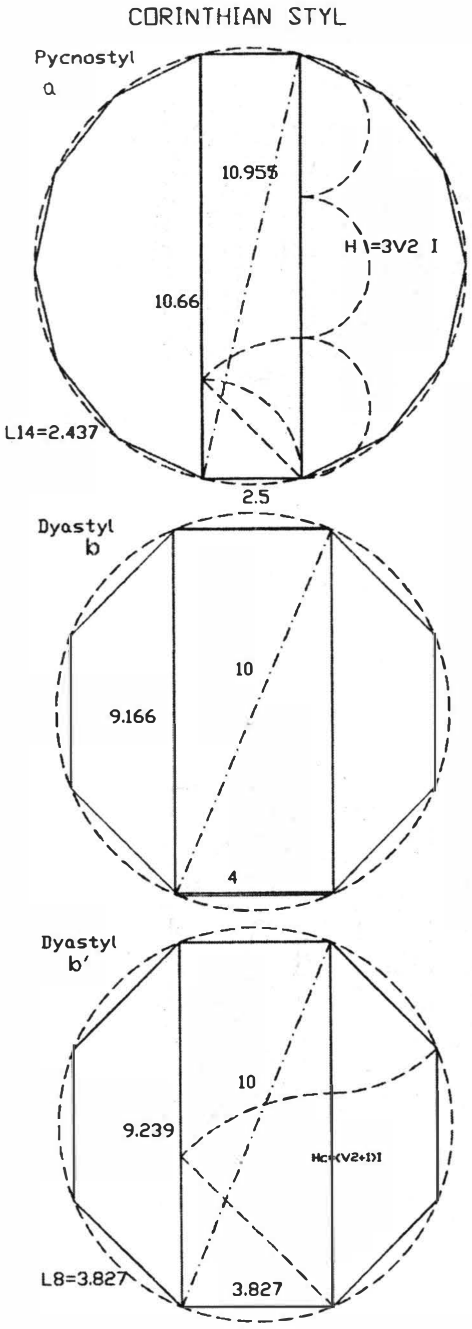

Fig. 18. Corinthian styl.
a) Pycnostyl. b-b') Dyastyl.

In this variant, there is a relation originating in a harmonic ratio between the height of the column and the dimension of the interaxis: $H/I=2.414=1+\sqrt{2}$ (Fig. 18b').

3. *Eustyl (I=3.25; H=10.166; [Di=10.672]) (Fig. 19a)*. It is a rhythm depending on the decagon inscribed in the circle of the fundamental rectangle. The diameter-diagonal measures 10.67D, which can be considered to represent, by rounding off, the value 10.6(6)D, in order to be adequate to the geometric support that it generates[45].

4. *Systyl (I=3; H=10.166; [Di=10.599])* (Figs.19b-b'). The geometric support is governed by the polygon with 11 sides ($L_{11}=2.986D\approx3D$). If the diameter-diagonal rounds off from 10.599D to a measurable value of 10.5D, to an interaxis of 3D corresponds a column height of 10.06D, that is ≈10D (Fig. 19b).

5. *Areostyl (Fig. 20) H=8.6(6)D; I>4D.* The limit value can be considered the one in which the dimension of the interaxis is identical to the size of the side of the pentagon inscribed in the fundamental rectangle. The theoretical repartition I / H / Di in this case is 4.07D /8.66D / 9.374D. Therefore, the height of the column can be considered to be obtained by rounding off from a diagram in which the diameter-diagonal is measurable, namely L_7 / 8.584D / 9.5D.

a) *I=4.25D; H=8.6(6)D; Di=9.652D* (Fig.20a); the difference in the size of the interaxis as related to the side of the inscribed heptagon (L_7) is 0.03D. Supposing an arithmetically measurable diameter-diagonal Di=9.6(6), the dimensions can be considered a result of rounding off an interaxis found in a simple diagonalizing relation with the modular unit: $I=3\sqrt{2}D= 4.242D\approx4.25D$ (Fig. 20a').

b) *I=4.5D; H=8.66D; Di=9.75D* (Fig.20b). The cord joining the tips of the two contiguous sides of the polygon with 13 sides directs the size of the interaxis.

c) In the variant *I=5D; H=8.66D* the interax depends on the sides of the inscribed hexagon; this simplicity of the construction accords a significant value of the diameter-diagonal (Di=10D) (Fig. 20c).

D. A FEW EXAMPLES OF REAL ARCHITECTURE

The fact that the grill referring to the rhythm of the columns was generated by a geometric matrix ignored by Vitruvius[46], does not mean that the realization of the architectural composition in general, according to a geometric support, has disappeared as a designing tool in the Roman world. The geometric

[45] In the ideal case, to a diameter-diagonal of 10.6 (6)D and an interaxis of 3.25D corresponds a column height of 10.158D (that is ≈10.16D).

[46] Gros, 117. This ignoring could be understood simply as a lack of interest in the theoretical level of the architecture design. As H. Geertman says, in the Vitruvian presentation, one does not start from theory but from considerations validated by a long application into practice, the rules of the designing being presented in an artisanal type systematization (Geertman, 30).

support continued to remain an essential tool for working out the design[47]. It might be said that in comparison with the earlier ages[48], inside the procedures of activating the geometry in the composition, the dynamics of the searching mitigated its diversification sensitivity, a process announced as early as the Hellenistic Age[49].

1. At the Temple of *Apollo in Circo*[50] (Fig. 21) we encounter the grill corresponding to the Ionic *pycnostyl* applied to a Corinthian ordinance: I≈2.5D; H≈10D; the circle of the fundamental rectangle tips (H:I) correlates it to the entablature, as it is tangent to the second facies of the architrave. The height of the shaft, and, implcitly, of the capital, observes a procedure that we will encounter frequently: it adheres to the horizontal cord joining cord tips of the polygon. The *ad quadratum* succession (square/circle) (Fig. 21a') incorporates the rectangle of the height of the capital (Hcap.:I)[51] with the module; the pentagon succession (circle-pentagon-stelar pentagon), links the rectangle (Hcap.:I) to the intercolumniation and to the distance between the abacuses (through the circle circumscribed to the homothetic pentagon) (Fig. 21a"). The geometric figures involved in these successions are themselves correlation elements between the entablature and the fundamental rectangle of the rhythm; through the starting square (Fig. a') the column shaft, the capital and the entablature simultaneously become commensurable as related to the modular unit. We retain the equality between the size of the height of the capital and the side of the circumscribed octogon (L_{8c}), an aspect we shall come across in other cases as well.

2. At the *Hadrianaeum* in Rome[52] (Fig. 22) we find a rhythm close to the *pycnostyl*:I≈2.6D; H=10.05D; the size of the interaxis is settled by the side of the polygon with 12 sides (Fig 22a'). An *ad quadratum* succession – square-circle – starting in two turned round squares (determining the octogon circumscribed to the circle (Hcap.:I), settling through its side the height of the capital) simultaneously accords as related to the module the interaxis, the height of the capital and the entablature elements.

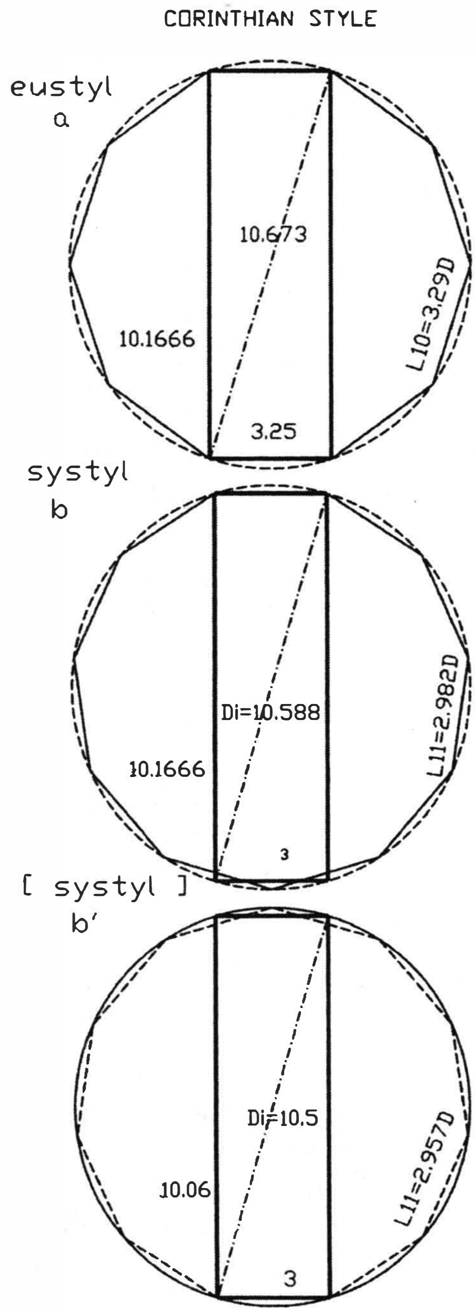

Fig. 19. Corinthian styl.
a) Pycnostyl. b-b') Dyastyl.

3. *Pantheon (Rome), exterior portico*[53] (Fig. 23); I=≈3D; H=9.567D; Di=10.02D. The Vitruvian grill of the Ionic *systyl* applied to the Corinthian style. It is no wonder that the geometric support is integrated into the decade symbols, one of the perfect numbers quoted by Vitruvius[54]: they govern the rhythm through the decagon settling by its side the interaxis of the columns: the height of the capital is equal to the side of the decagon circumscribed to the circle comprising the rectangle (Hcap.:I) (Fig. 23b); the inscribed pentagon/stelar pentagon, homothethic pentagon succession, simultaneously incorporates the interaxis, the column elements, capital, entablature, in the module. The

[47] On geometry as a geometric support of Roman architecture, in Jones, 2000.
[48] See also Mărgineanu-Cârstoiu, 2001.
[49] Ibidem.
[50] Gros, I. 162/Fig. 167.
[51] We call that the rectangle formed of the interaxis and the height of the capital.
[52] According to the layout by A. de Sangallo il Vechio, *apud* Gros, I, 205, Fig. 215; for dimensions Jones, 225.
[53] Jones. 147/ Fig. 7.24; .225.
[54] Jones, 42.

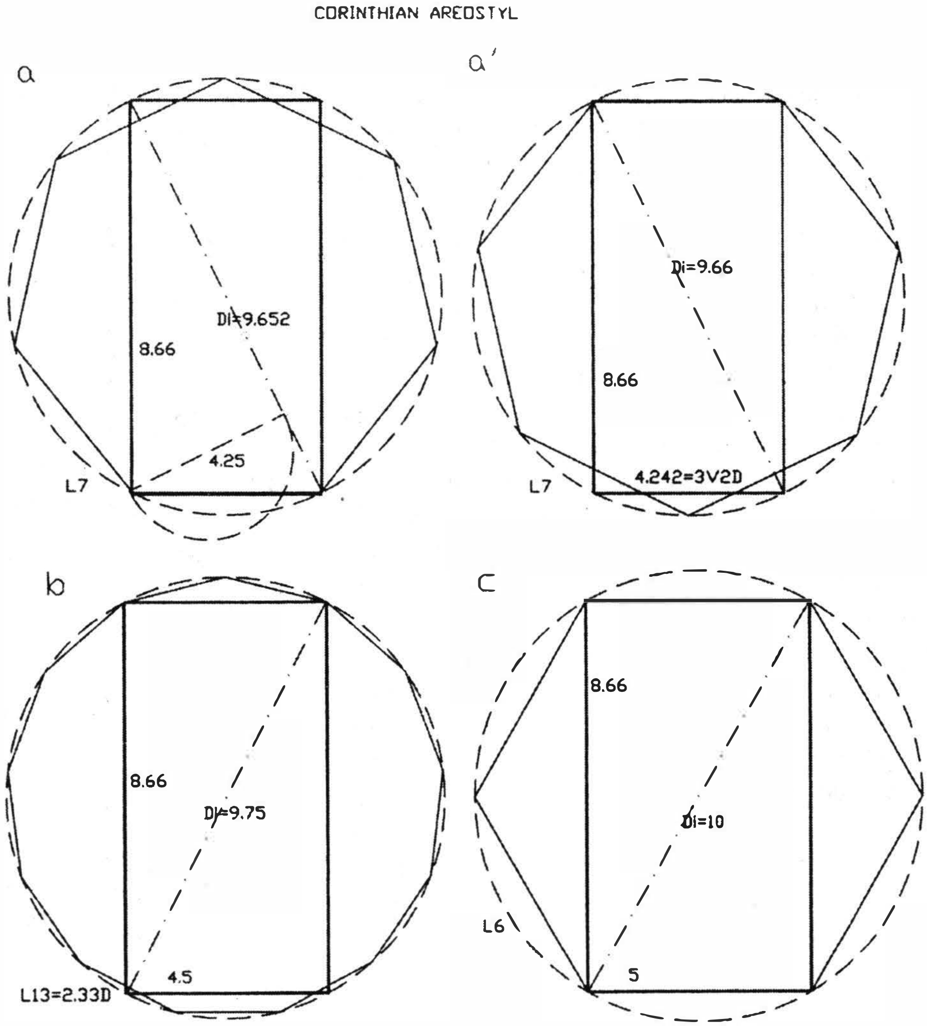

Fig. 20. Corinthian areostyl.

ad quadratum succession (circle–square–circle) incorporates the height of the capital (implicitly the height of the shaft), and intercolumniation (Fig. 23c).

4. *The Temple of Antoninus and Faustina*[55] *(Rome)* (Fig. 24): I≈2.5D; H=9.581D. As in the case encountered at Hadrianeum, the inscribed polygon with 12 sides settles the fundamental rectangle of the interaxis. Considering the rectangle (Hcapitel : I), the side of the circumscribed octogon (L_{8c}) is equal to the height of the capital. The number 6 seems to have a particular significance: a succession of hexagons inscribed in the circle (H:I) settles the end line of the column shaft, and, implictly, the height of the capital. The *ad quadratum* succession followed by the pentagon incorporates the module and the intercolumniation (Figs. 24a'-a").

5. *The Temple of Mars Ultor*[56] (Fig. 25). I≈2.5D; H=10.02D. The polygon with 13 sides settles the fundamental rectangle of the *interaxis*. The height of the column shaft is congruent with the cord subextending three polygon sides. The integration of the capital in the geometry of the rhythm – up to the last instance of the modular square – can be done by *ad quadratum* successions followed by hexagons (Figs. 25b-c). The height of the capital plays a particular role, being close to the distance between the capitals and, by the double of its size, settling the height of the entablature; the corresponding *ad quadratum* succession (Fig. 25a-a') finds its elements in the distribution of the entablature elements, like the geometric figures involved in the other successions mentioned.

[55] Jones, 42.

[56] Jones, 143, Fig. 7.18, p. 224.

[57] Jones, 143, Fig. 7.18, p. 224.

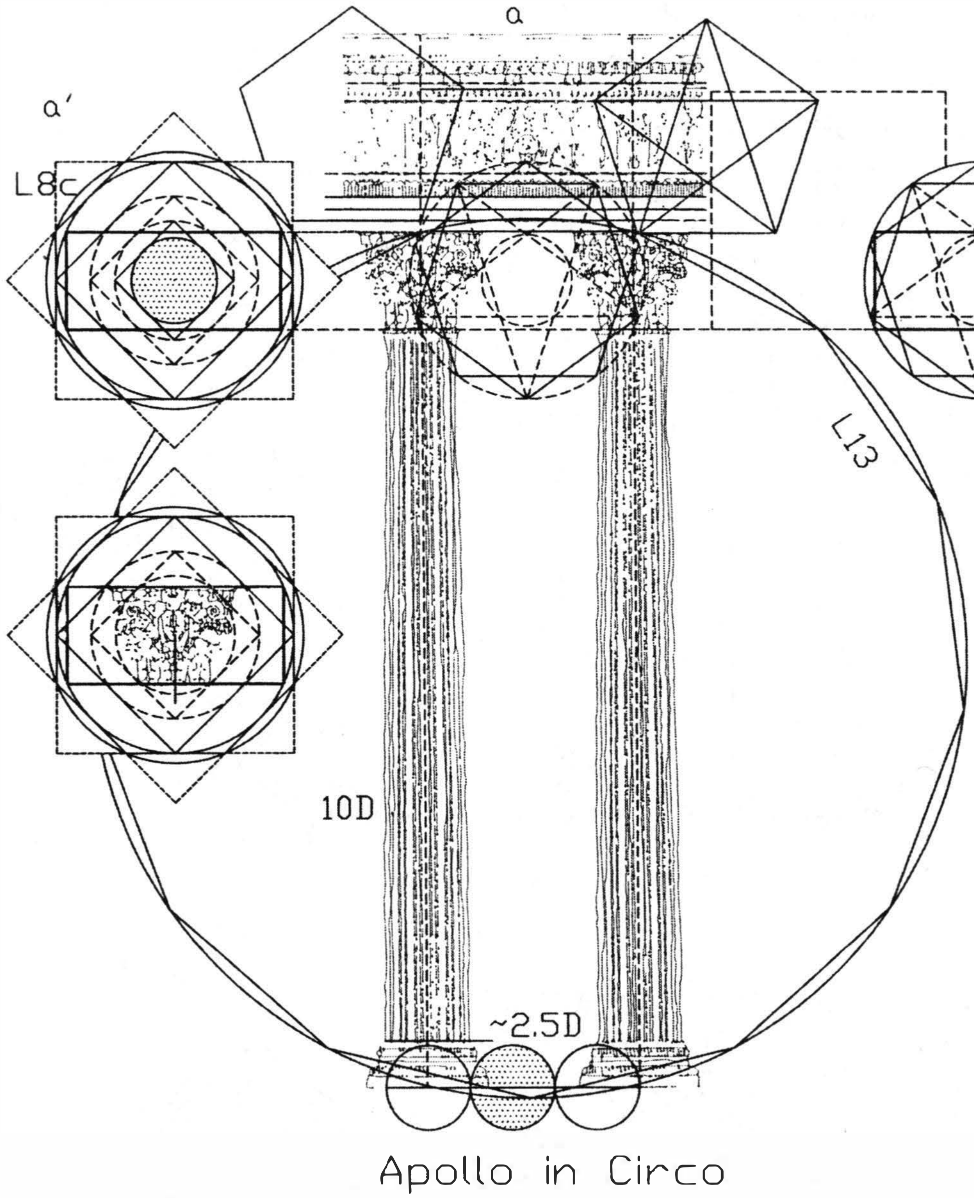

Fig. 21. Temple Apollo in Circo (geometric support).

6. *The Temple of Vespasian and Titus*[57] (Fig. 26). I≤2.33D; H=10.06D. The size of the interaxis is analogous to a side of the polygon with 14 sides circumscribed to the fundamental rectangle (H:I)[58]. The *ad quadratum* succession, directing the integration of the capital, interaxis, intercolumniation is analogous to that at Pantheon; to incorporate the module also the pentagon is involved. The squares involved direct the distribution of the entablature elements.

7. *Maison Carré (Nimes)*[59] (Figs. 27-28). The norms observed by the composition of the façade are considered to be a reflection of the official Orthodox type of Roman temple, as they were settled especially in the case of the Temple of Mars Ultor[60]. In spite of that, the mutation of the forms reflects a transformation linked to the geometry of the interaxis. The increased width of the interaxis[61] is reflected in

[58] It is possible that here might be a numerological significance: the double of 14 (the number 28) is considered by the Romans a perfect number, like the number 10; it belongs to the restrained numbers equal to the sum of their factors (Jones, 183).

[59] Jones, 66-68, Fig. 3.39-3.30; 224.

[60] Jones, 66.

[61] At the *Temple of Mars Ultor*, I≈2.5D for a column height of 10. 02D; at *Maison Carré* for the height of 10.07D the width of the interaxis is much higher, 2.5D<I<3D (Jones, 67, Figs. 3.30 and 3.28).

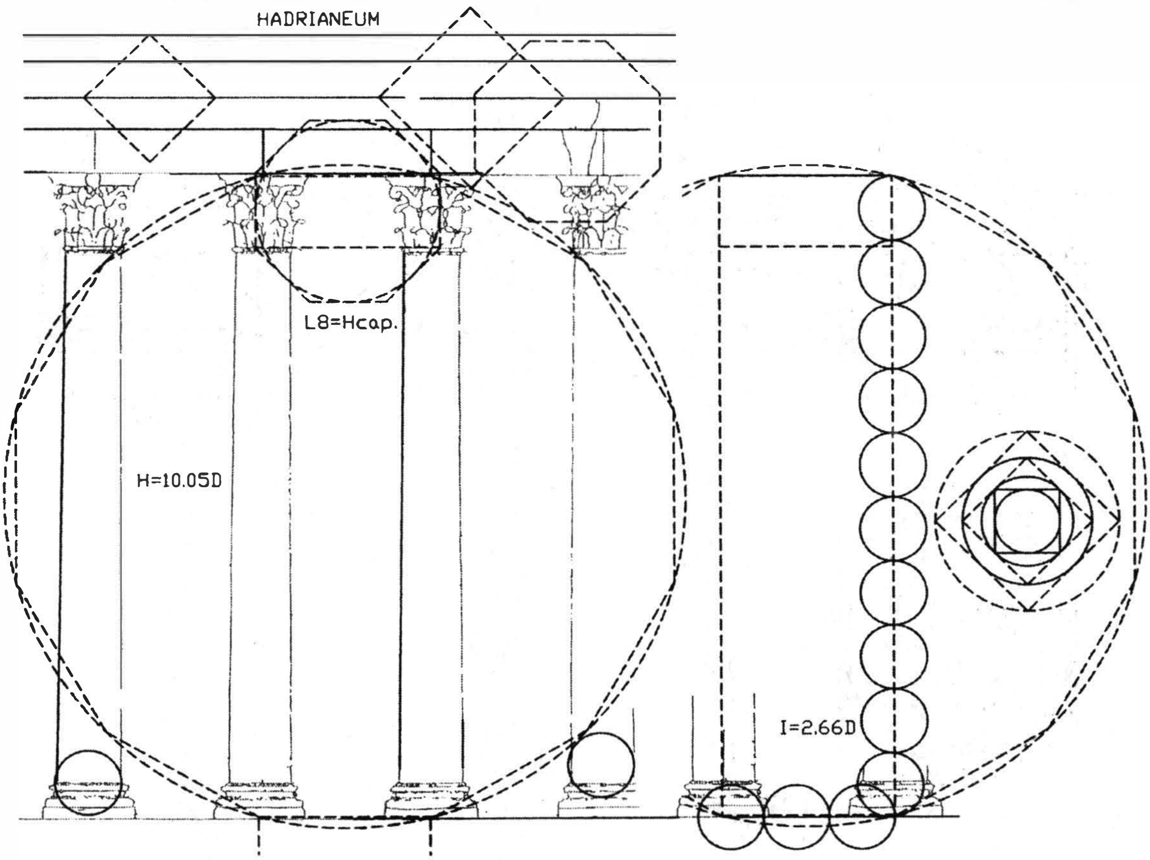

Fig. 22. Hadrianaeum (geometric support).

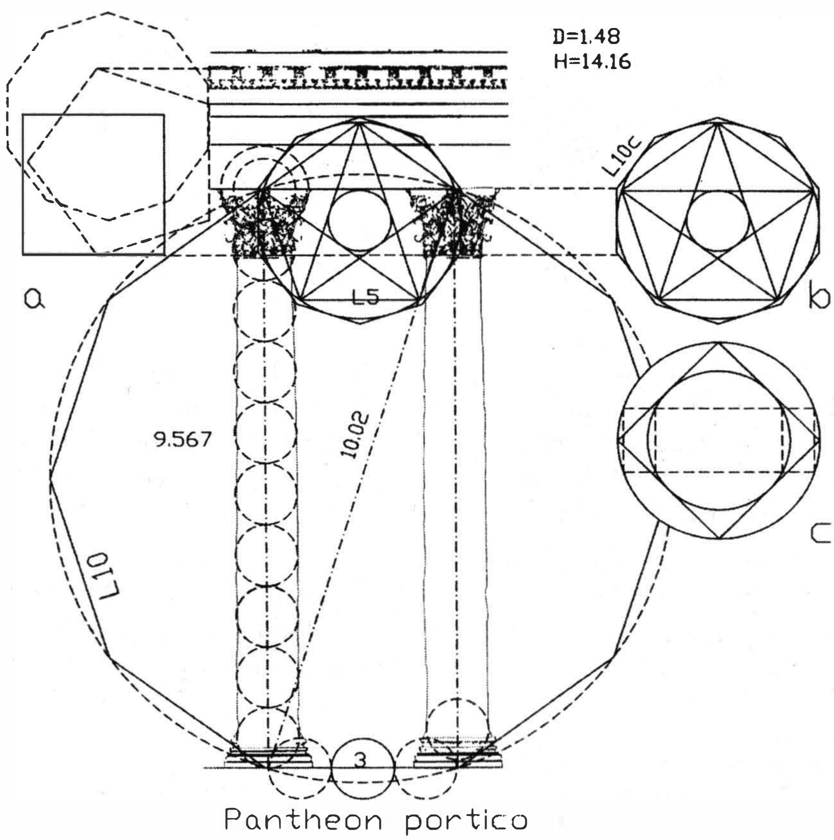

Fig. 23. Pantheon-Portico (geometric support).

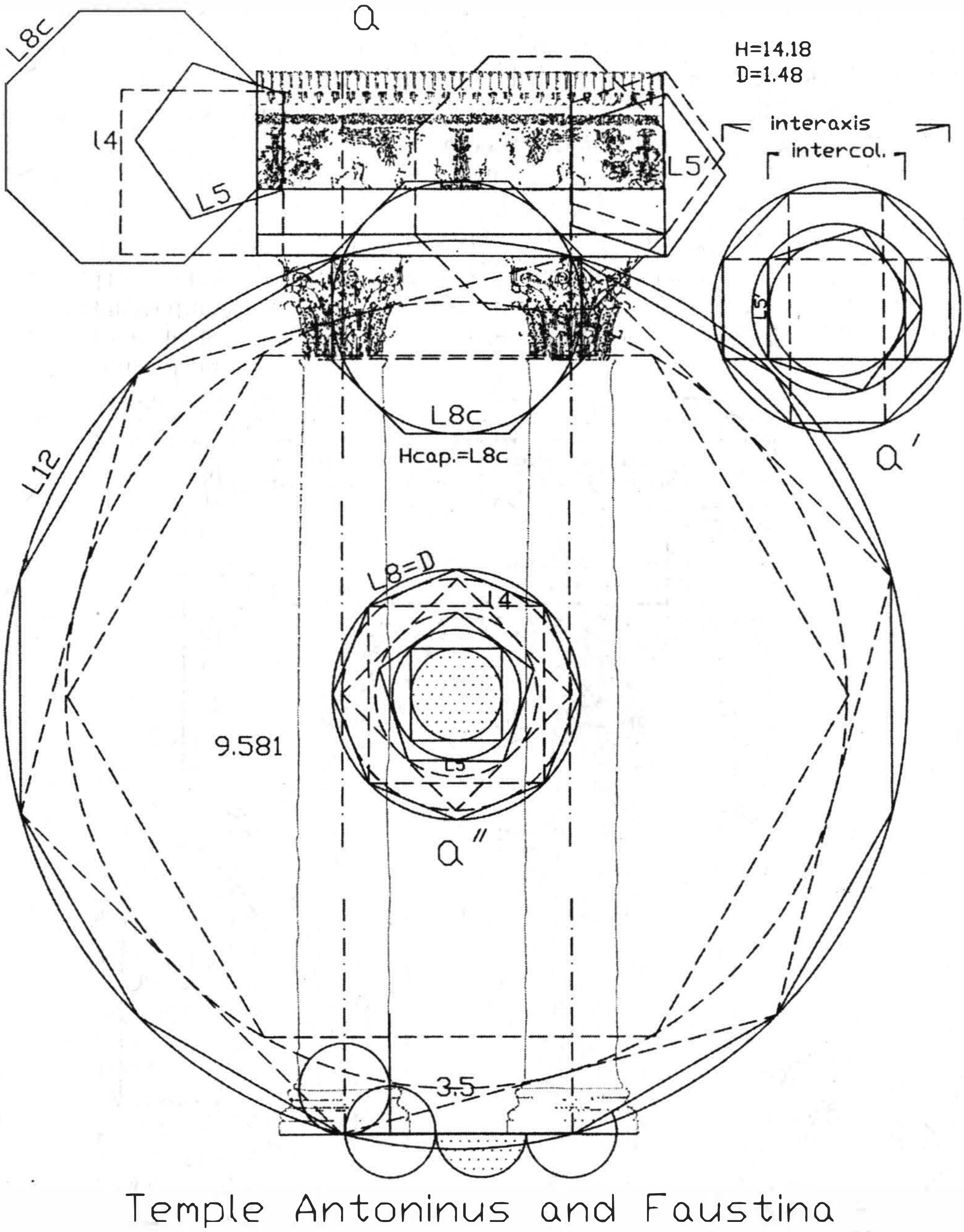

Fig. 24. Temple Antoninus and Faustina (geometric support).

the type of the polygon which characterizes the rhythm: if at Mars Ultor there was a regular polygon with 13 sides *inscribed* to the circle comprising the rectangle tips (H:I), at Maison Carré there is a polygon with 12 sides *circumscribed to the* corresponding circle. According to a principle already encountered at Mars Ultor, the Temple of Antoninus and Faustina and at Apollo in Circo, the upper line of the column shaft is settled by the horizontal cords of the generating polygons. The interdependence between the *ichnographia and the orthographia* is reflected in the squares/circles succession directing them simultaneously joining them in a common compositional body *ad quadratum* uniform succession (Figs. 28a'-a"): starting from the most important circle (no. 6) overlapping the inner rectangle of the cella, by *ad quadratum* successions the size of the dodecagon side (L12') harmonizes with the compositional circles/squares (Fig. 28 a'); this succession is linked by the dodecagon to the circle 1j, and by according with two hexagons achieves the entire correlation up to the size of the interaxis (I) (Fig. 28a''). Therefore, these successions are made up of circles and squares generating the distributions of elements in the plan and façade. In Figs. 27-28 there are a few of the most important circles/squares, through which are found also the two noticed by M.W. Jones (the circles corresponding to them are mentioned with the letter "j"). The dodecagon inscribed in the circle (1j) determines the height of the fronton (Fig. 28) and the height at the base of the capital up to the tip of the fronton; the side of the dodecagon corresponding to the interaxis settles also the height of the frontonal triangle; the square no. 4, circumscribed to the circle (H:I) – determines the depth of the pronaos to the inner wall of the cella. It is likely the position of the door should be dependent on the side of the inscribed octogon (that is by two squares turned round at 45°). The square inscribed in the circle (3j) determines the position of the lateral interaxises (noticed by Jones) and also the depth of the podium and the pronaos. The circle no. 6 (inscribed in the dodecagon inscribed in its turn in the circle 1j) is – in the plan – overlapping the inner rectangle of the cella: the square circumscribed to it determines the depth of the cella, and in the façade rests with its upper side on a line of the horizontal cornice; the length of the plan (without podium) is made up of two sides of such squares (Fig. 28 a'').

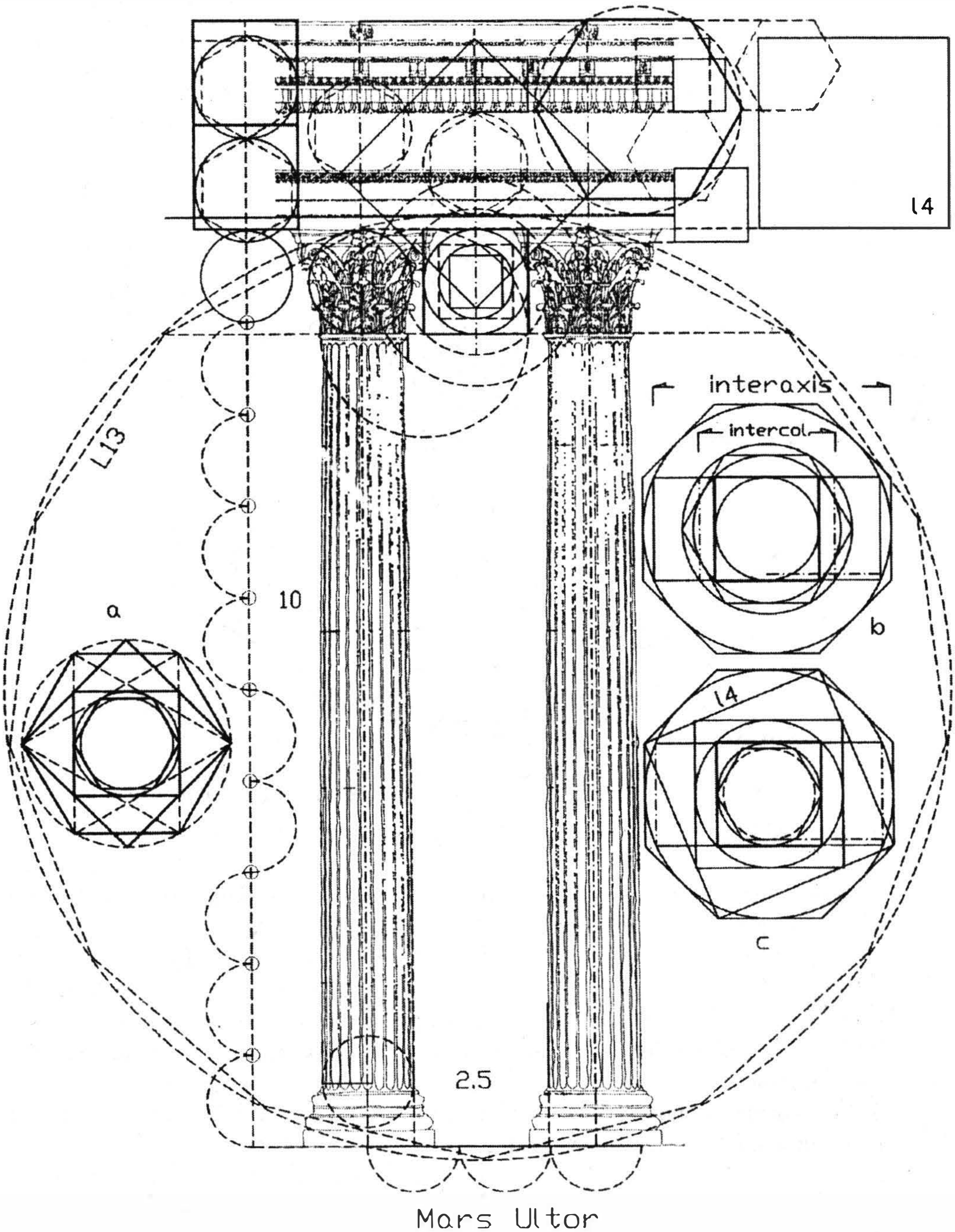

Fig. 25. Mars Ultor (geometric support)

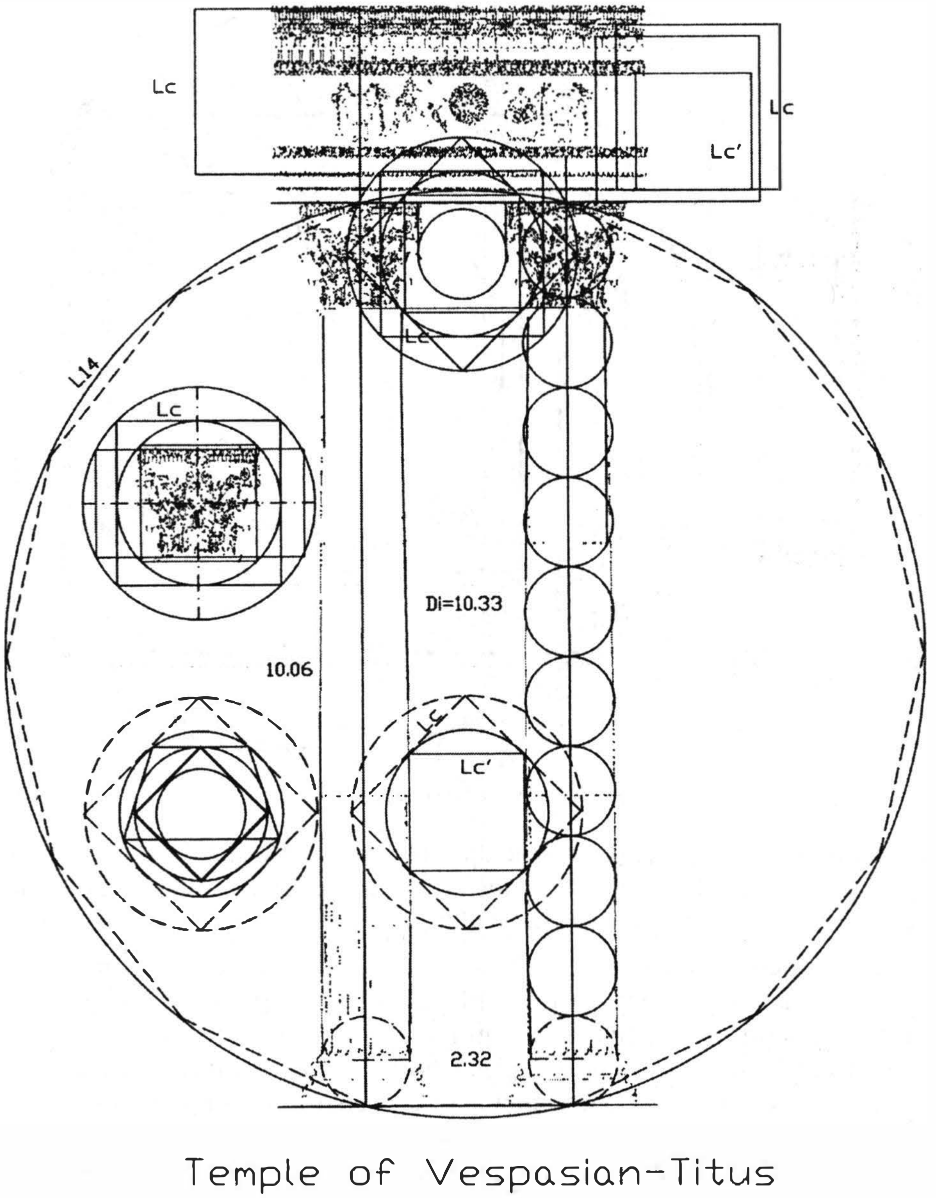

Fig. 26. Temple of Vespasian-Titus (geometric support).

8. A*d quadratum* successions followed by other geometric figures can be found also in the Ionic style, as can be seen at the *Temple of Portunus*[62] (Figs. 29-30) in Forum Boarium (1st century BC). The rectangle of the interaxis and the column (H:I) is governed by the number 10, through the circumscribed decagon (circle no.3). Again we encounter the procedure of correlating the upper line of the shaft with tips of the corresponding polygon (decagon). The squares noticed by M.W. Jones[63] (inscribed in the circles nos.1-2) are correlated through the decagon[64] – directing also the slopes of the fronton – and integrates by the circle no. 1 also the tip of the frontonal triangle. Another fundamental square of the façade is the one equal to the height of the columns and to the distance between the marginal columns (inscribed in the circle no. 5 and circumscribed to the circle no. 4): In Figs. 29a'-b'-b") are described the polygon successions correlating all the squares and circles corresponding to them as related to one other. The polygon-circle successions describe the numerical series 4-5-6-10, as the *ad quadratum* succession is not dominating. The same geometric figures harmonize the *ichnographia* with the *orthographia* (Fig. 30). For instance: the circle 1 overlaps the cella plan (Fig. 30); the square inscribed sprouts the depth of the cella

[62] Jones.65, Figs. 3.27 and 3.26.
[63] Jones, 65, Fig. 3.27.
[64] Circumscribed to the circle no. 2 is at the same time inscribed in the circle no. 1.

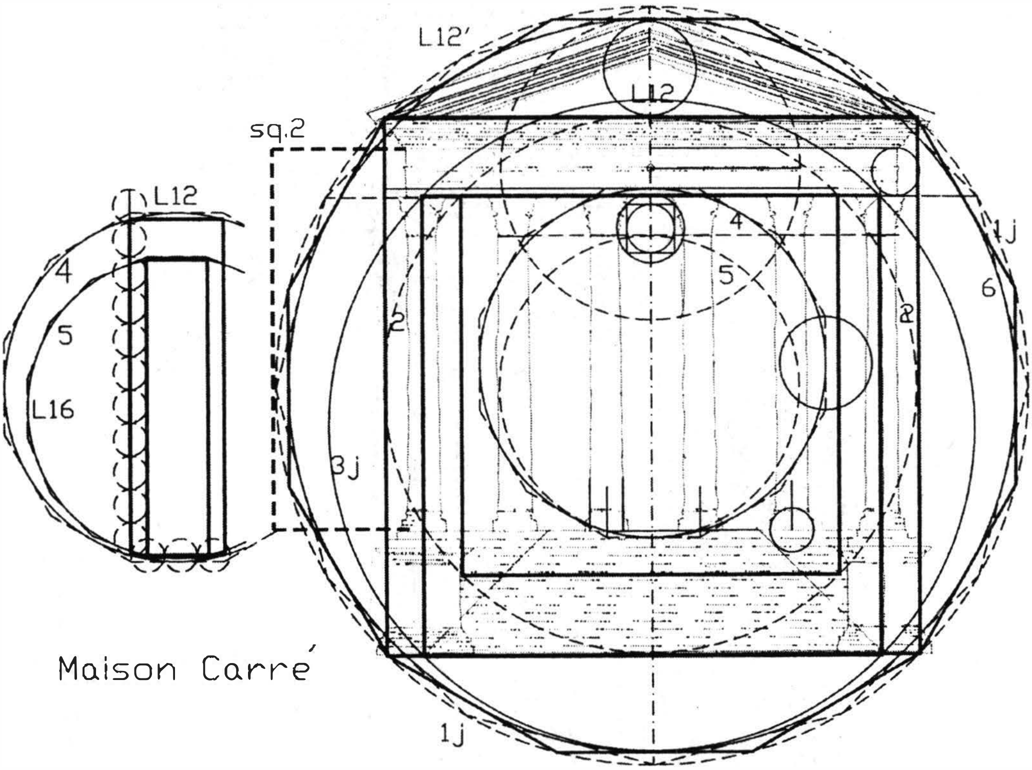

Fig. 27. Maison Carré (geometric support).

between the inner wall from the pronaos and the opposite outer wall; the decagon inscribed governs the lateral walls of the cella (on the inside) and the quadrilateral of the cult statue socle; the decagon circumscribed to the circle (3) correlates also the width of the door, etc. (Fig. 30).

The geometric compositions noticed in the studied cases are not very diversified, as they are all based on essentially common procedures: the distribution of the rhythm elements (the rectangle H:I) is directed by polygons inscribed or circumscribed to the overlapping circle, and the incorporation of the intercolumniation, capital and entablature elements, of the *ichnographia with the orthographia*, is directed by polygon successions. In the chronological perspective of the examples discussed, the *ad quadratum* succession does not play a prevailing role in the early Roman geometric matrix (the Temple of Portunus/1st century BC), but it becomes dominating later, followed or not by successive series made up of other types of polygons. The handling peculiarities of these successions can be varied. The common denominator of all these geometries is the fact that by them the correlation of all the parts with the modular unit is achieved. One may say that they condense those "privileged mathematical relations"[65] ensuring the harmony of the parts with the whole, but also of all of them with the starting unit (in the studied case the mod), thus participating in the realization of the phenomenon of commensurability. They are part of the abstract theoretical part, of the design, that renders possible ultimately the *symmetria*[67]. It is commonplace to say that the geometric procedures frequently entail dimensions arithmetically expressible by irrational numbers. In order to make them conveyable in practice it is necessary to perform adjustments generating the differences between the "ideal' geometry of the design and the real architecture, adjusted in order to be compatible with the unit of measure[67], namely for it to be conveyable for the execution of the design.

[65] Frey, 168.

[66] "Symmetria was clearly related to the Latin term proportio, but it evidently meant more than proportion alone" (Jones, 41). Vitruvius defined symmetria 'as a proper agreement between the members of the work itself, and relation of the parts and the whole general scheme, in accordance with a certain part selected as standard' (Jones, 41); *symmetria est ex ipsius operis membris conueniens consensus ex partibus separatis ad universae figurae speciem ratae partis responsus* (*apud* Gros, XI-XIII).

[67] To these operations refers prevailingly M.W Jones when he describes the realization of the interim *symmetria* and *eurythmia* by mathematical and formal conflicts resolved (Jones, 59-61).

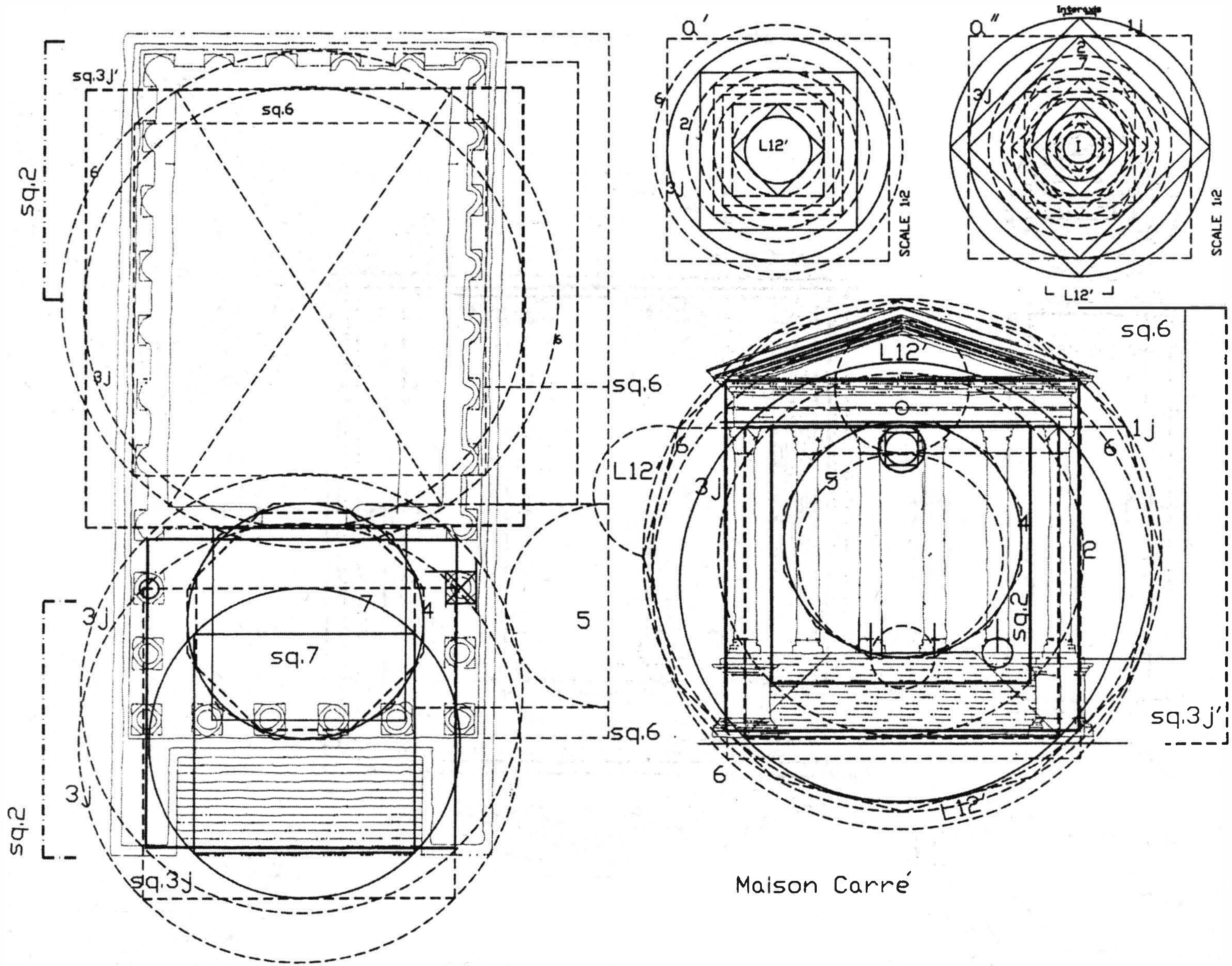

Fig. 28. Maison Carré (geometric support): plan and elevation.

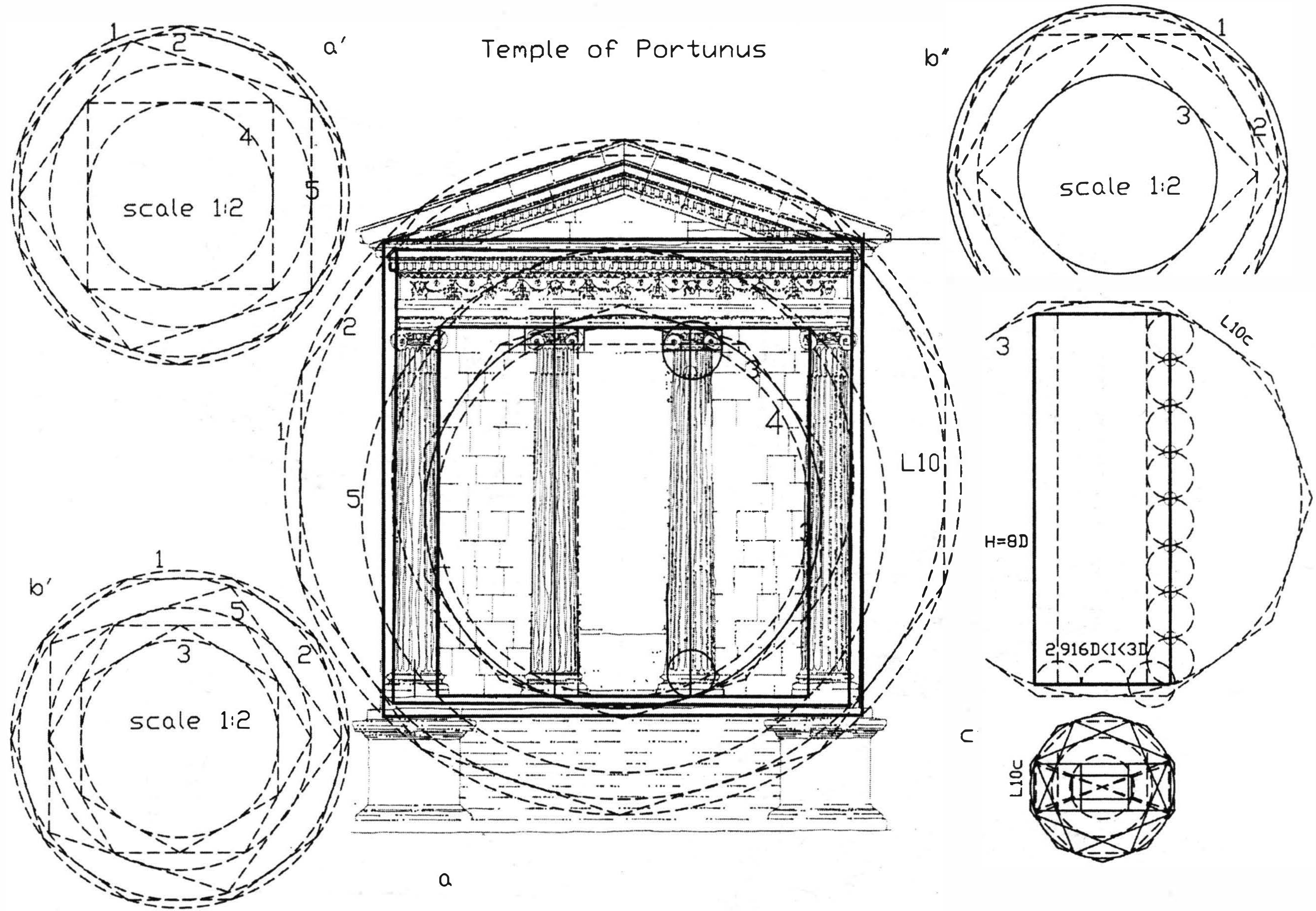

Fig. 29. Temple of Portunus (geometric support).

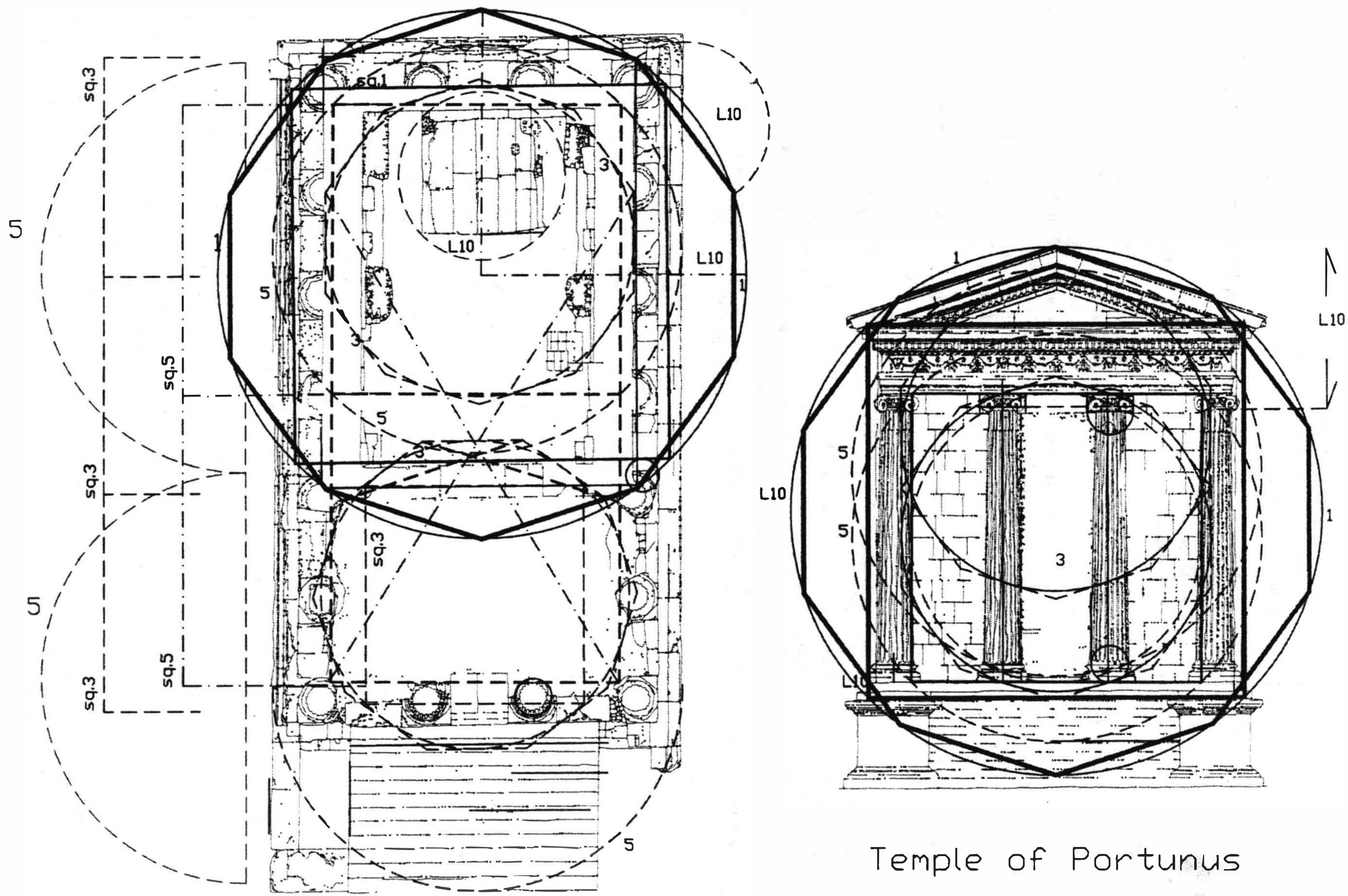

Fig. 30. Temple of Portunus (geometric support): plan and elevation.

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