STUDII

MUSIC AND MATHEMATICS (I)

The idea of an international symposium on music and mathematics in Bucharest arose in February 1992 in Seattle, during a conversation with John Rahn. We were not able to organize it in 1993, but it took place on May 29th and 30th, 1994, during the International Week of New Music.

The organizing Committee included John Clough from the New York State University at Buffalo, together with John Rahn and myself.

Only someone acquainted with the economic and organizational difficulties existing at this moment in Romania can understand the effort to make the symposium take place in appropriate conditions, to make it take place at all, considering that it did not have an official status and that it was not included in the scope of a sponsoring state institution with a scientific profile.

The twelve presentations lasted two days, following a certain logical course. There had not been a pre-established thematic range; still, the issues got structured almost by themselves. The first day they focused on classical topics connected with diatony, though treated with contemporary means; there is a remarkable shift of interest in this direction in musical composition and musicology; it reflects the stylistic moves which happened in music in the past two decades. New mathematical models are being tested now in this field. One realizes now that diatony, which seemed to represent simplicity in music, is actually a complex phenomenon.

If the first day dealt mainly with numerical aspects of mathematics involved in music, on the second day the non- numerical ones predominated; the evolution of computer science, the semiotic experience and especially the progress of musicology itself presently allow treating some aspects in music with non- numerical mathematics.

We publish the presentations following their sequence at the symposium.

Anatol Vieru

The Musical Signification of Multiplication by 7. Diatonicity and Chromaticity.

Anatol Vieru
Bucharest Academy of Music

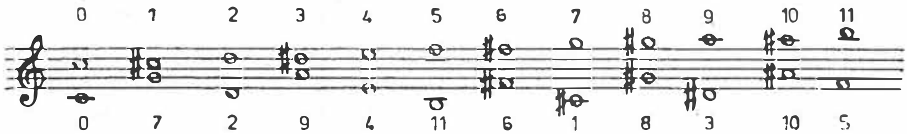
For a long time but particularly in the '50s, the avant-garde believed that music advances in a strict way, step by step, from simple to more and more complicated stages. It was considered that one of the main attributes of complexity is the progressive chromatisation of music; after chromatisation, hyperchromatisation (microtony) was meant, filling in the whole sonorous space.

It is true that in the latest 20 years one can observe a certain change of musical taste; more and more islands of diatonicity appeared in the new music. Nowadays the idea of complexity is not necessarily related to the notion of chromaticity.

There is a theoretical explanation of the relation between diatonicity and chromaticity: it is linked to multiplication by 7 (and 5).

1. In the multiplication table of the elements module 12, only the multiplications by 1, 11, 5 and 7 cross all the elements of the algebraic group.

Multiplying by 7 the elements of the group module 12 we are in the presence of the row of the fifths:

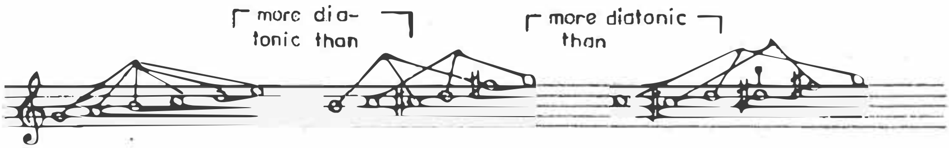


2. We can define as a *perfect diatonic mode* any mode having only one connection of perfect fifths, e.g.

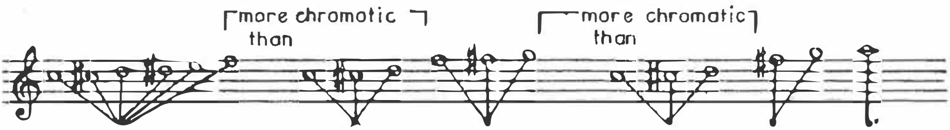
and we can define as a *perfect chromatic mode* any mode having only one connection of halftones, e.g.

3. Any mode can be decomposed in connections of perfect fifths and in connections of halftones.

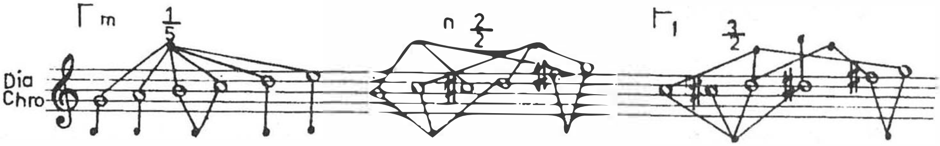
If a mode has more connections of fifths, it is less diatonic

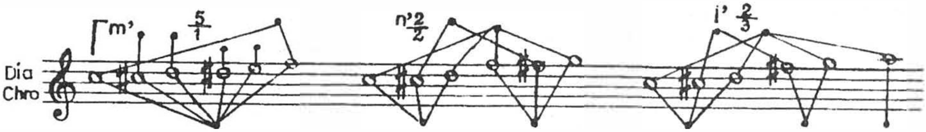


If a mode has more connections of halftones, it is less chromatic



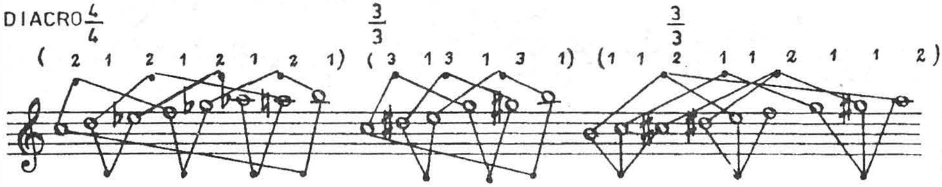
4. Any mode has its own degree of diatonicity and of chromaticity, that is its knots of perfect fifths and knots of halftones





We can call DIACRO the measure (the comparison) of the diatonicity and chromaticity of different modes.

Messiaen's modes "aux transpositions limitées" are all balanced as concerns DIACRO; any of them has the same number of perfect fifths and halftones connections:



5. Multiplication by 7 (or by 5 which is its symmetrical element in the group module 12) is just the monitor of diatonicity and chromaticity. Multiplication by 7 reverses the DIACRO measure of a mode.

(We can perform the multiplication of each element of any mode by substituting it with its neighbour in the other row)

So:

m x 7 = m' and DIACRO $\frac{1}{5}$ becomes $\frac{5}{1}$

n x 7 = n' and DIACRO $\frac{2}{2}$ becomes $\frac{2}{2}$

1 x 7 = I' and DIACRO $\frac{3}{2}$ becomes $\frac{2}{3}$

We can conclude: diatonicity and chromaticity are not a question of simplicity and complexity as it was thought before. That is rather a question of unity of contraries in the system module 12 (the classic European music); it is the contrast between light and darkness.

An example denying the permanent evolution music from diatonicity to more and more chromaticity is the apparition of "Parsifal" after "Tristan" in Wagner's work; "Parsifal" seems to be an astonishing return to diatony; but this creative gesture is full of significance: Wagner is beyond the understanding of the evolution of music through permanent sharpening of the chromatic world. "Parsifal" closes the strata of musical thinking from the oldest pentatony and prepentatony (see the motive of the bells), to the most chromatic ones: the beauty of the music lies just in the articulation of these different strata. Berg followed this way in his "Wozzeck" and "Violin Concerto".

A wonderful example of perfect diatonicism is the "Promenade" from the "Pictures From An Exhibition". Bartók used with great intensity the contrast between perfect diatonic and perfect chromatic modes. The second part of his Concerto No 2 for piano and orchestra is a brilliant example; the first section with its superposed perfect fifths make a striking contrast with the median section Presto which is built only with semitone connections.

The amateurs of archetypes could perhaps consider the perfect fifth and the halftone as musical archetypes. The perfect fifth is the most typical interval for the natural resonance and it is essentially instrumental, whereas the semitone is essentially vocal: the human sigh, breathing. (As a matter of fact: the hexatonal mode, Deabussy's mode in whole tones, Messiaen's mode No 1 is neither diatonic, nor chromatic; it is, so to say, "asexual" from this point of view: it includes no perfect fifth and no halftone.)

This theoretical understanding of diatonicity and chromaticity as "monitorized" by the multiplication by 7, allowed me to introduce a new criterion of classification in my catalog of modes; to the grouping of modes together with their complementaries we can add now a new grouping: every modal structure will be together with its DIACRO contrary, the result of its multiplication by 7.

Eytan Agmon

Diatonicism and Farey Series

Consider the following series of fractions:

$$\frac{0}{1}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{1}{1}.$$

In this series, known in number theory as the *Farey series of order 5* (F_5), we have in ascending order every irreducible fraction between 0 and 1 (inclusive) of which the denominator does not exceed 5. F_5, of course, is a special case. For any integer n>0 there exists a Farey series F_n, namely, the ascending series of irreducible fractions between 0 and 1 with a denominator not exceeding n. Farey series have a number of interesting properties. A particularly well-known property is the following: if $\frac{c}{d}$ immediately follows $\frac{a}{b}$ in some Farey series F_n, then the relation ad - bc =-1 holds (for example, in F_5 we have $0 \cdot 5 - 1 \cdot 1 = -1$, $1 \cdot 4 - 5 \cdot 1 = -1$, $1 \cdot 3 - 4 \cdot 1 = -1$, $1 \cdot 5 - 3 \cdot 2 = -1$, $2 \cdot 2 - 5 \cdot 1 = -1$, and so forth); conversely, if a, b, c, and d are integers satisfying the relation ad-bc=-1, then $\frac{c}{d}$ immediately follows $\frac{a}{b}$ in a Farey series whose order is the larger of the two denominators.

One area of musical research where Farey series may be encountered is diatonic intonation. A problem that often arises in the theory of diatonic intonation is approximating rationally the irrational pure intervals, for example, the pure "fifth" $\log_2 1.5$. Approximating irrationals by rationals is a classic problem in number theory. As is well known, there is no limit to how closely one may approximate rationally a given irrational μ. In particular, if μ is between 0 and 1, one can write an infinite series of fractions beginning with 1/2 where each successive fraction is a more accurate approximation of μ. Since the denominator can only increase from one such fraction to the next, any two successive fractions in the series, say $\frac{a}{b}$ and $\frac{c}{d}$, are by definition adjacent terms in F_d. The following is an example of such a series, where $\mu=\log_2 1.5$ (the pure "fifth"); the reader may easily verify that the relation ad-bc=±1 is satisfied for any two adjacent terms:

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \frac{7}{12}, \frac{17}{29}, \frac{24}{41}, \ldots$$

The appearance of Farey series in connection with diatonic intonation is far from surprising, given the nature of the problem involved (i.e., approximating irrationals by rationals). In the present article, however, quite a different connection between diatonicism and Farey series shall be considered. As this connection concerns a level of diatonic reality other than the familiar level of log frequency ratios, some preliminary discussion is necessary.

It is readily demonstrated that log frequency ratios do not suffice in order to capture the intuitive sense of the notion "diatonic interval" (e.g., "minor third,"